CURRENCY DEPRECIATION, INCOME, AND THE BALANCE OF TRADE

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I. INTRODUCTION

Existing discussions of the effects of depreciation on the balance of trade are framed in terms of a model in which incomes do not appear explicitly in the demand equations for imports and are thus implicitly assumed to be constant. Some writers have contended that the conditions for improvement in the balance of trade, obtained from such a model, are directly applicable to "Keynesian" models in which national incomes are variable. The purpose of the present paper is to investigate the validity of this contention; to make more precise the types of models for which various statements about the effects of depreciation on the balance of trade are valid; and to specify, for cases in which incomes are variable, the conditions under which income will increase as a result of depreciation.

In the discussion which follows, it will be assumed that the exchange rate is held fixed by means of national or international stabilization funds and is subject to change only through an alteration of the (national or international) policy governing the use of these funds. This assumption is in clear conformity with current institutional arrangements, and it permits us to consider a variation in the exchange rate as an independent cause leading to changes in the other variables of the system (as opposed to the alternative assumption of free exchanges, under which movements of the exchange rate are always induced by other causes, which also may have independent effects on the other variables of the system).

Furthermore, I shall assume that there are only two countries, each of which produces only one good. Income will be defined as the quantity of the national good currently produced (=production). For each country the quantity demanded of both its home-produced and the foreign-produced good will be taken to depend on current income (as defined above) and upon the ratio of the prices of the two goods (the terms of trade). I shall assume that all relations treated in this paper are linear. For convenience,

3 The conditions of supply of each national good will vary among the models considered. Implicit in the assumed conditions of supply in the models treated are assumptions regarding the monetary policy, the production function, and the supply function for labor in each country. As each model is discussed, a set of such implicit conditions will be specified which is sufficient to generate the assumed supply function of the national good. Though sufficient, the specified conditions may not be necessary, and the reader may find alternative sets of assumptions which would equally justify the models presented.

4 The assumption of linearity is justified by the fact that most functional relations can be approximated by linear relations when the range of contemplated variation is relatively small. The relevant linear approximation in all cases should be that one which connects the initial and ultimate equilibrium points.
commodity units will be so chosen that the price of each good in terms of the currency of its country of production is initially equal to unity, and units of currency will similarly be chosen to make the initial exchange rate equal to 1.

II. A MODERN "FULL-EMPLOYMENT" MODEL

I shall first consider a case in which national income is held constant by government policy. This case conforms closely to an "ideal" policy, in that it presupposes that each government succeeds in stabilizing both the "full-employment" level of national output and the price of the good produced in its country. Such a policy could be pursued by the creation of a commodity standard in the national good in each country. Even if the supply curve of labor is a function of the terms of trade, so long as the desired output is attainable for all values of the exchange rate considered, the maintenance of demand at the desired output and of prices at fixed levels will lead to a wage adjustment sufficient to produce the desired output.

Under these conditions the demand for imports in each country can be written as a function of the exchange rate alone, thus:

\[ x_1 = a_1 + \frac{b_1}{k} \text{ (demand of country 2 for the exports of country 1),} \]  
\[ x_2 = a_2 + \frac{b_2}{k} \text{ (demand of country 1 for the exports of country 2),} \]

where \( x_1 \) is exports of country 1; \( x_2 \) is exports of country 2; and \( k \) is the exchange rate (defined as that number by which the price of a good in country 1 must be multiplied to yield its price in country 2, with units of currency so chosen that, at the point of depreciation, \( k = 1 \)). If we now let \( p_i \) equal the stabilized price of the product of country 1 (\( i = 1 \)), and \( p_2 \) equal the stabilized price of the product of country 2 (\( i = 1 \)), we can write the balance of trade of country 1 in terms of the currency of country 2 as

\[ B_1 = x_1 p_1 k - x_2 p_2 . \]  

(3)

The change in this balance of trade as a result of depreciation is then given by

\[ \frac{dB_1}{dk} = x_1 + \frac{dx_1}{dk} - \frac{dx_2}{dk} = x_1 + b_1 + b_1 . \]  

(4)

We can now readily translate the price slopes (\( b \)) into price elasticities of demand (\( \eta \)), defined as < 0. Thus:

\[ \eta_1 = \frac{b_1}{x_2} \text{ Price elasticity of demand for imports of country 1}, \]
\[ \eta_2 = \frac{b_2}{x_1} \text{ Price elasticity of demand for imports of country 2}. \]

Then the change in \( B_1 \) resulting from depreciation can be written:

\[ \frac{dB_1}{dk} = x_1 + x_1 \eta_2 + x_2 \eta_1; \]  

(5)

and, if we let \( s \) equal the initial ratio of import expenditures to export receipts in country 1 = \( (x_2 p_2)/(x_1 p_1 k) = x_2/x_1 \), we have

\[ \frac{dB_1}{dk} = x_1 (1 + \eta_2 + s \eta_1). \]  

(6)

On our definition of the exchange rate, depreciation by country 1 implies a decrease in \( k \), so that, for improvement of the balance of trade of country 1 to result \( dB_1/dk \) must be negative. Thus our condition for improvement in the balance of trade in terms of foreign currency is that \( |\eta_2 + s \eta_1| > 1 \). If trade were initially balanced, the critical value of the (ab-

\^1\ Since both prices are, in this instance, assumed constant at unity, eq. (3) could alternatively be written \( B_1 = x_1 k - x_2 \).
solute) sum of the two elasticities of demand \(|\eta_i + \eta_j|\) would be 1, while, if country \(i\) had an initial deficit \((s > 1)\), this critical value would be somewhat less than 1.

These results are the same as those obtained by Marshall and Joan Robinson.\(^7\)

It is interesting to note, however, that these "traditional" conditions for improvement in the balance of trade follow

\[^4\] Only when trade is initially balanced is the critical value the same, regardless of whether the balance is written in terms of foreign currency or of home currency. For the case of an initial deficit, for example, the critical value is somewhat greater than 1 for improvement in the balance in terms of home currency, but less than 1 for improvement in terms of foreign currency (cf. A. O. Hirschman, "Devaluation and the Trade Balance: A Note," Review of Economics and Statistics, XXXI [February, 1949], 50–53). The decision to consider the balance of trade in terms of foreign currency in this paper was not arbitrary, since it was felt that a deficiency in the supply of foreign currency is one of the primary incentives to depreciation. The other major incentive to depreciation is considered to be the desire to increase home income. While past discussions have used the change in the balance of trade in terms of foreign currency as an indicator of how home income will change as a result of depreciation, the change in income is given explicitly in our models with variable incomes. Thus, for balance-of-trade problems, we consider \(B_i\) (in terms of foreign currency) to be the relevant object of policy, while, for problems of income and employment, we consider income, and not the balance of trade in terms of domestic currency, as the relevant object of policy.

\[^7\] Cf. Alfred Marshall, *Money, Credit, and Commerce* (London: Macmillan & Co., Ltd., 1933), pp. 355–56; and Joan Robinson, *Essays in the Theory of Employment* (2d ed.; Oxford: Basil Blackwell, 1947), pp. 142–43. Mrs. Robinson's more complicated expression for the effect of depreciation arises because she permits elasticities of supply of exports to be other than infinite. I shall discuss such a case below, where I permit the price of each national good to be a function of production. If each nation is assumed to produce more than one good, the conditions envisaged by Robinson might be still more closely approximated, and models which show the validity of Robinson's results when national incomes are assumed to be fixed by government policy can easily be constructed. However, when incomes are permitted to vary, the introduction of more than one commodity produced in each country leads to exceedingly complex models.

so directly from the assumption that production is stabilized by Keynesian means. We can, alternatively, obtain a quasi-stability in national output by the use of a "classical model."

### III. The Classical Model

In a simple classical model for one country, output is determined by the equilibrium in the labor market, in which both supply of and demand for labor are functions of the real wage. In a two-country model, however, such precise determination cannot result, since not only the price of the home-produced good but also the price of the foreign good enter into workers' calculations of their "real wage." However, by considering the locus of equilibrium outputs for varying terms of trade, we can obtain output in each country as a function of the terms of trade. The terms of trade can then be looked upon as determining the level of national income in each country, and these levels of national income and the terms of trade as determining the total demand for the product of each country. Varying the terms of trade can succeed in equating total demand with total supply of one of the two national goods, but not both (except accidentally), since one price cannot generally be expected to equilibrate two markets.

The classical system avoided this overdeterminacy by the assumption that in both countries all income is spent (Say's law). If this assumption holds true at all times (both \(ex\ ante\) and \(ex\ post\)) and in both countries, there will be some exchange rate which equilibrates the markets for both nations' goods; for now the demand functions for home goods and for imports in each country are related in a specific way, and we can eliminate the demand functions for, say, the product of country \(i\) by using Say's law and can
find an exchange rate which will satisfy the equilibrium condition in the market for the product of country 2. Now, because Say’s law implies not only that all income be spent but also that there is equilibrium in the balance of trade (see below), this exchange rate will also equilibrate the market for the product of country 1.

Thus the classical two-country system can be regarded as determinate, once Say’s law is introduced. But the introduction of Say’s law implies equilibrium in the balance of trade; for the identity of income received with income spent implies equality of receipts from exports with payments for imports. Thus \( x_p\cdot k = x_p\cdot k \), or \( w = x_s/x_i \). Clearly, if trade is to be balanced in both the equilibrium position existing before and in that existing after depreciation, depreciation cannot improve the balance of trade. It will, in fact, lead only to a countervailing change in the ratio of the prices of the two goods, so as to leave the equilibrium terms of trade unchanged.

The fact that the classical system, if stable, implies that depreciation cannot affect the balance of trade does not mean that the Marshall-Robinson conditions have no applicability at all, for they remain as the conditions of stability in the system. This can be shown most easily by demonstrating that they characterize the case of “neutral” equilibrium (coincidence, over a certain range, of the reciprocal demand curves of the two countries). When this is true, trade will remain balanced even if the terms of trade change within the relevant range. For this range, then, \( x_iw = x_i \), and

\[
\frac{dx_i}{dw} \cdot \frac{w}{x_i} + 1 = \frac{-dx_i}{d(1/w)} \cdot \frac{(1/w)}{x_i}.
\]

This last identity can be written, in the notation used above, \( \eta_i + 1 = -\eta_i \).

These elasticities are the same as those used elsewhere in this paper when the supply of labor is affected only by the real wage in terms of the home-produced good in each country. They differ from those used above, however, when the supply of labor is affected by the terms of trade as well. Then they must be viewed as elasticities of demand, which take account of the indirect effect, through labor supply and national production, of changes in the terms of trade upon the demand for imports.

IV. THE KEYNESIAN MODEL

Although the Marshall-Robinson conditions apply as conditions of stability in the classical model, the necessity that Say’s law hold for this model to be consistent considerably reduces its short-run applicability to the real world. A Keynesian model can be constructed, however, which does not lead to inconsistency in the absence of Say’s law. Such a case would arise if the supply of labor in each country were infinitely elastic at the prevailing real wage rate (in terms of the home-produced good) and if each national good were produced under competitive conditions with constant returns to labor input. Under these conditions we can, without loss of generality, con-
sider the price of each national good to be the unit of currency in its country of production and to be stable. Production in each country would be determined by the equilibrium of total demand for its national good with its infinitely elastic supply, and, given the exchange rate, total demand for each good would be determined by the levels of production in both countries. The possibility of underemployment equilibrium, and of continued disequilibrium in the balance of trade, both characteristic of the "Keynesian" system, clearly exists here.

Such a case has been treated by Joan Robinson and A. J. Brown. They have reached the conclusion that the introduction of variable production leads to no change in the critical value of the sum of the elasticities of demand for imports but acts only to lower the absolute magnitude of the effect of depreciation on the balance of trade.\(^9\) I propose to show that this conclusion does not follow from assumptions which I believe will be generally acceptable.

Consider any case in which, if production were stable, the elasticities of demand for imports would be precisely such as to leave the balance of trade unchanged as a result of depreciation. If, with such an initial situation, production were permitted to vary, the immediate effect of depreciation would be an increase in production in the depreciating country and a diminution of production abroad, both resulting from the substitution of the relatively cheaper product of the depreciating country for the relatively more expensive product of the nondepreciating country. These changes in production would, in turn, lead to an induced increase in demand for imports by the depreciating country and to an induced decrease in the demand for imports of the rest of the world. There would therefore ensue an induced deterioration of the trade balance of the depreciating country. Since in our example the initial effect of depreciation on the trade balance is, by hypothesis, zero, the "normal" total effect (initial plus induced) must be to worsen the balance.\(^9\) Therefore, in order for depreciation to have no net effect on the trade balance when production is variable, the sum of the elasticities of demand for imports of the two countries must normally be greater in absolute value than when production is fixed.

The "abnormal" cases in which our generalization would not hold follow readily from this example. Thus, if the induced effects led to a return of production in both countries to its initial levels, the critical value of the sum of the elasticities of demand for imports would be the same for the case of variable output as for the case of fixed output. But this would imply that production is not effectively variable as between "equilibrium" positions. Likewise, if the total effect of depreciation were to decrease production in the depreciating country and/or to increase production abroad, it might be true that the presence of variable production would lower rather than raise the critical value in question. But it would require a very special constellation of values of the marginal propensities to hoard and to import to invalidate the commonplace dictum that "unemployment can be exported by depreciation."

These conjectures can be proved rigor-

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\(^9\) This analysis assumes that the elasticity of total (domestic plus foreign) demand for each country's product is negative. For a qualification of this assumption, which will be retained in the remainder of this paper, see the appendix.
ously within the framework of the following model:

\[ y_1 = z_1 + x_1 \]  
Income definitions and equilibrium conditions; \( (7) \)

\[ x_1 = a_2 + b_2 k + c_2 y_2 \]  
Demand for imports; \( (8) \)

\[ z_2 = a_1 + b_1 k + c_1 y_1 \]  
Demand for home goods; \( (9) \)

where \( y_1 \) and \( y_2 \) represent production of the national goods of countries 1 and 2; \( z_1 \) and \( z_2 \) represent home demand in each country for its home-produced commodity; \( x_1 \) and \( x_2 \) represent exports of countries 1 and 2; \( c_1 \) and \( c_2 \) represent the marginal propensities to import in countries 1 and 2, and \( f_1 \) and \( f_2 \) represent the marginal propensities to consume home goods in countries 1 and 2. I shall additionally define \( h_1 = b_1 x_1 \) and \( h_2 = b_2 x_2 \) as the elasticities of demand for imports in countries 1 and 2; \( h_1 = 1 - f_1 - c_1 \) and \( h_2 = 1 - f_2 - c_2 \) as the marginal propensities to hoard in countries 1 and 2, and \( s = x_1/x_2 \) as the initial ratio of value of imports to value of exports in country 1.

From our assumptions it follows that production in each country of its national good is infinitely elastic (within the relevant range) with respect to the price of that good in terms of home currency. Thus, as before, prices do not appear explicitly in our model: their ratio (choosing commodity units so as to make each price = 1) is necessarily equal to the exchange rate. Again, as before, we choose units of currency so as to make the exchange rate \( k = 1 \) at the point of departure. In drawing conclusions from this model, I shall assume that all the marginal propensities involved are positive.

From the definition of \( B_1 \) in equation (3) and from equations (8) it follows that

\[ \frac{dB_1}{d k} = x_1 + b_2 + b_1 + c_2 \frac{dy_2}{d k} + c_1 \frac{dy_1}{d k} \]  \( (10) \)

We can determine \( dy_1/d k \) and \( dy_2/d k \) either directly (see n. 15 below) or indirectly. The latter approach has been chosen as a better indicator of the behavior relationships which underlie the system under consideration. The first fundamental assumption involved in the analysis which follows is that the marginal propensities to hoard, to import, and to consume home goods operate not on national production per se but on “real consumer income,” of which national production is an adequate measure only when the exchange rate is constant. Let us therefore define \( y_1 = y'_1(y, k); y_2 = y'_2(y, k) \), where \( y' \) represents real consumer income and is made commensurable with production \( y \) by the requirement that \( \partial y'/\partial y_1 = \partial y'/\partial y_2 = 1 \) (i.e., that, as long as the terms of trade do not change, production can be used as a measure of real consumer income). Then

\[ \frac{dy'_1}{d k} = \frac{dy_1}{d k} + \frac{\partial y'_1}{\partial k}, \quad \frac{dy'_2}{d k} = \frac{dy_2}{d k} + \frac{\partial y'_2}{\partial k} \]  \( (11) \)

The second fundamental assumption involved in the present analysis is that hoarding is a function of real consumer income alone. This assumption, together with the fact that the balance of trade in terms of home currency must be offset by hoarding (voluntary foreign investment

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Footnote: Both the above model and the present paper were suggested by a pioneering article by J. Tinbergen: “Modelli di commercio internazionale,” Giornale degli economisti, VII (1948), 627-48. Valuable comments by J. J. Polak led to the present formulation of the argument which follows.
is assumed to be included in the demand for imports, makes it possible for us to write

\[
\frac{dB_1}{dk} = \frac{h_1}{h_2} \frac{dy_1'}{dk}, \quad \frac{dB_2}{dk} = h_2 \frac{dy_2'}{dk} \tag{12}
\]

where \( B_1 \) and \( B_2 \) are the trade balances of countries 1 and 2 expressed in terms of home currency. From the definitions of \( B_1 \) and \( B_2 \), it is evident that \( \frac{d^2 B_1}{dk^2} = -\frac{dB_1}{dk} \), and that \( \frac{d^2 B_2}{dk^2} = (\frac{d}{dk}) (x_1 p_i - x_2 p_i) = \frac{dB_1}{dk} + x_1 - x_2 \). We can therefore substitute expressions in \( \frac{d^2 B_1}{dk^2} \) and the initial balance of trade for \( \frac{dy_1'}{dk} \) and \( \frac{dy_2'}{dk} \) in equation (11), solve for \( \frac{dy_1'}{dk} \) and \( \frac{dy_2'}{dk} \), and substitute the results in equation (10) to obtain

\[
\frac{dB_1}{dk} = x_1 + b_2 + b_1 - c_2 \frac{dB_1}{dk} + c_2 \frac{dy_2'}{dk} - \frac{c_1}{h_1} \left( \frac{dB_1}{dk} + x_2 - x_1 \right) + c_1 \frac{dy_2'}{dk} \tag{13}
\]

In order to be able to express \( \frac{dB_1}{dk} \) in terms of the parameters and initial values of the variables of the system, we now need only to determine \( \frac{dy_1'}{dk} \) and \( \frac{dy_2'}{dk} \). It is easy to show, on the assumptions made above, however, that the change in “real consumer income” which would result if the exchange rate were to change while national production remained constant is equal to initial income times the percentage of income initially spent on imports times the change in the exchange rate (i.e., \( \frac{dy_1'}{dk} = x_1; \frac{dy_2'}{dk} = -x_2 \)). Equation (13) now becomes

\[
\frac{dB_1}{dk} = \frac{h_1 h_2 \left[ x_1 + b_2 + b_1 + c_2 x_1 + c_1 x_2 - \frac{c_1}{h_1} (x_2 - x_1) \right]}{h_1 h_2 + c_1 h_2 + c_2 h_1}
\]

which, when trade is initially balanced, reduces to

\[
\frac{dB_1}{dk} = \frac{h_1 h_2 x_1 (1 + \eta_2 + \eta_1 + c_2 + c_1)}{h_1 h_2 + c_1 h_2 + c_2 h_1}
\]

Thus for the case of initial balance the critical (absolute) value of the sum of the elasticities of demand for imports is not unity when production is variable but unity plus the sum of the marginal propensities to import. Furthermore, since,

\[
\begin{align*}
\frac{\partial x_1}{\partial k} + \frac{\partial x_2}{\partial k} + \frac{\partial x_1}{\partial k} - x_1 &= 0, \\
\text{or} \quad c_1 &= -b_1 + h_1 \frac{\partial y_1'}{\partial k},
\end{align*}
\]

But \( c_1 \) and \( b_1 \) can be divided into substitution effects \( (c_1, b_1) \) and real-income effects \( (f_2, \partial y_2'/\partial k, c_1, \partial y_1'/\partial k) \), yielding the equation

\[
(c_1 - b_1 + h_1 \frac{\partial y_1'}{\partial k}) + c_1 \frac{\partial y_1'}{\partial k} = x_1 = 0.
\]

Because \( c_1 \) and \( b_1 \) are slopes representing the pure substitution effect between home goods and imports, they must be equal (cf. Hicks, Value and Capital [2d ed.; Oxford: Clarendon Press, 1946], p. 370). Now, since \( f_2 + c_1 + h_1 = 1 \), our equation reduces to \( \frac{\partial y_1'}{\partial k} = x_1 \). Similarly, it can be shown that \( \frac{\partial y_2'}{\partial k} = -x_2 \).

It may be noted that the critical values discussed thus far are critical values for elasticities of demand for imports \( (\eta) \) which contain income effects as well as substitution effects. We can alternatively consider critical values for elasticities \( (\eta') \) which contain substitution effects alone, by utilizing the equations

\[
\begin{align*}
b_1 &= b_1 - c_1 \frac{\partial y_1'}{\partial k} = b_1 - c_1 x_1; \\
b_2 &= b_2 + c_2 \frac{\partial y_2'}{\partial k} = b_2 - c_2 x_2,
\end{align*}
\]

and defining \( \eta' = b_1/x_1; \quad \eta' = b_2/x_2 \). The critical value for \( \eta' \) in the variable-production case

\[
\frac{dB_1}{dk} = \frac{h_1 h_2 x_1 \left[ 1 + \eta_2 + \eta_1 + c_2 + c_1 \right]}{h_1 h_2 + c_1 h_2 + c_2 h_1}.
\]
when trade is initially balanced, \( \frac{dB}{dk} = dB/\frac{dk}{dk} \), this critical value also applies to the change in the balance of trade expressed in terms of domestic currency and to the change in "real consumer income" resulting from depreciation.

The effect of an initial deficit is, as in the fixed-production case, to reduce the critical value of the sum of the elasticities of demand for imports. It can further be stated that, for fixed values of the marginal propensities, there will be some initial deficit for which the critical value is the same for the fixed and variable production cases. This critical deficit is given by

\[ s = 1 + \left( \frac{c_i}{c_t} \right) h_t / (1 - h_t), \]

which can be found by setting \( c_t + c_h - (c_t/h_t) (s - 1) = 0 \). Should \( s \) exceed this value, a set of elasticities of demand for imports such that depreciation, with fixed incomes, would lead to no change in the balance of trade, with variable incomes, provide improvement in the balance of trade. This outcome would ensue because of a "perversely" effect of depreciation on the depreciating country's output \((y_t)\), which would, in the new equilibrium established after depreciation, have fallen rather than risen from its initial value.\(^{14}\)

The above example is probably unrepresentative of frequently encountered practical situations, but it points up clearly the fact that, under "reasonable" assumptions regarding production and tastes, it is possible for output to decline and the balance of trade to improve as a result of depreciation. The converse—an increase in output together with a deterioration of the balance of trade—is also possible, since, when trade is initially balanced, \( \frac{dy_t}{dk} = (1/h_t) \left( \frac{dB}{dk} \right) - x_t \) and can therefore be negative even when \( dB/\frac{dk}{dk} \) is positive. It is not possible, however, for production in both countries to move "perversely" as a result of depreciation, i.e., for production in the depreciating country to fall and for production in the rest of the world to rise.\(^{15}\)

If the balance of trade in terms of domestic currency \((B'_t)\) is used in place of the balance in terms of foreign currency, our conclusions require only moderate revision. It is possible for this balance of trade to improve while production increases.\(^{16}\)

\(^{14}\) For depreciation to be ineffective in the fixed-production case, \( s + h_t + h_{tt} \) must equal \( 0 \). Then for the variable-production case

\[ \frac{dB}{dk} = h_t x_t \left[ c_t + c_h - (s - 1) c_t/h_t \right] \]

which will be \( s \) for \( s > (1 + [c_t/c_h][1/(1 - h_t)]) \). But

\[ \frac{dy_t}{dk} = \frac{dB}{dk} + \frac{x_t - x_h - x_h}{h_t} \]

which must be \( > 0 \) for \( s > (1 + [c_t/c_h][1/(1 - h_t)]) \).

\(^{15}\) Direct solution of eqs. (7)–(9) yields alternative expressions for \( dy_t/\frac{dk}{dk} \) and \( dy_h/\frac{dk}{dk} \):

\[ dy_t = \frac{(c_t + h_t)(c_t + h_t) - c_t(c_t + h_t)}{h_t h_t + c_t h_t + c_t h_t} \]

\[ dy_t = \frac{(c_t + h_t) c_t - (c_t + h_t)(c_t + h_t)}{h_t h_t + c_t h_t + c_t h_t} \]

For output to decrease in the depreciating country \((dy_t/\frac{dk}{dk}) < 0\) and to increase abroad \((dy_h/\frac{dk}{dk}) > 0\) is impossible, since it requires that \( (c_t + h_t) > (c_t + h_t) \) and \( (c_t + h_t) > (c_t + h_t) \).
CREASES, or to deteriorate while production either increases or decreases, as a result of depreciation by country 1. It is impossible, however, for both an improvement in the trade balance in terms of domestic currency and a decrease in production to result from depreciation, since, from equations (11) and (12) and note 12, \( dy_1/dk = h_1(dB^*_1/dk) - x_1 \) and can therefore not be positive when \( dB^*_1/dk \) is negative.

When the trade balance is expressed in terms of domestic currency, the effect of an initial deficit is to raise rather than lower the value which the absolute sum of the elasticities of demand for imports must exceed if the balance is to improve with depreciation. Conversely, an initial surplus will lower this critical value, and if this surplus is great enough \( s < (1 - h_1)/(1 + e_1h_1/c_1) \), the critical value will be lower rather than higher for the variable-income case than for the fixed-income case.\(^{16}\)

The fact that an initial deficit has effects on the change in the balance of trade due to depreciation which are opposite in sign depending on whether the balance is expressed in terms of domestic or of foreign currency is not surprising. In this respect our conclusions merely extend the variable-income case of results which Hirschman has already shown to be true for the fixed-income case. Indeed, these relationships can be seen intuitively, once it is realized that \( B^*_1 = -B_1 \) (i.e., that the depreciating country’s balance of trade in terms of its own currency is merely the negative of the nondepreciating country’s balance of trade in terms of foreign currency).

\(^{16}\) These statements can be proved by deriving expressions analogous to eqs. (6) and (14) for \( dB^*_1/dk \), and following, by analogy, the proof in n. 14.

V. A GENERAL MODEL

We can now generalize the Keynesian model specified by equations (7)–(9) by permitting the absolute price of each country’s product to be a (presumably increasing) function of total production of that product. Such conditions would arise if the supply of labor in each country were infinitely elastic at the ruling money wage and if diminishing returns to labor input prevailed or if, for any other reason, the supply of labor were characterized by “money illusion” on the part of workers. We must then substitute for \( k \) (the exchange rate), wherever it appears in equations (7)–(9), an expression for the terms of trade, \( w(= p_1/k/p_0) \), and add two equations expressing the price-income relationship:

\[
\begin{align*}
  p_1 &= h_1 + m_1y_1, \quad \text{Supply of products} \quad (15) \\
  p_2 &= h_2 + m_2y_2, \quad \int (m_1, m_2 > 0).
\end{align*}
\]

We now have released our assumption of constant prices in each country and can assume, instead, only such choice of units of commodities and currency that at the start \( p_1 = p_2 = k = 1 \). Differentiating the first six equations of this system with respect to \( w \), we obtain the same expressions for \( dy_1/dw \) and \( dy_2/dw \) as we obtained in note 15 for \( dy_1/dk \) and \( dy_2/dk \), respectively. Thus, so long as \( dw/dk \) is positive, only the magnitudes, but not the signs of the changes in incomes resulting from depreciation, will be affected.

If trade is not initially balanced, however, the expression for \( dB_1/dw \) will differ from that obtained for \( dB_1/dk \) in equation (14). This may be seen by rewriting equation (3) as

\[
\begin{align*}
  B_1 &= x_1w - x_2 \quad \text{(16)}
\end{align*}
\]
Then
\[
\frac{dB_1}{dw} = x_1 + \frac{dx_1}{dw} - \frac{dx_2}{dw} + B_1 m_2 \frac{dy_2}{dw}.
\tag{17}
\]
If trade is initially balanced \((B_i = 0)\) or if the price of the good produced in country 2 is stable \((m_2 = 0)\), the last term in this equation will equal 0, and the identity of \(dB_i/dw\) with the expression for \(dB_1/dk\) obtained earlier will be complete. Otherwise, in the "normal" case of a decrease in income in country 2, the critical value of the sum of the elasticities of demand for imports will be lower if an initial deficit, and higher if an initial surplus, prevailed in the balance of trade of country 1. However, the possibility of opposite directions of change in income and the balance of trade and the probability of a critical value other than that given by equation (6) for the sum of the elasticities of demand for imports remain as characteristics of this generalized model.

We must now investigate the conditions under which \(dw/dk\) will, in fact, be positive, i.e., in which the movement in terms of trade resulting from a change in the exchange rate will be in the same direction as the movement in the exchange rate itself. We can write the expression for \(dw/dk\) thus:
\[
\frac{dw}{dk} = 1 + \frac{dp_1}{dk} - \frac{dp_2}{dk} + \left( m_1 \frac{dy_1}{dw} - m_2 \frac{dy_2}{dw} \right) \frac{dw}{dk}
\]
\[
= 1 + m_2 \left( \frac{dy_2}{dw} - \frac{dy_1}{dw} \right) \frac{dw}{dk}.
\tag{18}
\]
Obviously, this is uniformly positive for the normal case of an increase in income in country 1 \((dy_1/dw) < 0\) and a decrease in income in country 2 \((dy_2/dw) > 0\) as a result of depreciation by country 1. Only for cases in which incomes in both countries move in the same direction does any possibility exist that \(dw/dk\) might be negative.\textsuperscript{17} Now in the normal case \(dw/dk\) is always \(< 1\). In the other admissible cases it may be \(> 1\), because here \(m_1 (dy_1/dw) - m_2 (dy_2/dw)\) may be \(< 0\). In these cases we should expect the effects of depreciation to be magnified rather than diminished by price flexibility, as \(1 + m_1 (dy_1/dw) - m_2 (dy_2/dw)\) approaches zero. If this expression were equal to \(0\), \(dw/dk\) would equal \(\infty\); and we should expect the system to be characterized by explosive instability. Yet the case of the negative multiplier (marginal propensity to consume > 1 implying explosive upward instability) leads to the conjecture that any \(dw/dk < 0\) might imply explosive instability, the "multiplier", involved being "more than infinite."\textsuperscript{18} L. A. Metzler and Svend Laurersen, in an unpublished paper,\textsuperscript{19} have indeed shown this to be true. Thus the fact that such explosive instability is not observed can be accepted as demonstrating that, if the present model is considered sufficiently realistic for practical use, the sign of \(dw/dk\) can be taken as uniformly positive.

The introduction of flexible prices as functions of national production there-
fore destroys none of the qualitative conclusions which we reached in considering the Keynesian case above. In fact, all the models considered earlier are special cases of the present one, the Keynesian model arising when \( m_s = m_t = 0 \) and \( dw/dk = 1 \), the classical fixed-income model when \( m_s = m_t = 0 \) and \( dw/dk = 0 \), and the case in which prices and incomes were fixed by government policy \( m_s = m_t = 0 \), \( dw/dk = 1 \), and \( dy/dk = dy_s/dk = 0 \).

VI. MODELS AND THE REAL WORLD

Though the application of the simple models considered in this paper to the real world is hazardous, it is nevertheless tempting. If we aver, with the classical economists, that the fundamental equilibrium relations of an economic system (national or international) are, within limits, determined by real variables and if we hold, with Pigou, that Keynesian underemployment equilibrium is, in fact, no equilibrium at all but only an important stage in one possible process of adjustment to disturbances, then we must retain the classical model set out in Section III as a fundamental guidepost to our economic thinking. The other models considered would then show methods, consistent with the policy and taste assumptions underlying them, of achieving equilibrium when, at the start, disequilibrium prevails or, in the hands of "hardhearted" governments, means of achieving a favorable (and reasonably lasting) disequilibrium in place of a pre-existing equilibrium.

If one is willing to grant that (in an approximate sense) governments are likely in the future to succeed, by monetary-fiscal measures, in maintaining both full employment and a stable price level, then the model of Section II becomes appropriate for analysis. Comparing this with the model of Section IV, which, in the same approximate sense, appears best to explain our experience in the 1930's, one can state that, if a significant fall in international demand for a country's exports should occur in spite of the successful maintenance of full employment at home and abroad, this fall in demand will no longer be partially equilibrated by movements in incomes at home and abroad. Rather, it will be reflected solely in the form of a balance-of-payments deficit—a deficit which will be larger than the one which would have arisen in the absence of policy-determined employment levels. Thus solution of the employment problem implies greater fluctuation in balances of payments arising out of shifts in international demand than would otherwise occur.

But the menace of balance-of-payments problems is not without compensation; for we have seen that a given amount of depreciation will generally be more effective in equilibrating the balance of payments if incomes are held stable than it would be if it were permitted to affect incomes as well as the balance of payments. In fact, so long as the ultimate equilibrium position is determined by real variables and so long as prices are stable in both countries, the amount of depreciation which will be necessary to reach this ultimate equilibrium after a given disturbance (e.g., a fall in foreign demand) will be the same, regardless of whether full-employment policies are successfully pursued in the countries concerned. The requisite amount of depreciation will, in the full-employment case, merely equilibrate the balance of payments. In the variable-income case the same amount of depreciation will succeed both in rectifying the (smaller) balance-of-payments deficit and
in relieving the concomitant unemployment resulting from the postulated disturbance.

Now if, in the future, full-employment policies are less than ideally successful and if unemployment is generally accompanied by a declining price level, while higher employment is found to be best stimulated by permitting the price level to rise, the general model of Section V becomes the appropriate tool of analysis. To the extent that full employment and stable price-level policies are effective, the generalizations made in the preceding paragraph will hold. However, to the extent that they do not hold, countries contemplating depreciation to eliminate balance-of-payments deficits should realize that depreciation will have repercussions on prices and employment which will tend to lessen its efficacy in equilibrating the balance of payments. When these repercussions are taken into account, the countries involved will find that the amount of depreciation necessary to restore equilibrium will be greater than would be estimated in abstraction from such repercussions.

The conclusions thus far presented could have been derived from the analyses of Robinson and Brown, with certain modifications for the case of variable price levels. I have as yet drawn no implications from the substantive conclusion of this paper that the critical value of the sum of the elasticities of demand for imports is lower, and may be substantially lower, when incomes are maintained by policy than when incomes are permitted to fluctuate. On this point I should like to suggest that the reluctance of so-called "practical" men in certain countries to contemplate depreciation may have been in part determined by the relative or complete ineffectiveness of exchange depreciations in the 1930's. This ineffectiveness was due partly to the fact that the depreciations of the 1930's were so widespread as to amount to little more than a world-wide change in the price of gold, and partly to the fact that the effect of the depreciations on incomes operated to lessen the absolute magnitude of the resulting changes in the balance of payments. But it may also be true that in some cases (e.g., where the relevant marginal propensities to import were high) the volatility of incomes made the critical value of the sum of the elasticities of demand for imports so high that depreciation led to a deterioration, not to an improvement, of the balance of payments. If this was the case with respect to certain countries in the 1930's, it should be pointed out to those countries that their situations need no longer be considered hopeless; for, if full-employment policies are successfully pursued at home and/or abroad, the critical values in question may now be significantly lower than they were in the 1930's; and depreciation, though ineffective in the last decade, may constitute the solution of their balance-of-payments problems in the present one.

VII. LIMITATIONS

The present simplifications of the theory of international trade reveal certain relationships of macro-economic magnitudes but conceal others, especially through the assumption of a homogeneous product and a single price level in each country. These limitations suggest that fruitful results might ensue if models were formulated which took into account (a) restraints on the substitution of one commodity for another in production, especially at levels of less than full employment,\(^{19}\) and (b) the effects of differential

\(^{19}\) The classical full-employment model can easily be generalized to release the one-commodity assump-
movements in sectional price levels on the process of adjustment to disturbances under various assumptions and on the equilibria which would ultimately be approached.

APPENDIX

It has been assumed throughout this paper that, with production in both countries constant, the total quantity demanded (at home and abroad) of each country’s good was a decreasing function of the relative price of that good. This need not be the case, however, since (Hicksian) income effects are present in the situation postulated. This can be illustrated in the accompanying indifference diagram (Fig. 1).

Since our price slopes $b_1$ and $c$, are defined as measuring the amounts by which consumption of imports and home goods, respectively, will change if $p_1k/p_2k$, changes with national production constant, we are justified in making the two price lines converge at $y_1$. Now one can easily see that, though the substitution effect will always lead toward negative price elasticities (point 1), the income effect will do the same only in the case in which the home-produced good is inferior (point 2). Under our assumption that neither good is inferior ($e_i > 0$), the income effect will lead toward $b_1 < 0$, $e_i > 0$ (point 3).

In order for the total demand for the product of country 1 to be an increasing function of its relative price (i.e., for $e_i + b_1$ to be $> 0$), the

\[^{28}\text{For simplicity, the hoarding effects, which are taken into account in the text, are neglected in the diagram.}\]
countries, and, even when other effects of depreciation are present, this tendency to parallel movements of production may dominate.

These conclusions can be verified by reference to the equations of note 15, which show that $dy/dk$ can be $>0$ if $c(e_1 + b_1) > (e_1 + h_1)e_1 + b_1$, and that $dy/dk$ can be $<0$ if $c(e_1 + b_1) > (e_1 + h_1)e_1 + b_1$. Thus an ultimate decrease in production in the depreciating country will be the more likely, the larger is the initial decrease in production abroad compared to the initial increase in production at home (i.e., the larger is $|e_1 + b_1|/e_1 + b_1$), the larger is the marginal propensity to import of the foreign country, and the smaller is the marginal propensity to hoard of the foreign country. Similarly, an ultimate increase in production abroad will be the more likely, the larger is the initial increase of production in the depreciating country compared to the initial decrease in production abroad, the larger is the marginal propensity to import of the depreciating country, and the smaller is the marginal propensity to hoard of the depreciating country.