IN VOLUNTARY UNEMPLOYMENT AND THE KEYNESIAN SUPPLY FUNCTION

I. INTRODUCTION

1. Within the framework of traditional Keynesian economics there are at least two basic issues which have not yet been settled. The first concerns about the frequently heard complaint that Keynesian models neglect the supply side of the market. The second is concerned with the very question which brought forth the General Theory: involuntary unemployment. Examination of the Keynesian theory shows that (even granted its argument) it explains primarily the level of employment; it is inadequate in providing either a criterion for the measurement of unemployment, or a justification for calling it "involuntary."

The only serious attempt to deal with this last question proceeds by assuming a special shape for the supply curve of labor, as in Fig. 1. Here $N$ represents employment; $w$, the money wage; and $w = f(N)$, the supply curve of labor. By assumption, this supply curve is horizontal at the "customary money wage," $w_0$, until the amount of employment $N_1$. In other words, it is assumed that until a certain point money wages are completely rigid. If the amount of labor employed is $N_0$, then involuntary unemployment to the extent $N_1 - N_0$ is said to exist—since this...

---

1 Research in connection with this paper was undertaken under a Social Science Research Council fellowship. It will be reprinted in Cowles Commission Paper No. 38.

I should like to express my general indebtedness to two of my former colleagues at the Cowles Commission for Research in Economics: to Lawrence R. Klein (now of the National Bureau of Economic Research), whose excellent book, The Keynesian Revolution (New York: Macmillan and Co., 1947), read in manuscript form, started me thinking about many of the problems discussed in this paper; and to Trygve Haavelmo (now of the Oslo Institute of Economics), who was a constant source of stimulation and encouragement.

I am also particularly indebted to the following individuals, who read earlier drafts of this paper and offered many valuable criticisms and suggestions: James Buchanan (University of Tennessee); Robert and Marianne Ferber (University of Illinois); Malcolm Hogg (University of Illinois); Everett E. Hagen (University of Illinois); and D. Gale Johnson, H. Gregg Lewis, and Jacob Marschak (University of Chicago). I am also grateful to many former colleagues of the University of Chicago Department of Economics and of the Cowles Commission with whom I have discussed the ideas presented in this essay.

many additional workers would be willing to work at the prevailing market wage. The artificiality of this definition is sufficiently demonstrated if one considers the case in which the supply curve, instead of being horizontal until the point $N_1$, rises at an extremely slow rate.

![Diagram of supply function](image)

In the following sections it will be shown that the two seemingly independent issues raised above are, in fact, vitally interrelated. In particular, it will be argued that the key to our difficulties lies in explicitly introducing supply functions into the standard Keynesian models: once this is done, the problem of defining and measuring involuntary unemployment is simultaneously solved.

II. The Supply Function

2. The traditional tools of Keynesian analysis are the consumption and investment functions. In the very simple Keynesian models these are assumed to depend only on the level of real national income; this is the situation in Fig. 2. Here $C$, $I$ and $E$ are “desired” real-consumption expenditures, investment expenditures and total expenditures, respectively. $Y$ is real national income, and $E^*$ is actual, as distinct from “desired,” total real expenditures. $G(Y)$ and $H(Y)$ are the “desired” consumption and “desired”-investment functions, respectively. The total-expenditures function, $F(Y)$, is obtained by the vertical addition of $G(Y)$ and $H(Y)$.

First of all, we must make clear the sense in which the term “desired” is being used. It refers to nothing more than the schedule of alternative actions (as represented by the demand and supply
curves) of traditional economic theory. Consider, for example, the "desired"-consumption function. This can be considered as the end-product of the following experiment: Every individual in the economy is approached, told that he must stay within his income and asked how much of each particular good he will buy at different sets of prices and personal income. This gives us the usual individual demand curves of Walrasian general-equilibrium economics. If we sum up these demand curves for all individuals and for all goods, we obtain the aggregate "desired"-consumption function. Under certain assumptions,\(^1\) this desired-consumption function will be of such a form that aggregate real desired-consumption will depend only on aggregate real income. These assumptions have been implicitly made in the preceding paragraph; consequently, our desired-consumption function there is written as \(C = G(Y)\). It tells us the aggregate amounts the economy desires to consume at different income levels.

Thus the desired behavior of consumers is defined as their behavior under certain specified conditions—a behavior that is described by their Walrasian demand curves. In a similar way we can specify certain conditions under which the firm must

---

\(^1\) Namely, (a) that the individual demand curves depend only on relative prices and real income; and (b) that in deflating money consumption expenditures and money national income we use a certain price index which emerges from the aggregating process itself. For an example of the derivation of the consumption function in this way see L. R. Klein, "A Post-Mortem on Transition Predictions," *Journal of Political Economy,* LIV (1946), 309-1.
operate (e.g., the transformation function) and derive the desired-investment function. The conceptual experiment here is the following one: We ask each firm how much it will invest at different sets of prices and national income, subject to the restriction that the firm's inputs and outputs are related in a specified way (i.e., subject to the firm's transformation function). The results of these experiments will, after aggregation, give us the desired-investment function, \( I = H(Y) \). (Admittedly, there are many more conceptual difficulties in doing this than in constructing the desired-consumption function; but these need not detain us now.) The total expenditures function, \( F(Y) \), being the vertical sum of the consumption and investment function, thus represents the total expenditure people desire to make at any given level of real income.

3. Once the total desired-expenditures function \( F(Y) \) is given, the equilibrium level of real national income is determined as follows: First, a 45° line through the origin is drawn. This line (whose equation is \( E^* = Y \)) represents the fact that, by definition, real national income is equal to actual real expenditures: no one can receive income except as a result of expenditures by someone else.¹ Now consider the income level \( Y_1 \). At this level people desire to spend only the amount \( E_1 \), which is less than \( Y_1 \). In other words, if the income \( Y_1 \) is to be maintained, people must continue spending more than they desire to the extent \( Y_1 - E_1 \). For example, inventories will be accumulated above the amount indicated by the desired-investment function. This undesired spending cannot continue indefinitely. To use again the example of inventories, people will attempt to reduce their undesired accumulations, and, as a result of this attempt, the level of real income prevailing in the system will change. Hence \( Y_1 \) cannot be an equilibrium position.

Similarly, the equilibrium level cannot be \( Y_2 \): for at that level people would try to correct the resulting undesired disinvestment that would be occurring. To use again the example of inventories, people will try to replenish inventories that have been drawn down below their desired levels, and by this action will change the level of income. By reasoning along these lines we see that the equilibrium level must be at \( Y_0 \). At this point—which is given by the intersection of the 45° line and the total desired-expenditure function—desired and actual aggregate expenditures are

¹ Many complications are hidden in this simple statement—such as the practice of considering inventory accumulation as purchases of the firm from itself. But these complications need not concern us here.
equal. Hence, within the framework of the given conditions, there exists no force acting to change the income level from $Y_0$.

4. So much for the standard Keynesian analysis. However, a little reflection will make it obvious that there are some missing links in the argument. Consider again Fig. 2. The careful reader will have noted that all that was said about the income level $Y_1$ or $Y_2$ was that they were not equilibrium levels; nothing was said about the direction in which the forces set up by the resulting disequilibria would cause the income level to move. In other words, no attempt was made to show that if the economy were, say, at the income level $Y_2$, automatic forces would be set up to push the income in the direction of the equilibrium level, $Y_0$.

If we were to try to follow through such a dynamic analysis, the omission of certain factors from the Keynesian analysis represented by Fig. 2 would immediately become evident. For the standard dynamic analysis runs along the following lines (cf. Fig. 2): If the income level were $Y_1$, the resulting accumulation of undesired inventories would drive prices down and therefore discourage production. As an immediate result, income payments, and hence national income, would decline. This process would continue until the income level $Y_0$ is reached. Similarly, if the income level were $Y_2$, prices would be driven up, and production increased.

The significant point about the preceding dynamic analysis is that it calls into play factors completely outside the analytical framework of Fig. 2. It presupposes some type of behaviour (e.g., responses to changes in inventories) from the supply side of the market which is never explicitly introduced into the analysis. This procedure is a sharp contrast with that employed in, say, the dynamic analysis of partial equilibrium in one particular market. There both sides of the market are represented by the traditional demand and supply curves. If the price is higher than the equilibrium one, the analysis itself indicates the force (viz., the excess supply) which drives prices down. There is no need to appeal to outside forces to explain the movement toward equilibrium.

From all this there follows but one conclusion: something must be done to complete the Keynesian picture; and that "something" must clearly be the explicit introduction of the

---

1 I abstract here from the difficulty that this equilibrium is only aggregative, and may be disturbed by the disequilibrium of individual firms. Cf. Arthur F. Burns, Economic Research and the Keynesian Thinking of our Times (Twenty-sixth Annual Report of the National Bureau of Economic Research, 1946), p. 9.
supply side into the analysis. It is to this task that we turn in the section which follows.

5. To start from fundamentals, it is clear that a complete explanation of the economic system can be presented only through a Walrasian general-equilibrium system. Unfortunately, for practical analytical purposes such a system is entirely too large and cumbersome. Aggregation is necessary to reduce it to manageable terms. Thus, we have seen that in the preceding Keynesian models all the Walrasian demand curves for finished goods are aggregated into two functions: the consumption and investment functions. But these represent only one side of the market—the demand side. The system presented in Fig. 2 provides no aggregate counterpart to the supply functions of the general-equilibrium system. But clearly such an aggregate function can be built up from the Walrasian supply functions in exactly the same way that the consumption and investment functions were built up from the Walrasian demand functions. Furthermore, the interpretation of such an aggregate function will be completely analogous to that of, say, the aggregate desired-consumption function. The Walrasian supply functions from which the aggregate supply function is constructed represent the behaviour of suppliers within a specified framework (profit maximisation, given transformation curves, etc.); these supply functions, by definition, represent the desires of suppliers. Hence, the function we get by aggregating them also represents the desires of suppliers. We call this function “the aggregate desired-supply function.” It shows the aggregate amount of finished goods and services suppliers desire to provide at different levels of income.

In other words, the equilibrium position of the Walrasian system is determined by the joint influence of the demand and supply functions. In the process of aggregation we do lose a lot of detailed information. This we are willing to sacrifice for the sake of manageability. But one characteristic of the general-equilibrium system which we should carry over to the aggregate (or macro-) system is that supply factors, as well as demand factors, influence the equilibrium position. No such influence is provided for in our Keynesian model of Fig. 2. In order to bring it in, our macro-system must provide an aggregate desired-supply function (corresponding to the aggregate desired-consumption and investment functions) which should play a co-equal role in the determination of the equilibrium position.

What might the form of this aggregate desired-supply function be? By making assumptions similar to those made for the con-
sumption function we can have aggregate real desired supply, \( S \), depending only on real income, \( Y \). In Fig. 3 this function is drawn as \( S = Q(Y) \).  

![Graph showing aggregate supply function](image)

**Fig. 3.**

What is the form of \( Q(Y) \)? It can be shown that the assumptions made in aggregating the supply function imply that the real return to productive services is constant; that is, the price of finished goods is always proportionate to the price of productive services.\(^1\) This should lead us to suspect that the function \( Q(Y) \) might have a special form. In particular, since the real return is constant, suppliers might desire to provide the same amount of goods regardless of the level of income. In that case the aggregate supply function would be a horizontal line at, say, the level \( \eta \). This extreme position was not taken in Fig. 4; but some allowance for it was made by giving \( Q(Y) \) a small slope with respect to \( Y \). In other words, it was assumed that the desired supply does not vary much with the level of income.

Lest there be any misunderstanding on this point, it should be made clear that the assumption of a constant real wage is regarded as completely unrealistic. We shall, in fact, remove this assump-

---

\(^1\) Limitations of space prevented the publication of the mathematical appendix to this article in which this was proved. The general argument is related to footnote 1 on p. 362, above. The price index that comes out of the aggregation here is for finished goods only; but the price index resulting from the aggregation for productive services as well. If the supply function is to depend on the same measure of real income, \( Y \), that the expenditure function does, then the two price indexes must be the same, which means that the prices of finished and productive services must be proportionate.

This mathematical appendix is available upon request.
tion later (§ 14). However, the main point here is that for the purposes of this article it is only the existence, and not the form, of the aggregate supply function which is of importance. We shall return to this point again (§ 14).

6. Let us for the moment ignore the expenditure function in Fig. 3 and concentrate on the relationship between the 45° line and the desired-supply function. Introduce the variable \( S^* \) which equals actual, as distinct from desired, aggregate real supply of finished goods and services. Previously we used the fact that national income is equal to aggregate expenditures. Now we note that, by definition, real national income is also equal to the actual real value of finished output, \( S^* \). So the 45° line also represents this equality, i.e., \( S^* = Y \).

In a manner completely analogous to that of the discussion of the expenditure function, it can now be showed that the level of income \( \eta \) (Fig. 3) is the equilibrium level determined by supply conditions. For consider the income level \( Y_4 \). At this level firms desire to supply the amount \( S_4 \), which is greater than \( Y_4 \). In other words, if the income level \( Y_4 \) is to be maintained, firms must continue supplying less than they actually desire, to the extent \( S_4 - Y_4 \). Consequently, an effort will be made (via price reductions) to correct this disappointment of desires, and in the process the level of income will change. Similarly, \( Y_3 \) cannot be an equilibrium value; for to maintain that level firms must supply more than they actually desire. The only possible equilibrium from the supply side is \( Y = \eta \). At this level, since desired and actual supply are equal, there is no stimulus for any change in the system.

If we now consider Fig. 3 as a whole, it is immediately evident that the macro-system we have built up from our Walrasian system is one which can never be at equilibrium. For the income level, \( Y_0 \), which equilibrates the demand side of the economy, leaves the supply side in disequilibrium. Conversely, the income level \( \eta \), which equilibrates the supply side, leaves the demand side in disequilibrium. There is no level of income which will simultaneously equilibrate both of these sets of forces in the economy. What is the economic interpretation of this inability to reach a consistent equilibrium position? This is the problem discussed in Part III. There it is argued that the inconsistency created by the explicit introduction of the aggregate supply function into Keynesian systems provides the key to the theory of involuntary unemployment implicit in Keynesian economics.

7. One more point must be made, an important one for our
later analysis. In the discussion of Fig. 3 it was tacitly implied that there is no way of resolving the inconsistency of having two "equilibrium" positions. There are, however, a priori grounds (formulated by Pigou) for claiming that the initial duo-equilibrium itself brings into play automatic market forces which tend to remove the inconsistency.¹ Specifically, suppliers at, say, the income level \(Y_0\) in Fig. 3, will find themselves supplying less than they desire. Hence they will reduce the general price level in an effort to increase their sales. The fall in the price level will in turn increase the real value of the cash holding of individuals. As a result, their willingness to spend out of income will increase so that the whole expenditure function in Fig. 3 will shift upwards. (In other words, the expenditure function is now assumed to depend on the absolute price level, \(p\), as well as the level of real income, \(Y\).) Under certain assumptions, if the price decline continues long enough, the expenditure function can be shifted so far up that it intersects the 45° line at the same income level, \(\eta\), at which the supply function intersects. (This is the situation pictured in Fig. 5 below.) Thus, a unique equilibrium position is determined and the inconsistency removed.

In addition to these forces there is the more traditional Keynesian effect on the expenditure function through variations in the interest rate. An excess supply will drive interest rates down and thereby raise the expenditure function. If it has sufficient interest elasticity, the expenditure function might eventually be driven up to its position in Fig. 5. It is clear that for the purpose of this paper the interest rate and the price level play completely equivalent roles. Hence, whenever in the subsequent argument the reader will find the phrase "price level," he can add "and interest rate."

As we shall see later, this argument makes no fundamental change in our analysis. At most, it requires that we shift from a static to a dynamic viewpoint. The full implications of these remarks will become clearer in the exposition which follows.

III. THE CONCEPT OF INVOLUNTARY UNEMPLOYMENT

8. Involuntary unemployment involves what might be called "relative coercion": people cannot fulfill their desires as freely as under some other situation which serves as a norm of reference;

¹ The argument of this paragraph is presented in its barest details, since it has already been discussed at length in my article "Price Flexibility and Full Employment," *American Economic Review*, XXXVIII (1948), 643–65. This will be referred to henceforth as "Price Flexibility."
hence in order to give concreteness to the concept of coercion we must first define this norm of reference. Thus, it is theoretically meaningless to speak of involuntary unemployment without introducing a comparison between two alternative models: the actually existing one and some designated norm. The extent of involuntary unemployment is then measured by the difference between the existing amount of employment, and the amount that would have existed under the norm.

I must emphasise that coercion and freedom are defined in a relative sense only. People acting with the "normal" freedom (i.e., under the restrictions to be found in the norm of reference) will (for the sake of brevity) be defined as fulfilling their desires freely. People acting under more than the "normal" restrictions will be said to be coerced and prevented from fulfilling their desires. In what follows our norm of reference is defined as a model in which perfect competition reigns and the economic unit is restricted only by the budget restraint and technological relationships (e.g., the production function). Thus, by definition, our norm is a system of equations. Within this norm of reference the individual will be defined as fulfilling his desires—though he may be poor and unhappy. In other words, an individual will be said to be acting freely as long as he is on his Walrasian demand and/or supply curves.

Partial equilibrium analysis provides an illustration. Consider the classical demand (D) and supply (S) curves for labor (N) in terms of the real wage w/p (Fig. 4). By definition, the demand curve, D, represents the desires of employers under normal restrictions, and the supply curve, S, represents the desires of workers under normal restrictions. Now, if the real wage, w/p, is always at the intersection of D and S, then within this model there can be no involuntary unemployment. No matter how far D shifts over to the left and employment drops, workers will be working as much as they desire: Workers and employers will be fulfilling their desires as long as the equilibrium wage and employment are always at the intersection of the curves. Only if some force entered which established the equilibrium value at, say, the wage (w/p)2 and the employment N2, a point off the supply curve of labor, could coercion, and therefore involuntary unemployment (to the extent N1 − N2), be said to exist in the system.

One fundamental qualification must be introduced into the discussion of Fig. 4. If the wage (w/p)2 has been set and maintained by monopolistic tactics of a trade union, it is clearly a distortion to say that "involuntary" unemployment exists in the
economy. (Of course, those workers unable to find jobs because of the union wage policy might be said to be "involuntarily" unemployed; but this involves a completely different usage from the customary one, which implies that the workers are unemployed neither through their fault, nor through that of their brethren.) This paper does not deal with any of these difficulties; it is concerned solely with the definition of involuntary unemployment within an actual competitive framework. Within this framework we can think of the wage \((w/p)_2\) being maintained despite wage flexibility if, for example, the price level were to fall proportionately

with the wage level and so keep the real wage constant. In such a case, workers would truly be involuntarily unemployed—despite all their efforts to correct the situation by money-wage reduction.¹

More generally, in a dynamic framework, we can think of the price decline as being less than proportionate to the money-wage decline, so that the real wage falls only slowly. During the time it takes for it to fall, workers are involuntarily unemployed; but the amount of involuntary unemployment is continuously decreasing. Eventually, if the real wage falls to \((w/p)_1\) (cf. Fig. 4) a full-employment condition may be re-established.

9. Let us apply the concepts of § 8 to the analysis of Fig. 3. Clearly a full-employment level of income in this model must mean a level of income at which suppliers are able to supply exactly what they desire—in the sense of § 8. In other words,

¹ This is clearly the argument of Keynes in his *General Theory*, pp. 11-13.
suppliers must be employed to the full extent they desire. From the analysis of Fig. 3 it immediately follows that \( \eta \) is the full-employment level of income; for at any other level there would exist a discrepancy between the amount sellers desire to supply, and the amount they actually do. Hence we choose as our norm of reference a model in which this level of income could be maintained indefinitely; i.e., a model whose equilibrium level of national income (for both the demand and supply sectors) is \( \eta \). This norm of reference is drawn in Fig. 5.

![Diagram](image)

**Fig. 5.**

Assume now that, in contrast with this norm, the actual desires of individuals are represented by the expenditure and supply curves in Fig. 3. This figure reveals an initial incompatibility of interests— with demanders desiring the income level \( Y_\theta \), and suppliers the level \( \eta \). Several possibilities now present themselves. First consider the case in which the prevailing level of national income is always \( Y_\theta \). In other words, only the desires of demanders influence the determination of the national income, while the desires of suppliers are completely ignored. We would then have as a measure of the extent of involuntary unemployment (\( U \)) in the system

\[
U = \eta - Y_\theta
\]

That is, involuntary unemployment is measured by the difference between the level of national income in the norm of reference, \( \eta \), and the level actually prevailing, \( Y_\theta \) (cf. § 8).
What are the implications of assuming that the income level will remain at $Y_0$? It is with respect to this question that the remarks of §7 are pertinent. It will be recalled that at the income $Y_0$ suppliers will reduce their prices in an attempt to eliminate the discrepancy between the quantity they are selling and the quantity they desire to sell. Now, if these price reductions have no effect on spenders—that is, if the expenditure function does not shift upwards at all, despite all price reductions, no matter how far down they go or how long they are maintained—then the income level will remain indefinitely at $Y_0$. In the case of such insensitivity of spenders to general price declines we can say that suppliers are in a “weak strategic position.” At the other extreme is the case where the slightest price decline instantaneously shifts the expenditure function to the position it has in Fig. 5, so that full employment is established. In this case we can say that suppliers are in a “strong strategic position.” Midway between these extremes is the case illustrated in Fig. 6. Here it is assumed that by price reductions we can shift the expenditure function upwards; but that (to repeat the phrase of the preceding paragraph) despite all price reductions, no matter how far down they go or how long they are maintained, it is impossible to shift the expenditure function up to its position in Fig. 5. That is, say, the expenditure function cannot be pushed above its position in Fig. 6. Under these circumstances the income level $Y_0$ will be maintained; correspondingly, the amount of
involuntary unemployment is measured by $\eta - Y_0$. In this case we can say that suppliers have an "intermediate strategic position."

Thus, the strategic position of suppliers is essentially a measure of the sensitivity of spenders to changes in the absolute price level (i.e., to changes in their real cash balances) and interest rate. The strategic position is stronger the smaller the price (interest) decline required to shift the expenditure function a given amount within a given period. Similarly, it is stronger the shorter the time required to shift the expenditure function to a given position by a given price (interest) decline. And, of course, it is stronger the greater the upward shift in the expenditure function corresponding to a given price (interest) decline maintained for a given period. Finally, there is the case where the initial price (interest) decline creates expectations of further price (interest) declines and causes the expenditure function to drop even further downwards.\footnote{Cf. "Price Flexibility," \S\ 11.} In this case suppliers are clearly in an extremely weak strategic position: all their attempts to extricate themselves from the unemployment situation will perversely plunge them ever deeper into it. Corresponding to each of the above cases, the unemployment will be defined as permanent, temporary or prolonged, according to the strategic position of the suppliers.\footnote{Ibid., p. 564.}

There is one further (and, perhaps, even more fundamental) sense in which we can speak of suppliers as being in a weak strategic position. Assume that we start out with the incompatibility of Fig. 3, but that by a series of price declines we are finally able to reach the situation of Fig. 5. We can conceive of two opposite ways in which this dynamic adjustment might take place. The first is the one that has been assumed up to now in this section. The income level starts at $Y_0$ (cf. Fig. 3); as the price level falls, the desired-expenditure function rises, and for each period of time the actually prevailing income is determined by the intersection of this function with the 45° line, until finally the income level $\eta$ is reached. At the other extreme we might consider the income starting out at $\eta$ and remaining there all the time the expenditure function is moving upwards. In other words, under the first method the prevailing income is always at a level desired by demanders, throughout the adjustment process; while under the second method it is always at the level desired by suppliers.

In our economy it is easy to think of suppliers selling less than they desire; but it is difficult to conceive of demanders buying...
more than they desire. Hence, when we start off from a situation such as that of Fig. 3, it is the first type of adjustment that takes place. Thus, throughout the period of adjustment, even if income finally succeeds in reaching \( \eta \), demanders are obtaining an income level they desire, while suppliers are not. This is another very real sense in which suppliers are in a weak strategic position relative to demanders.

10. Until now we have dealt with involuntary unemployment; but using exactly the same concepts of \$8 we can also define involuntary over-employment. Consider the case where the desired expenditure function is \( E = W(Y) \) as in Fig. 7. This situation implies that despite price increases suppliers are unable to bring the expenditure function down to the point where it intersects the 45\(^{\circ} \) line at \( \eta \). In brief, the level of national income desired by spenders (\( Y_\eta \)) is greater than that desired by suppliers. If the level of national income is actually \( Y_\eta \), then a measure of the extent to which suppliers are over-employed is the negative quantity

\[
U = \eta - Y_\eta
\]

That is, involuntary over-employment is measured by the difference between the level of national income in the norm of reference, and the level actually prevailing. Clearly, the same concept applies when suppliers do succeed in bringing the expenditure function down somewhat, so that an intermediate level of national income—between \( \eta \) and \( Y_\eta \)—prevails.
A situation of over-employment may well have existed during the War. Here the Government provided almost an unlimited demand for goods, which was not diminished by higher prices. Then it resorted to patriotic appeals to persuade the supplier to produce more than they really desired. It might be argued that this patriotic appeal caused an upward shift in the supply function itself, so that suppliers were "really not" involuntarily over-employed. The danger in this type of argument is that eventually it will define away the whole concept of involuntary action. It leads to such nonsense statements as: a man held up at the point of a gun "voluntarily" gives up his wallet because he "desires" to save his life! This example simply points up the necessity of stating a norm of reference (arbitrary as it may be) whenever we wish to speak of involuntary actions. In our economic norm suppliers are presumed to be acting like "economic men," completely devoid of any nationalistic motivations. Hence, when they are influenced by patriotic appeals, they can properly be said to be acting involuntarily.

Another way in which suppliers can meet a situation of over-employment is by rationing. That is, if price increases prove ineffectual in reducing the expenditure function sufficiently, suppliers may decide that nevertheless they will produce only the output they desire, and allocate it among consumers on some arbitrary rationing basis (first come first served, fixed percentage of purchases in previous years, etc.). In this case it is the spenders who are forced into involuntary actions: they must buy less than they actually desire.

The elimination of peacetime involuntary over-employment seems to be a simpler task than the corresponding elimination of involuntary unemployment. (Once again the reader is reminded that we are abstracting here from all monopolistic forces in the economy.) This follows from the supposition that price rises are a much more effective means of shifting the expenditure function down, than price declines of shifting it up. There is no inconsistency here, as in many other places in economic theory, there is no reason to expect symmetry of reactions. Thus, under conditions of over-employment the strategic position of suppliers, in the first sense of § 9, is stronger than that under unemployment. The suppliers' strategic position is also stronger in the second sense; for the income level may remain at $\gamma$ during the whole period of adjustment in which the expenditure function is being forced down. For example, suppliers may resort to rationing during this whole period.
IV. Keynes and the Classics

11. From the perspective of the preceding analysis it is now possible to examine, and contrast, the assumptions of the Keynesian and classical positions. Consider first the conventional Keynesian analysis of §§ 2–3. Explicitly, the supply function is not introduced at all. Implicitly, it is assumed that under unemployment conditions suppliers are in a weak strategic position, in both of the senses of § 9. First of all, spenders respond little, if at all, to price-level and interest-rate reductions. Secondly, the actually prevailing level of income is always determined by the intersection of the expenditure function and the 45° line. Hence, there is no need to introduce the supply function, since the prevailing level of income is determined by demand factors alone. This level of income is then compared with an arbitrarily selected level, designated as the full-employment income, and the difference used as a measure of unemployment. The advantage of the preceding argument is that it makes this element of arbitrariness unnecessary: the full employment level of income is defined by the same analytical apparatus which determines the actually prevailing level of income.

In addition, we differ from the usual Keynesian analysis in saying that the income level need not remain at the original intersection of the expenditure function with the 45° line. True, for each period of time the level of income is determined by the intersection of the expenditure function with the 45° line. But this level increases over time as the expenditure function is pushed upwards.

It is important to understand the dynamic theory implicit in this interpretation of the Keynesian theory. Essentially we divide the economy into two markets: one for finished goods and services, and the other for productive services. From the preceding paragraph it is clear that we assume that equilibrium is rapidly restored in the first market. In other words, if a disturbance should suddenly shift the expenditure function downwards, the level of income would quickly fall to the new intersection point with the 45° line. All this means is that undesired inventories are rapidly eliminated from the system (cf. § 3). Thus no stimulus for any further movement in prices comes from the finished-goods market. Nevertheless, the system continues with its dynamic

---

1 Actually, Keynes never considered the effect of the price level on the expenditure function; but, as I have argued elsewhere ("Price Flexibility," pp. 563–4), it does not seem too difficult to extrapolate his position on this matter.

2 In the formulation of this paragraph I have benefited from discussions with Kenneth J. Arrow of the Cowles Commission for Research in Economics.
adjustment due to the fact that the market for productive services is not in equilibrium. By analogy, one might say that the involuntary unemployment in this market represents "undesired inventories of productive services." In any event, the presence of unemployed productive services drives the price level down as long as equilibrium is not re-established in this market too.

12. In the classical position, involuntary unemployment could not arise in this way. For a basic assumption of that position was Say's law; and under this assumption full employment was always assured.

![](image)

Fig. 8.

The meaning of Say's law is that regardless of the level of income, people desire to spend their entire incomes; or, what is the same thing, people will not use their money incomes to add to their cash holdings. In the words of J. S. Mill, in his chapter on Say's law: ¹

Could we suddenly double the productive powers of the country, we should double the supply of commodities in every market; but we should, by the same stroke, double the purchasing power. Everybody would bring a double demand as well as a supply: everybody would be able to [and, presumably, would] buy twice as much, because everyone would have twice as much to offer in exchange.

What does this mean as far as the shape of the aggregate expenditure function is concerned? Under Say's law the expenditure function must always coincide with the 45° line, as in Fig. 8. This

coinciding is the graphical counterpart of the statement that at every level of income people want to spend their entire income. Analytically this means that the expenditure function has the form \( E = Y \). Under this assumption it is clear that there exists only one equilibrium level of the national income, a level jointly desired by both spenders and suppliers. Therefore, in the classical system full employment is always established; the income of the community is limited only by supply factors. Thus, Say's law, far from precipitating the economy into a state of unstable equilibrium (as is sometimes assumed), instead removes a possible inconsistency from the system, and insures the rapid achievement of a unique, stable, full-employment equilibrium.

13. Keynes, of course, violently disagreed with the assumption of Say's law. This is the sum and substance of his liquidity-preference theory: Out of any given income, people may have a net desire to add to their cash balances. Hence, the expenditure function need not coincide with the 45° line; and hence, the level of income established by the intersection of the 45° line and the expenditure function need not be the full-employment level. Thus Keynes denied the basic proposition of classical economics: namely, that the economic system would automatically generate full employment.

It should, however, be emphasised that the classical position as revised by Pigou no longer needs Say's law. In this system automatic full employment is brought about by interest, wage and price flexibility regardless of the form of the expenditure function. Correspondingly, as I have argued elsewhere, the Keynesian attack shifts its concentration from Say's law to the dynamic instability of our economic system—its inability to restore full employment within a reasonable time after being subjected to a shock of one type or another.

Actually it can be said that everyone simultaneously accepts and rejects Say's law: rejects it, in the sense that no one believes the short-run-expenditure function must have the form it has in Fig. 8; accepts it, in the sense that everyone recognises that in the long run people want goods, and not money.

V. THE ARGUMENT GENERALISED

14. There have been many occasions in this paper where the analysis has pressed hard against the confines of our oversimplified model. Essentially, the trouble is that the model explicitly pro-

---

2 Ibid., § 14.
vides for only one price level (finished goods), while implicit in the analysis is that of yet another (productive services). This is one reason why the aggregate supply function appears like such a monstrosity. Actually, it should depend on both of these prices; however, our oversimplified assumption that these prices are proportionate (cf. § 5, above) forces us to write it as dependent on neither. Nevertheless, it was claimed in § 5 that the general analysis developed on the basis of this oversimplified model could be readily extended to more realistic ones. This and the following sections attempt to make good this claim.

First, we shall briefly sketch the way in which the concepts of this article appear in a somewhat more extended model. Let \( Y \) = national income, \( r \) = rate of interest, \( p \) = absolute price level, \( E \) = aggregate demand for finished goods, \( N^p \) = demand for labor, \( N^s \) = supply of labor, \( M^p \) = demand for cash balances and \( M^s \) = supply of cash balances. It must be emphasised that these last five variables represent desired quantities demanded or supplied—in the sense used in § 2 and throughout this paper. Consider now the following system:

1. \( E = \phi (Y, r, p) \)
2. \( Y = \phi (N^p) \)
3. \( E = Y \)
4. \( N^p = f(w/p) \)
5. \( N^s = g(w, p) \)
6. \( N^p = N^s \)
7. \( M^p = L(Y, r, p) \)
8. \( M^s = \text{const.} \)
9. \( M^p = M^s \)

This model divides up the economy into three markets: finished goods, labor and money.\(^1\) For each market there are two behavior equations and one equilibrium condition. Starting from the last triplet of equations, we have in (7) the desired-demand for cash balances (i.e., the liquidity preference equation); in (8), the desired-supply of cash balances (assumed for simplicity to be a constant); and, in (9), the condition that the money market is not in equilibrium unless desired supply and demand for money are equal. The next triplet furnishes corresponding information for the labor market. Here we have followed Keynes in assuming that the

\(^1\) Implicit in the model is yet a fourth market: that for bonds. This can be ignored since, by "Walras' law," it is considered here as residual. Cf. D. Patinkin, "The Indeterminacy of Absolute Prices in Classical Economic Theory," Econometrica, XVII (1949), §§ 6 and 14.
supply of labor depends on money, and not real, wages; but this is not a necessary part of the argument which follows. The demand for labor is assumed to depend on real wages; this is not essential to the argument either.

In the first triplet, the first and third equations are quite familiar. The only difference between equation (1) and the expenditure function used throughout this paper is that (1) provides explicitly for the possible influence of the rate of interest. Equation (3) is the equilibrium condition for the finished-goods market, as explained in § 3. Equation (2) is the only new-comer: this is the production function. In this model the production function is completely interchangeable with the aggregate supply function. This follows from the relationship between equations (2) and (4). If suppliers of finished goods are faced with a given real wage rate, the amount of labor they will purchase is determined by (4). If we insert the resulting labor input in (2), we get the output of finished goods that suppliers will provide for this labor input, i.e., at the designated real wage. In other words, by substituting (4) into (2) we come out with the familiar aggregate supply function. It differs from the one used in this paper in that it depends on the real wage rate, now no longer (cf. § 5) assumed to be constant.

The use of the production function in (2) instead of the equivalent aggregate supply function was deliberate. There is no doubt that the former function is a much more familiar and accepted tool of economic analysis. In fact, one can find in the literature several examples of models very similar to the one above. Consequently, these models have implicit in them the aggregate supply function developed in this paper. My only objection to these other presentations is that they make no satisfactory interpretation of the supply function; in particular, they fail to see the relationship between it and involuntary unemployment. This is the problem we shall now examine.

Assume, for simplicity, that our model is consistent: that is, it has a solution. Let the solution values for income and employment be \( Y = \eta \) and \( N^u = N^s = \sigma \), respectively. Then, by definition of the functions of our model, these are the full-employment values of the respective variables. Assume now that a sudden disturbance in the economy causes a downward shift in the ex-

---

1 For simplicity we are assuming the amount of capital to be held constant. Hence the production function depends only on the rate of labor input, \( N^u \).


3 The implications of an inconsistent model are discussed below, § 10. On this whole paragraph see in particular § 11, above.
penditure function. This sets up a whole chain of dynamic events. During this process there is no reason why any of the equilibrium conditions—(3), (6) and (9)—should be satisfied. We assume, however, that equilibrium in the finished goods and money markets is quickly re-established. The only pressure for continued movements of the variables comes from the failure to satisfy (6). (Cf. above, § 11.) Correspondingly, as long as this equilibrium condition is unsatisfied, the level of employment is less than \( \sigma \). Hence, by definition, there is involuntary unemployment within the system. (Cf. above, § 8.) The level of this unemployment will continue to fluctuate as the system tries to correct the disequilibrium in the labor market. In some cases it is possible that the system will finally succeed in restoring a full-employment equilibrium.

15. The preceding section extended the analysis by applying it to a model with more equations. Another method of extension is to go back to the maximising behavior from which the equations themselves are derived. In this way it is possible to make the concept of involuntary action even more vivid.

Consider, for example, the demand function of an individual for a certain commodity. Assume that the norm is given by the condition that he must stay within his budget. Then inability to be on his demand curve (for example, the commodity may be rationed by the government) means, in mathematical terms, that besides being subject to the (normal) budget restraint when maximising utility, the individual is also subject to at least one additional restraint or side condition (viz., that arising from the rationing). This additional restraint is an indication of the extent to which he must act involuntarily. On the other hand, an individual who maximises his utility subject only to the (normal) budget restraint (say, an individual exempt from the rationing regulations), is said to be acting freely and fully satisfying his desires. This interpretation can readily be generalised to any type of maximising activity.

16. So far we have dealt with particular extensions. Let us now see how the basic concepts presented in this article can be, extended to a perfectly general system.

Consider a model with \( n \) equations. For the moment, assume that the model is a static one. Its \( n \) equations will be of various types: some will be behavior equations for various sectors of the economy, some equilibrium conditions and some definitions. Assume that the behavior equations represent behavior under the restrictions of the norm, in other words, assume that they represent the desires of the respective sectors, in the sense used in
this paper. Mathematically, there are two possibilities: the static system may be consistent, or it may be inconsistent. To say that the system is mathematically inconsistent is equivalent to saying that the desires of people, as reflected by this system, are incompatible: they cannot all be satisfied simultaneously. Let us examine in detail the implications of this proposition.

If the system is inconsistent, then there exists no point (i.e., no set of prices, quantities, incomes, etc.) which will simultaneously satisfy all the equations of the system. In other words, no matter at what point the economy may be, some of its behavior equations cannot be satisfied; that means, no matter what happens some people must be off their (desired) behavior curves, and hence must be acting involuntarily. From this it follows that the system can never be in equilibrium; for no matter at what point it is, it cannot remain there: the members of the economy left dissatisfied at that point will try to achieve their desired behavior, and thereby move the system away from whatever point it happens to be at.

An example of such an inconsistent static system is provided by Fig. 3—under the assumption that nothing can be done to move any of the curves presented in it. As pointed out in the discussion of this figure (§ 6), no matter at what point the system happens to be—that is, no matter what the income and the price level (for by the preceding sentence we are assuming that the expenditure function is completely insensitive to the price level)—it is impossible for both demanders and suppliers simultaneously to fulfill their desires; at least one of them must be forced into involuntary action. Correspondingly, the system can never be at equilibrium.¹

Now assume that the static system is consistent. Does that mean that everyone in the society will always fulfill his desires? In order to answer this question we must turn to dynamic analysis. The general proposition that can be made is: The existence of a consistent equilibrium position for the static system is a necessary, but not a sufficient condition for the elimination of involuntary action within the economy.² In other words, the argument of the preceding paragraph has shown that the system cannot be free of involuntary action unless it is consistent; but that does not mean that once it is consistent we can be sure of the elimination of involuntary action. For example, assume that the system does have the consistent solution represented in Fig. 5. Assume further

---

¹ Another example of an inconsistent static system is provided by Fig. 3 and § 3 in "Price Flexibility."
that due to a certain disturbance in the system the expenditure function falls to its position in Fig. 3. This sets up price-level and interest-rate declines, and we now assume that these declines shift the expenditure function upwards. But in certain cases it may be that, due to dynamic expectation factors, no matter how far the price level and interest rate falls, it is impossible to shift the expenditure function back to its position in Fig. 6.¹ Under these assumptions we may continue to have involuntary action within the system for an indefinitely long period.

But we need not go to such an extreme case—again, a mathematical, not necessarily a realistic, extreme. Assume now that by price-level and interest-rate declines the expenditure function is eventually brought back to its position in Fig. 5. But all this takes time; and during this period of movement and adjustment some individuals must be off their behaviour curves; that is, they must be forced into involuntary action.² Here, too, we can define our concept of strategic position. The strategic position of any behavior group is defined in terms of how much it must give up its desires, for what period of time. The one that can stay "closest" to its desired behavior curve during the period of adjustment is the "strongest."

From the framework just described, we can appreciate the major significance of the classical position. It is, in brief, that the behavior equations are so sensitive to price and interest changes that the market will automatically and quickly establish a position in which everyone's desires are satisfied. That is, the market will make the desires of people consistent. Correspondingly, we can appreciate the nature of the Keynesian argument: its denial of the efficacy, and even existence, of these delicate balancing operations; and its insistence that the end result of leaving the market to its automatic functioning must inevitably be the frustration of desires somewhere in the system.

Don Patinkin

Hebrew University, Jerusalem.

¹ This particular argument is developed in much greater detail in "Price Flexibility," §12.
² Cf. above, §9; "Price Flexibility," §14.