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CAPITAL ACCUMULATION AND THE END OF PROSPERITY*

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A. The Problem

Over the business cycle, investment plays a double role: on the one hand income generated by it raises the level of total national income and employment and creates prosperity; on the other, the resulting rapid accumulation of capital uses up existing investment outlets and eventually causes a breakdown and a depression.

This is the essence of several business-cycle theories developed by a number of economists, such as Marx, Hobson, more recently Keynes, Hansen, Kalecki, Kaldor, Harrod, Paul Sweezy, and others.[1] The purpose of this paper is to examine some of these theories by means of a few simple economic models and to develop some of their ideas somewhat further. Owing to limitation of space, we shall devote most of our attention to the logic of the argument, its economic implications having been published elsewhere.[2]

The discussion will refer to a private capitalist society in which the government plays a minor part. Therefore we can disregard the difference between national income and national product, the tax collections being very small. It is also assumed that the general price level is kept constant.

Notations: Y = national income; C = consumption; S = saving (including corporate saving); I = investment; P = productive capacity of

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[1] Marx's writings are well known; J. A. Hobson, *Economics of Unemployment* (London, 1922), and *Rationalization and Unemployment* (New York, 1930); J. M. Keynes, *The General Theory of Employment, Interest and Money* (New York, 1936); A. H. Hansen, *Fiscal Policy and Business Cycles* (New York, 1941), and *Economic Policy and Full Employment* (New York, 1947); Michal Kalecki, *Essays in the Theory of Economic Fluctuations* (New York, 1939), and "Full Employment by Stimulating Private Investment?," *Oxford Economic Papers*, No. 7 (March, 1945), pp. 83–92; Nicholas Kaldor, "Stability and Full Employment," *Economic Journal*, Vol. 48 (December, 1938), pp. 642–57; R. F. Harrod, *The Trade Cycle* (Oxford, 1936); P. M. Sweezy, *The Theory of Capitalist Development* (New York, 1942).

[2] See my papers "Capital Expansion, Rate of Growth, and Employment," *Econometrica*, Vol. 14 (1946), pp. 137-147; "Expansion and Employment," *American Economic Review*, Vol. 37 (1947), pp. 34-35; and particularly "The Problem of Capital Accumulation," *American Economic Review*, Vol. 39 (1948), pp. 777-794.

the economy at full employment; K = capital; E = exogenous causes; t = time; s, a, α, β, r = constants. Y, C, S, I, P, s are in net terms (over and above depreciation) per unit of time.

B. A Simplified Keynesian System[3]

(1.1) $C + S = Y$,
(1.2) $S = F(Y)$,
(1.3) $I = G(Y, E)$,
(1.4) $I = S$,
(1.5) $Y = P$ (P constant).

The stock of capital (as an integral of investment) does not enter Keynes's system explicitly. The possibility of a breakdown or of chronic unemployment is due to the contradiction between the value of Y given by the first four equations and the requirement that income equal productive capacity expressed in (1.5). It is a static system, and in the form presented here cannot be used as an explanation of the cycle.

C. A Simplified Hobson-Marx System

This system represents an attempt to express the theories of these two writers by an explicit model. Such an undertaking is always hazardous, particularly in the case of Marx whose work is so complex. There is no doubt that an alternative system, or even several alternative systems, could be derived from his writings with equal accuracy.

Capital accumulation, absent in Keynes's formal system, appears here explicitly, but its role is still not entirely clear. So long as the stock of capital remains below some vaguely defined "critical" magnitude K_n, the system takes the following form:

CASE I. For $K < K_n$:
(2.1a) $C + S = Y$,
(2.2a) $S = F(Y)$,
(2.3a) No investment function,
(2.4a) $I \equiv S$,
(2.5a) $Y = P$ (P constant),
(2.6a) $\dfrac{dK}{dt} = I$.

This is essentially a restatement of Say's Law. The saving-investment problem, so prominent in Keynesian literature, is entirely absent here. Investment and saving are identically equal, and full employment is automatically maintained.

[3] The interest-rate and money equations are omitted because they are not essential for our purposes.

As soon, however, as K reaches its critical magnitude K_n, the profitability of further additions to capital stock (*i.e.*, investment) drops radically, and prosperity abruptly ends. We have then the second case.

CASE II. For $K \geqslant K_n$:

(2.1*b*) $C + S = Y$,
(2.2*b*) $S = F(Y)$,
(2.3*b*) $I = a$,
(2.4*b*) $I = S$,
(2.5*b*) $Y = P$ (P constant),

$$(2.6b) \quad \frac{dK}{dt} = I,$$

where a is some small constant. The system is overdetermined; the magnitude of I given by (2.3*b*) is presumably much too small to yield the Y required by (2.5*b*), and a depression comes in. The two cases could be combined by making $I = \Phi(K)$ such that $I \equiv S$ for $K < K_n$, and $I = a$ for $K \geqslant K_n$. The essence of the Hobson-Marx system lies in the idea that capital accumulation has a strong negative effect on profitability of investment, an idea which is accepted, explicitly or otherwise, by Keynes and his followers, as well as by some non-Keynesians. But the character and magnitude of K_n requires further exploration.

D. An Expanded System

This system contains some of the characteristics of the preceding ones; it emphasizes the relation between capital and productive capacity.

I. No Contradictions

(3.1) $C + S = Y$,
(3.2) $S = F(Y)$,
(3.3) No investment function,
(3.4) $I \equiv S$,
(3.5) $Y = P$,

$$(3.6) \quad \frac{dK}{dt} = I,$$

(3.7) $P = H(K, s)$,

where s is some constant to be discussed presently. We should also note that P is no longer constant.

The system is not overdetermined and is capable of a solution. If we take the simplest possible case[4]

[4] It would be more general and realistic to take $S = \beta + \alpha Y$ ($\beta \leqslant 0$). The particular assumption that $\beta = 0$ simplifies the argument to such an extent, as to make this assumption worthwhile even at the expense of generality.

(3.2) $S = \alpha Y \quad (0 < \alpha < 1)$, and

(3.7) $P = Ks$,

where s is the ratio between productive capacity and capital required by existing technological and other conditions — a concept similar to that (or rather to its inverse) used in connection with the acceleration principle, the solution takes the form of

(3.8) $Y = Y_0 e^{\alpha s t}$.

II. Possible Contradictions

(a) If the investment function is reintroduced as

(3.3) $I = G(Y, E)$

and (3.4) is changed to $I = S$, we have the familiar Keynesian case. The latter, however, would not make use of all possibilities of this system.

(b) A more interesting case arises when (3.7) is made subject to the restriction that

(3.9) $P = Ks$ only for $\alpha s \leqslant r$,

where r is the maximum rate of growth the economy can achieve.

If $\alpha s = r$, then, with investment and saving functions being identical, the economy is in equilibrium in the sense that not only full employment (of labor) is preserved, but that no excess accumulation of capital develops as well. If we introduce an additional assumption that capital income (profits and interest) remains a constant fraction of national income, then (with s remaining constant), it can be shown that the average yield from capital remains unchanged in spite of continuous capital accumulation. As far as our system is concerned, this state of affairs can continue indefinitely. On the other hand, if the magnitude of s gradually diminishes (the so-called "deepening" of capital), full employment and full utilization of capital are maintained, but the profitability of investment will gradually fall off. It appears to me, however, that in the absence of additional assumptions, such as lags, the cyclical significance of this gradually diminishing rate of profit should not be great.

The restriction (3.9) implies that the required rate of growth αs may not be achieved owing to limited quantities, or more exactly, owing to insufficient growth of factors of production other than capital; of these labor is probably the most important. If, in the presence of this restriction, national income grows at the maximum rate r, full employment of *labor* is maintained by definition of r, but a part of capital stock becomes idle. If we define $\sigma = dP/dK$, then $\sigma < s$. This case has interesting cyclical implications. If (3.4) becomes an equation, and

if in addition (3.3) is changed to

(3.10) $I = \theta(Y, E, \gamma)$,

where $\gamma = s/\sigma$ (or possibly $\gamma = s - \sigma$) and naturally $\theta_\gamma < 0$, the system acquires a strong cumulative force. If for some reason (not necessarily even due to labor shortage) income fails to grow at the αs rate, idle capital develops (γ increases). This reduces the profitability of investment; its rate of growth slackens (or becomes even negative), and the corresponding fall in the rate of growth of income makes an additional part of the capital stock idle. This in turn depresses the profitability of investment, and so on. A dynamic model along these lines could easily be constructed, though it should be indicated that it could only be used as a partial explanation of the downturn (there may be other causes as well), and, as presented here, does not explain the upturn at all.

It should be made clear that the presence of idle capital observed in our economy so often does not prove by itself that the restriction (3.9) has been operative. Idle capital will develop whenever income fails to grow at the αs rate, irrespective of the cause. There is nothing in the investment function used here or in the one that could be obtained empirically, that indicates that investment and income will grow at the required rate. Thus besides the obvious hypothesis that the αs rate is impossible physically, another one can be suggested—that even in the absence of this limitation the required growth will not materialize owing to institutional factors, or more precisely, to the peculiarities of the investment process in a capitalist society. An empirical verification of the two hypotheses is highly desirable.

III. Possible Objections

(a) Rejection of the whole approach on the grounds that the parametric treatment of s is illegitimate. The existence of a relatively rigid ratio (or even a reasonably small range) between output and the stock of capital needed for its production is by no means an established fact, though it has been assumed in economic literature many a time. The rejection of s will deprive our system of most of its cyclical significance, because a slowly declining rate of profit resulting from $d^2Y/dK^2 < 0$ will hardly exert strong depressing effects over some five or seven years, and few, if any, of our prosperities lasted longer. Yet there are reasons to believe that the relation of capital stock to its productive capacity over relatively short periods is less flexible than it is usually supposed in the traditional theory. The current investigations of Professor W. W. Leontief of Harvard should shed more light on this problem.

(b) Idle capital is not likely to be distributed evenly over all industries and firms. Its presence in some sectors of the economy need not deter others from expanding their capital facilities.

(c) If exogenous factors, such as development of new methods of production or new products, changes in tastes and habits, population growth, aggressive competition, etc., play an important role in the investment function, the past accumulation of capital, idle or otherwise, will have little effect on profitability of investment. Professor J. A. S c h u m p e t e r is likely to take this point of view.

The processes described in (a) and (b) may involve heavy capital losses. And if the economy is to continue expanding in spite of them, it is necessary that those on whom they are inflicted be unable to prevent the construction of new capital. The institutional structure of the economy, the presence of powerful business combinations and financial interconnections between firms is of utmost importance here.

IV. Policy

The purpose of policy here is to bridge the gap between the required rate of growth of income and the actual one.

(a) If (3.9) is effective, there are three possibilities:

i. Reduction of α.

ii. Reduction of s by developing industries requiring large capital outlays per unit of output.

iii. A continued rise in the general price level (excluded from consideration here).

(b) If (3.9) is not effective, *i.e.*, if the required rate of growth is physically possible, there are other possibilities in addition to those listed in (a). They consist in various methods of encouraging private investment, such as low interest rates, incentive taxation, liberal loss offsets for income-tax purposes, etc. A guaranteed growth of income as a method of creating investment opportunities should be explored in this connection. And there is no inherent reason why public investment should not play a more important role as well.