THE USE OF ECONOMETRIC MODELS AS A GUIDE TO ECONOMIC POLICY*

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It is desirable to provide tools of analysis suited for public economic policy that are, as much as possible, independent of the personal judgments of a particular investigator. Econometric models are put forth in this scientific spirit, because these models, if fully developed and properly used, eventually should lead all investigators to the same conclusions, independent of their personal whims. The usual experience in the field of economic policy is that there are about as many types of advice as there are advisors (sometimes even more!).

Statistical models of the working of the economy are not proposed as magic formulas which divulge all the secrets of the complex real world in a single equation. The statistical models attempt to provide as much information about future or other unknown phenomena as can be gleaned from the historical records of observable and measurable facts. To the extent to which people maintain their past behavior patterns in the future, the statistical models provide information about the quantitative properties of economic variables in the future. However, econometricians do not operate in a vacuum; their methods are not purely mechanical in the sense that they do nothing but substitute in formulas. Any information of a qualitative nature that is available should be used by the econometrician in drawing inferences about the real world from his models. For example, suppose that an econometrician is called upon to forecast next year's level of employment and suppose further that this econometrician knows that war will break out next year. Would the econometrician merely substitute into his equations of peacetime behavior patterns in order to forecast employment in a period during which there will be war? Obviously, any qualitative information (e.g., the outbreak of war next year) must be taken into account in order to make a proper forecast.

The nonstatistical economist has only qualitative information from which to make judgments. The statistical economist has this same qualitative information plus a thorough knowledge of historically developed behavior patterns; hence it may be said that the latter is better equipped.

* It must be emphasized that the forecasts for fiscal year 1947 in this paper were all made during the week of November 10, 1946.

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TYPES OF POLICIES

The ideas that grew out of the discussion on the Full Employment Bill showed clearly the close relationship between forecasting and economic policy. As the Bill was originally drafted, it called for a periodic forecast of the deflationary or inflationary gap. A predicted deflationary gap would call for one type of policy and a predicted inflationary gap for another type of policy. Thus the first step in carrying out the provisions of the Bill is to make a forecast. The success of the public policy will depend vitally on the accuracy of the forecast. An important use of econometric models, as will be demonstrated below, is to make forecasts.

The second step in the implementation of a Full Employment Bill is to wipe out the forecasted deflationary or inflationary gap. This step will also have to be quantitative. How much employment will be created by an x-per-cent cut in taxes? By how much will prices be expected to rise if government expenditures rise by a known amount with constant tax rates? These are only a sample of the types of questions that must be answered in order to decide among alternative policies. It is evident that the answers to such questions depend upon consumer and business spending-saving habits. The statistical approach is to examine the spending-saving habits of past periods in order to get some idea of the pattern of future habits.

Suppose that the population's habits are going to change in a known way. We may want to know the effect of this change upon the entire system. If we have econometric models, we can often predict the results of such changes. An example can easily be given. If it is known that the introduction of a social security program will raise the marginal propensity to consume by y per cent because people will be more certain of the future, the quantitative effects of the program can be estimated in advance from statistical models. The estimate may very well have some influence on the decision whether or not to adopt the social-security scheme.

A similar use of models arises in the study of the effects of technological change. A change in the technique of production can often be translated into an exact quantitative change in some of the parameters of the production function that has been statistically determined from data which referred to the old process of production. It is possible to calculate the change in several relevant variables of the system such as employment, output, wages, prices, etc., as a result of the technological change, provided we have an appropriate econometric model. On the basis of the limited number of observations available for testing different economic models from which to form policy decisions, it is not yet possible to select an unique model. More than one model are
consistent with the observations. In this paper we shall present three plausible models, and methods of forming policy will be studied with each alternative. The reader is free to choose among the models, all of which rest on different hypotheses. Other models, in addition to those presented here, have also been studied by the author, but they are not demonstrated in order to avoid repetition.

**MODEL I—EXOGENOUS INVESTMENT**

A simple model in which there is little possibility of confusion is useful to demonstrate some very specific applications of econometric models in policy formation.

The following variables comprise the system:

\[ C = \text{consumer expenditures measured in billions of current dollars.} \]

\[ G^* = \text{gross private capital formation measured in billions of current dollars.} \]

\[ G = \text{government expenditures on goods and services measured in billions of current dollars.} \]

\[ Y = \text{disposable income measured in billions of current dollars.} \]

\[ GNP = \text{gross national product measured in billions of current dollars (expenditure concept).}^1 \]

\[ \Sigma = \text{government receipts + corporate savings + business reserves} - \text{transfer payments} - \text{inventory profits, all measured in billions of current dollars.} \]

\[ p = \text{cost-of-living index, 1935–1939: 1.00.} \]

\[ N = \text{population of the continental United States measured in billions of persons.} \]

\[ u = \text{normally distributed random disturbance.} \]

The economic model connecting these variables is:

\[ \frac{C}{pN} = \alpha_0 + \alpha_1 \frac{Y}{pN} + \alpha_2 \left( \frac{Y}{pN} \right)^{-1} + u, \]  \hspace{1cm} \text{(1.1)}

\[ GNP = C + G^* + G, \]  \hspace{1cm} \text{(1.2)}

\[ Y + \Sigma = GNP. \]  \hspace{1cm} \text{(1.3)}

Equation (1.1) is an economic-behavior equation which relates consumer spending to income, current and past. Since this is an equation

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1. Gross national product may be computed directly as the sum of consumer, business, and government spending (expenditure concept) or as the sum of income payments to factors of production and indirect taxes plus business reserves (income concept). The two measures should be equal but there is now a sizeable discrepancy of about $5 billion in the official estimates. The symbol $\text{GNP}$ will refer everywhere in this paper to the expenditure concept.
of economic behavior, it is subject to random disturbance. Equations (1.2) and (1.3) are definitions. They hold exactly and are not subject to random disturbance.

The endogenous variables are $\mathcal{C}/pN$, $\mathcal{G}N\mathcal{V}/pN$, and $\mathcal{T}/pN$. The exogenous variables are $\mathcal{I}/pN$, $\mathcal{G}/pN$, $\mathcal{C}/pN$. There is some question whether the truly exogenous variables of capital formation, government spending, and taxation, should be aggregates in current dollars or per capita variables in constant dollars. For the statistical estimation of the parameters of the system we shall proceed as though $\mathcal{I}/pN$, $\mathcal{G}/pN$, and $\mathcal{C}/pN$ are the exogenous variables rather than $\mathcal{I}$, $\mathcal{G}$, and $\mathcal{C}$, although the most correct solution is far from obvious.

Haavelmo\[^{1}\] has shown that the system (1.1)–(1.3) may be solved to get

\begin{equation}
\frac{\mathcal{T}}{pN} = \frac{\alpha_0}{1-\alpha_1} + \frac{\alpha_2}{1-\alpha_1} \left( \frac{\mathcal{T}}{pN} \right)_{-1} + \frac{1}{1-\alpha_1} \left( \frac{\mathcal{I} + \mathcal{G} - \mathcal{C}}{pN} \right)_{-1} + \frac{1}{1-\alpha_1} u,
\end{equation}

and that the parameters of this latter equation can be consistently estimated by the method of least squares if we make the assumptions that we have made about endogenous and exogenous variables. It will be observed that a knowledge of the estimates of the parameters of (1.4) leads to a knowledge of the estimates of the parameters of (1.1).

Statistical estimates of (1.4) yield the results

\begin{equation}
\frac{\mathcal{T}}{pN} = 202.54 + 0.37 \left( \frac{\mathcal{T}}{pN} \right)_{-1} + 2.39 \frac{\mathcal{I} + \mathcal{G} - \mathcal{C}}{pN},
\end{equation}

\begin{equation}
S = 821.21, \quad \delta^2/S^2 = 1.14.
\end{equation}

The numbers in parentheses below the coefficients are standard errors; $S = $ square root of the estimate of the variance of $u/(1-\alpha_1)$; $\delta^2/S^2$ = the ratio of the mean-square successive difference to the variance of the residuals. The distribution of this ratio has been tabulated,\[^{2}\] and we conclude that the probability is slightly less than 3 per cent that we could get a sample value for $\delta^2/S^2$ as small as 1.14 if the population values of the $u$'s were independent in time (i.e., nonautocorrelated).

The sample value of $\delta^2/S^2$ is slightly less than the size usually required (the value corresponding to the 5-per-cent significance level) in order


to be confident that the disturbances are random. The statistical methods assume that the disturbances are nonautocorrelated, and this assumption is not quite fulfilled in (1.5), although this is clearly a borderline case.

The numerical values of (1.5) imply the following numerical results for an estimate of (1.1):

\[
(1.6) \quad \frac{\bar{G}}{p_N} = 84.74 + 0.58 \frac{\bar{Y}}{p_N} + 0.15 \left( \frac{\bar{Y}}{p_N} \right). 
\]

Provided the relevant information concerning the size of \( G', G, \) \( T \) is known in advance, equation (1.5) can be used to make forecasts. We generally know, in advance, the size of \( G \) and \( T. \) Government expenditures are fixed by the budgets that are adopted by federal, state, and local governments; hence we can assign a definite value of \( G. \) It is not possible to assign an unique value of \( T \) because the observed value of \( T \) depends upon the level of income. We estimated the parameters of (1.4) as though \( T \) were an exogenous variable but there is an error committed in this approach. \( T, \) in fact, is a function of \( Y \) or \( GNP, \) and the parameters of this function are the exogenous elements. One of the major elements of \( T \) is taxes. Taxes vary with income, but the government autonomously sets the tax rates. The parameters of the function connecting \( T \) and \( GNP \) are averages of these tax rates. In practice, we proceed as follows: We assume several hypothetical values of \( GNP \) and estimate according to existing laws on taxes, unemployment compensation, etc., the corresponding values of \( T. \) We then determine from these hypothetical values of \( T \) and \( GNP \) a relation of the form:

\[
(1.7) \quad T = \beta_0 + \beta_1 (GNP).
\]

This could also be written as:

\[
GNP - Y = \beta_0 + \beta_1 (GNP), \\
Y = - \beta_0 + (1 - \beta_1) (GNP),
\]

or

\[
T = \beta_0 + \beta_1 (Y + T), \\
\frac{\beta_0}{1 - \beta_1} + \frac{\beta_1}{1 - \beta_1} Y,
\]

since \( GNP = Y + T. \) It is assumed that equation (1.7) is exact and not subject to a random disturbance.

\* See pp. 119–120 below for a more detailed discussion.
Since population (N) changes very slowly, it is not difficult to assign a numerical value to this variable for a few months or a year in advance. The remaining variables that are necessary in order to forecast \( \Upsilon \) or \( Gd/p \) are \( \Upsilon_{-1}, p_{-1}, N_{-1}, p, \) and \( \Upsilon' \). The lagged variables are known from historically recorded observations, but the other two variables are not known in advance with the same certainty that we know \( G, \Upsilon, \) or \( N \) in advance. Government agencies survey business firms in order to find out what the latter intend to spend on plant, equipment, and inventories. In this way, there is an attempt to assign a known value to \( \Upsilon' \), but such attempts are not entirely satisfactory because businessmen are in no way committed to spend what they intend to spend. At best, we can solve the system of equations for \( \Upsilon \), given \( G, \beta_0, \beta_1, N, p_{-1}, \Upsilon_{-1}, N_{-1} \), in terms of \( p \) and \( \Upsilon' \). For the fiscal year 1947 we assign the values:

\[
\begin{align*}
G & = 32.8 \text{ billion}, \\
\Upsilon & = -39.52 + 0.61 \Upsilon', \\
\Upsilon_{-1} & = 138.7 \text{ billion}, \\
p_{-1} & = 1.30, \\
N_{-1} & = 0.140.
\end{align*}
\]

Substituting these numerical values into (1.5), we get:

\[
(1.8) \quad \Upsilon = 70.32 + 27.51p + 0.97 \Upsilon'.
\]

For any pair of values corresponding to \( p \) and \( \Upsilon' \), there results a definite forecast of \( \Upsilon \). However, an error must be attached to this forecast. The quantity \( u/(1-\alpha_1) \) in (1.4) fluctuates about its mean (zero) and thereby causes the observed value of \( \Upsilon/pN \) to deviate from the value calculated from (1.5). On top of this error caused by the disturbance \( u/(1-\alpha_2) \), there is another error resulting from the fact that we do not know the exact values of the parameters, \( \alpha_0, \alpha_1, \alpha_2 \). The numerical (point) estimates given in (1.5) are subject to error, and only a range is known which encloses these parameters with a specified probability. A combination of the errors in the estimates of the parameters \( \alpha_0, \alpha_1, \alpha_2 \) and the disturbances \( u \) provides a range of error for any forecasts from this model.

Hotelling\(^4\) has given a very clear exposition of the theory underlying errors in prediction. Hotelling’s formula applied to our problem gives:

\[
(1.9) \quad \text{estimate of variance of forecast} = S^2 + S_{11} \left( \frac{\Upsilon}{pN} \right)_{-1}^2 + 2S_{12} \left( \frac{\Upsilon}{pN} \right)_{-1} \frac{\Upsilon'}{pN} + \left( \frac{G - \Upsilon}{pN} \right)_{-1}^2
\]

\[ + S_{22} \left( \frac{\mathbf{G}^* - \mathbf{G} - \mathbf{C}}{pN} \right)^2 + S_{22} \frac{1}{20}, \]

where

\[ S_{11} = \text{estimate of the variance of } \frac{\alpha_2}{1 - \alpha_1}, \]

\[ S_{12} = \text{estimate of the covariance of } \frac{\alpha_2}{1 - \alpha_1} \text{ with } \frac{1}{1 - \alpha_1}, \]

\[ S_{22} = \text{estimate of the variance of } \frac{1}{1 - \alpha_1}, \]

and the * sign denotes deviations from the sample mean. The numerical values of the estimated variance of forecast depend upon the values assigned to the predetermined and exogenous variables.

In carrying out the computations for (1.9), the value used for \( \mathbf{C} \) was the value obtained by substituting the point estimate of \( \mathbf{Y} \) into \( \mathbf{C} = -39.52 + 0.61Y \). However, since there is an interval of error to be attached to \( \mathbf{Y} \), it is not obvious that the correct value of \( \mathbf{C} \) for (1.9) can be estimated from the point estimates of \( \mathbf{Y} \). But it happens that the error committed by this procedure is negligible. This can be seen as follows: (1) Solve for the forecast error as a function of \( \mathbf{C} \). (2) Solve for \( \mathbf{C} \) as a function of the point estimate of \( \mathbf{Y} \) and the error attached to the point estimate of \( \mathbf{Y} \). (3) Substitute \( \mathbf{C} \) from step 2 into (1.9). This gives one equation in one unknown variable, namely, the error of forecast. (4) The error attached to \( \mathbf{C} \) may be positive or negative. Choose that sign which gives the largest possible range of forecast error and solve the equation from step 3 for the error of forecast. We find that the error of forecast obtained by computing \( \mathbf{C} \) from the point estimate of \( \mathbf{Y} \) alone is not different (in billions of dollars) from the worst possible error obtained by taking account of an error in \( \mathbf{C} \).

We shall denote the expression in (1.9) by \( S^*_p \). Probability theory tells us that there is somewhat less\(^6\) than a 70-per-cent chance that the true value of \( \frac{\mathbf{Y}}{pN} \) will be in the range

\[ \left[ -S^*_p + \left( \frac{\mathbf{Y}}{pN} \right)^0, \left( \frac{\mathbf{Y}}{pN} \right)^6 + S^*_p \right], \]

where \( \left( \frac{\mathbf{Y}}{pN} \right)^0 \) is the value of \( \frac{\mathbf{Y}}{pN} \) forecast from (1.5). If we want a

\(^6\) The value of \( t \) for the 30-per-cent significance level and 17 degrees of freedom is 1.069. The significance level corresponding to \( t = 1 \) is slightly larger than 30 per cent but is not given in all tables.
larger probability of being correct, we must widen the range of forecast. In general, our forecast will be of the form:

\[
\left( \frac{\gamma}{p_n} \right)^{\alpha} \pm t_{\alpha} S_F,
\]

where \( t_{\alpha} \) is taken from the table of the \( t \)-distribution at the \( \alpha \)th significance level.

The ranges of error in Table I refer to errors in the forecast of \( \gamma \) and \( \mathcal{G}N\mathcal{P} \) in current prices. Forecasts in constant dollars lead to much smaller ranges because the forecast equation (1.5) is in constant dollars, and when we multiply both sides of this equation by \( p \) to get forecasts in current prices, the forecast error is also multiplied by \( p \), which is greater than unity during the present period. It is evident that the percentage error is the same for forecasts of disposable income in current or constant dollars. It should be pointed out, further, that the ranges of error are calculated from (1.10) under the condition that \( t_{\alpha} = 1 \).

**Table I**

**Forecasts for Fiscal 1947: Model I**

<table>
<thead>
<tr>
<th>Disposable Income (( \gamma ))</th>
<th>Gross Capital Formation (( \mathcal{F} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Index ( p )</td>
<td>30</td>
</tr>
<tr>
<td>1.35</td>
<td>137 ± 6</td>
</tr>
<tr>
<td>1.40</td>
<td>138 ± 7</td>
</tr>
<tr>
<td>1.45</td>
<td>139 ± 7</td>
</tr>
<tr>
<td>1.50</td>
<td>141 ± 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gross National Product (( \mathcal{G}N\mathcal{P} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Index ( p )</td>
</tr>
<tr>
<td>1.35</td>
</tr>
<tr>
<td>1.40</td>
</tr>
<tr>
<td>1.45</td>
</tr>
<tr>
<td>1.50</td>
</tr>
</tbody>
</table>
The forecasts presented thus far refer to fiscal year 1947, but, at this writing (November, 1946) part of this fiscal year has already elapsed. If we subtract what has already been observed for fiscal 1947 from the values in the tables, we have a forecast of the remainder of fiscal 1947, an unobserved period.

Table I tells us that the present price level (1.43–1.45) and volume of capital formation (32–34) imply a value of $G\text{NP}$ for fiscal 1947 at about the present annual rate. If prices and private capital formation do not fall, the current rate of national product will be maintained, i.e., the national product will not fall or rise appreciably in the first half of calendar 1947. If price and capital formation fall, $G\text{NP}$ is likely to fall in calendar 1947 and if prices and capital formation rise, $G\text{NP}$ is likely to rise in calendar 1947. Later we shall attempt to forecast the behavior of capital formation in a model where this variable is endogenous.

It is easy to see how other types of policy decisions could be made on the basis of this model. Various levels of government spending can be inserted into the numerical system. Each value of $G$ will lead to a different set of values for $\gamma$ and $G\text{NP}$, hence we can judge the quantitative effects of government spending on income and national product. Similarly it is possible to calculate the effects of changes in tax rates and by altering the numerical estimates of $\beta_0$ or $\beta_1$ depending upon the type of tax change under consideration. A different set of values for $\beta_0$ and $\beta_1$ will generate a different set of values of forecasted $\gamma$ and $G\text{NP}$.

The change in the forecast as a consequence of the change in the tax laws, gives an evaluation of some of the important implications of the different tax policies.

There are many steps involved in the computation of (1.7) and it is not entirely correct to assume that this equation is not subject to random error. The computation of the values of the parameters involves some approximations and the linear form of the equation is, in itself, an approximation; hence it would be desirable to attach an error to the estimate of (1.7), but this has not yet been done. We shall outline the steps involved in estimating the parameters of (1.7) in order that the various sources of error can be properly exposed:

1. Assume an arbitrary value of $G\text{NP}$.
2. Subtract from $G\text{NP}$, government interest and wage-salary payments, to get privately produced $G\text{NP}$.
3. Estimate corporate profits before taxes as a function of private $G\text{NP}$.
4. Apply existing tax rates to the estimated corporate profits before taxes to get corporate taxes.
5. From corporate profits after taxes estimate the distribution between dividends and corporate savings.

6. For the hypothetical level of \( \mathcal{GNP} \) and existing tax rates for other business taxes, estimate the volume of other business taxes (not corporate taxes).

7. For the hypothetical level of \( \mathcal{GNP} \), estimate the total business reserves (depreciation, depletion, capital outlays charged to current expense).

8. For the hypothetical level of \( \mathcal{GNP} \), estimate the profit or loss on inventory revaluation.

9. From \( \mathcal{GNP} \) subtract corporate taxes, other business taxes, business reserves, and the negative of inventory profits. The result is a hypothetical level of national income corresponding to a hypothetical \( \mathcal{GNP} \).

10. Calculate transfer payments estimated at existing benefit rates for the assumed level of national income.

11. Compute the contributions to social insurance funds by applying existing rates to the hypothetical national income.

12. To national income add transfer payments and subtract the sum of corporate savings and contributions to social insurance funds. The result is income payments to individuals.

13. Apply existing personal tax rates to income payments to get personal tax and nontax payments to government.

14. Subtract personal tax and nontax payments from income payments to get disposable income.

It is obvious that all tax payments (personal, corporate, and other business) depend upon the distribution of income among taxing units as well as upon the sum of individual incomes. However, assuming little change in income distribution, it is possible to estimate approximately the tax revenues corresponding to any particular level of income. But it should be noted that this is an approximation and not an exact relation. However, there are other steps at which the approximations are less exact and subject to much wider errors. Steps 3 and 5 involve some estimates on which there undoubtedly could be improvement. Step 3 makes use of a behavior equation which should be part of the economic model and which should be subject to random disturbance. From past data, the regression of corporate profits on private \( \mathcal{GNP} \) is obtained, and this regression is used to estimate corporate profits corresponding to the assumed hypothetical levels of total and private \( \mathcal{GNP} \). The use of simple least-squares correlations between corporate profits and private \( \mathcal{GNP} \) can fortunately be justified, although many investigators have been using this correlation in the steps outlined above without knowing why it is correct. The models
which the author has constructed, have often contained the wage-bill as a linear function of the value of private output. The best relation also makes use of a trend and lagged as well as current output. It has been found in several cases that the least-squares regression of wages on private output is practically the same as the relation obtained by the use of more satisfactory methods of statistical estimation that are known to be consistent (i.e., roughly unbiased in large samples). Since the sum of wages and profits is approximately equal to the value of private output, it is also true that the least-squares regression of profits on output will not differ appreciably from a consistent estimate of the true relation between these two variables. But no matter how correct we find the statistical estimates to be, the equation used in step 3 is an equation of economic behavior and is subject to random disturbance.

Step 5 also involves the use of an economic-behavior equation, and unfortunately this equation is not nearly as well established as that used in step 3. In step 5, it is necessary to split net corporate profits after taxes into two components, dividends and corporate savings. How do boards of directors decide upon their dividend policies? No simple economic theory seems to have been adequately developed to explain the behavior pattern of corporate directors. Tinbergen\(^7\) has advanced a relation that is consistent with the observed data. He found that dividends are a linear function of net corporate income, current and lagged, and the accumulated surplus lagged. Until a theory of dividend distribution is more definitively established we shall have to work with Tinbergen's relation, but here, also, the relation is subject to error (probably a wide error), and consequently another inexactness is introduced into the model. In making up the table of the relationship between disposable income and assumed levels of \(GNP\), we should insert the latest observed figure of dividends corresponding to the observed levels of \(GNP\) and net corporate profits, and then from Tinbergen's formula calculate the change in the absolute level of dividends associated with any assumed change in the absolute level of net corporate profits. This procedure should be adopted because the lagged value of surplus (the other variable in Tinbergen's equation) is predetermined for our problem and does not vary with the different hypothetical levels of \(GNP\).

Finally, step 8 involves some very questionable relationships. Ordinarily, inventory revaluation might not be large, but in periods of rapidly changing prices, the capital gains (or losses) on stocks of goods

are large and play an important role in determining the level of effective demand. Inventory profits are especially large now, and cannot be neglected. For any hypothetical level of $GNP$, a figure for inventory revaluation must be estimated. Any estimates are subject to a large error. No systematic relationship has been used for the evaluation of this item other than some informed guesses as to future developments in the labor situation that will serve to determine wage rates and thus prices (assuming a fixed profit margin).

Forecasts from econometric models undoubtedly can be greatly improved, and one of the first points of attacks should be on the relationship between disposable income and $GNP$. In the estimation of the parameters of the models, it has usually been assumed that the difference, $GNP - Y = \xi$, is exogenous, but the above discussion has shown this to be an incorrect assumption. At best, we can assume that tax rates, unemployment compensation rates, pensions, government interest payments, and the like are exogenous. Further work is called for here.

**MODEL II—EXOGENOUS INVESTMENT AND LIQUIDITY**

Let us modify Model I by the addition of one new variable to the system, namely cash balances. One of the most frequently-heard explanations of the present boom is that private economic units have emerged from the War with large accumulations of liquid assets that cause people to spend at an abnormally high rate. This hypothesis can be tested from the data to see whether or not the total private holdings of liquid assets influenced spending habits in the past. Since we could not split total private holdings of cash balances for all past years into personal holdings and business holdings, we used the total private holdings of cash balances as the appropriate variable. For the pre-war years in which total balances have been split into private and business categories, there is a close linear relationship between the two components, so it seems that the total can be used as an index of either type in a linear system. The consumption function will now be written as

\[
\frac{C}{pN} = \alpha_0' + \alpha_1' \frac{Y}{pN} + \alpha_2' \left( \frac{Y}{pN} \right)_{-1} + \alpha_3' \left( \frac{M}{pN} \right)_{-1} + u',
\]

where $M_{-1} =$ currency outside banks + demand deposits adjusted + time deposits of the middle of the preceding year measured in billions of current dollars.

All the other equations of the system are unchanged. The forecast equation which is obtained by solving for \( \frac{\Upsilon}{pN} \) in terms of predetermined and exogenous variables is:

\[
\frac{\Upsilon}{pN} = \frac{\alpha_0'}{1 - \alpha_1'} + \frac{\alpha_2'}{1 - \alpha_1'} \left( \frac{\Upsilon}{pN} \right)_{t-1} + \frac{1}{1 - \alpha_1'} \frac{\Upsilon' + G - \Upsilon}{pN} + \frac{a_3'}{1 - a_1'} \left( \frac{\Upsilon}{pN} \right)_{t-1} u'.
\]  

(2.2)

The statistical estimate of (2.2) is:

\[
\frac{\Upsilon}{pN} = 186.53 + 0.30 \left( \frac{\Upsilon}{pN} \right)_{t-1} + 2.36 \frac{\Upsilon' + G - \Upsilon}{pN} + 0.13 \left( \frac{\Upsilon}{pN} \right)_{t-1},
\]

(0.13)  
(0.34)  
(0.10)

\[S = 20.69, \quad \frac{\hat{\sigma}^2}{S^2} = 1.28.\]

The statistic \( \hat{\sigma}^2/S^2 \), in this model, is large enough so that we cannot reject the hypothesis that the \( u \)'s are nonautocorrelated at the 5-percent level of significance.

From the estimates of the parameters of (2.2), we can derive estimates of the parameters of (2.1). They are:

\[
\frac{\Upsilon}{pN} = 79.04 + 0.58 \frac{\Upsilon}{pN} + 0.13 \left( \frac{\Upsilon}{pN} \right)_{t-1} + 0.06 \left( \frac{\Upsilon}{pN} \right)_{t-1},
\]

(2.4)

If to the estimates of the predetermined and exogenous variables used in Model I for fiscal 1947, we add the estimate \( \Upsilon_{t-1} = \$175.4 \) billion, the resulting forecast equation,

\[
\Upsilon = 69.96 + 31.13p + 0.97\Upsilon',
\]

(2.5)
is obtained. Corresponding to Table I used in conjunction with Model I, we now have Table II which gives the forecasts obtained from Model II. In spite of the fact that the coefficient of \( \frac{\Upsilon}{pN} \) is small in (2.3), the value of this variable is so high now that the forecasted GNP is raised considerably as compared with Model I. But the standard error of the coefficient of \( \frac{\Upsilon}{pN} \) is very large. As a matter of fact, the standard error of the coefficient of \( \frac{\Upsilon}{pN} \) in (2.3) is sufficiently large that we cannot reject the hypothesis that the true value of the parameter is zero. The past data do not contradict the hypothesis that spending habits are not influenced by the holding of liquid assets. In estimating the error of forecast, however, it is not sufficient to look at the estimated standard error of the coefficient of \( \frac{\Upsilon}{pN} \) without taking into consideration the covariance of this coefficient and the other coefficients of the forecast equation. The ap-
TABLE II  
Forecasts for Fiscal 1947: Model II  
Disposable Income (Y)  

<table>
<thead>
<tr>
<th>Price Index p</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>38</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>141 ±7</td>
<td>143 ±7</td>
<td>145 ±7</td>
<td>147 ±7</td>
<td>149 ±6</td>
<td>151 ±6</td>
</tr>
<tr>
<td>1.40</td>
<td>143 ±8</td>
<td>145 ±7</td>
<td>147 ±7</td>
<td>148 ±7</td>
<td>150 ±7</td>
<td>152 ±7</td>
</tr>
<tr>
<td>1.45</td>
<td>144 ±9</td>
<td>146 ±9</td>
<td>148 ±8</td>
<td>150 ±8</td>
<td>152 ±8</td>
<td>154 ±8</td>
</tr>
<tr>
<td>1.50</td>
<td>146 ±10</td>
<td>148 ±9</td>
<td>150 ±9</td>
<td>152 ±9</td>
<td>154 ±9</td>
<td>155 ±8</td>
</tr>
</tbody>
</table>

Gross National Product (GNP)  

<table>
<thead>
<tr>
<th>Price Index p</th>
<th>30</th>
<th>32</th>
<th>34</th>
<th>36</th>
<th>38</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>188 ±11</td>
<td>191 ±11</td>
<td>194 ±11</td>
<td>197 ±11</td>
<td>200 ±11</td>
<td>203 ±10</td>
</tr>
<tr>
<td>1.40</td>
<td>190 ±13</td>
<td>193 ±11</td>
<td>196 ±11</td>
<td>199 ±11</td>
<td>203 ±11</td>
<td>206 ±11</td>
</tr>
<tr>
<td>1.45</td>
<td>193 ±15</td>
<td>196 ±15</td>
<td>199 ±13</td>
<td>202 ±13</td>
<td>205 ±13</td>
<td>208 ±13</td>
</tr>
<tr>
<td>1.50</td>
<td>195 ±16</td>
<td>198 ±15</td>
<td>201 ±15</td>
<td>204 ±15</td>
<td>208 ±15</td>
<td>211 ±13</td>
</tr>
</tbody>
</table>

Application of Hotelling’s formula for the estimate of the variance of the forecast explicitly takes into account the covariances as well as the variances of the parameters. The range of error attached to the forecasts of Model II is larger than the corresponding range for Model I, but the former range is not as much larger as one might hastily conclude from looking at the variance alone. A 70-per-cent probability of being correct requires a range of ±$13 to $15 billion for forecasts of GNP in Model II; whereas the range for the same probability level in Model I is about ±$11 billion.

Equation (2.3) might be used to answer another type of question that is important in policy formation. In the presence of unemployment what is the effect, on the level of income, of deficit spending with a constant real money supply, or of an increase in the real money supply with a constant deficit? To simplify the problem, let us try to answer these questions for the equilibrium situation, i.e., the situation in which
all variables assume their equilibrium values. In equilibrium we have:

\[
\frac{\gamma}{pN} = \left( \frac{\gamma}{pN} \right)_0 = \left( \frac{\gamma}{pN} \right)_{-1} = \left( \frac{\gamma}{pN} \right)_{-1} = \left( \frac{\gamma}{pN} \right)_0,
\]

and

\[
\frac{\delta' + \tilde{G} - \bar{C}}{pN} = \left( \frac{\delta' + \tilde{G} - \bar{C}}{pN} \right)_{-1} = \left( \frac{\delta' + \tilde{G} - \bar{C}}{pN} \right)_0.
\]

The equilibrium solution for (2.3) becomes:

\[
(2.6) \quad \left( \frac{\gamma}{pN} \right)_0 = 266.87 + 3.37 \left( \frac{\delta' + \tilde{G} - \bar{C}}{pN} \right)_0 + 0.19 \left( \frac{\bar{M}}{pN} \right)_0,
\]

assuming the point estimates for the parameters to be correct. From (2.6) we can calculate the following two multipliers: (1) An extra dollar of deficit spending (with a constant money supply) creates $3.37 additional disposable income; (2) an extra dollar of money supplied (with a constant deficit) creates $0.19 additional disposable income.

**MODEL III—ENDOGENOUS INVESTMENT**

The forecasts made from the first two models were, of course, contingent upon the correct choice of exogenous variables, of which some of the principal ones were government spending, government receipts and transfers, and private capital formation. There is little that can be done to improve upon the assumed levels of government variables for purposes of prediction, but we may be able to make considerable improvement in the estimation of private capital formation. If the latter variable is assumed to be exogenous, our only known method for the estimation of this variable is to survey businessmen and ask them what they intend to spend in the future on capital goods. However, if we assume that business decisions are endogenous variables, we can study the historical records to attempt to discover the laws of business behavior. Model III will be one in which definite behavior patterns are assumed for many more variables in the system than was the case for Models I and II. In this paper, we shall not attempt to present a theoretical justification for all the equations of Model III, but we shall write explicitly all the equations. In a subsequent publication, there will be a lengthy justification for the model on theoretical grounds. At present, we shall merely attempt to show how such models can be used in forecasting.
In forecasting, especially under the conditions of November, 1946, certain equations of the complete model are suppressed because of government controls or other reasons. The full set of equations will be presented and the suppressions will be pointed out for the applications.

The notation is as follows:

$I = \text{net investment in private producers' plant and equipment, measured in billions of 1934 dollars.}$

$q = \text{price index of private producer's plant and equipment, 1934: 1.00.}$

$p = \text{price index of output as a whole, 1934:1.00.}$

$X = \text{output of the private sector of the economy, exclusive of housing services, measured in billions of 1934 dollars.}$

$\xi = \text{excise taxes, measured in billions of current dollars.}$

$K = \text{end-of-year stock of private producers' plant and equipment, measured in billions of 1934 dollars.}$

$H = \text{end-of-year stock of inventories, measured in billions of 1934 dollars.}$

$\tilde{W}_t = \text{private wage-salary bill, measured in billions of current dollars.}$

$Y = \text{disposable income, measured in billions of 1934 dollars.}$

$C = \text{consumer expenditures, measured in billions of 1934 dollars.}$

$D_t = \text{gross construction expenditures on owner-occupied, single-family, nonfarm residences, measured in billions of 1934 dollars.}$

$r = \text{index of rents, 1934:1.00.}$

$q_t = \text{index of construction costs, 1934:1.00.}$

$\Delta F = \text{thousands of new nonfarm families.}$

$D_e = \text{gross construction expenditures on rented, nonfarm residences, measured in billions of 1934 dollars.}$

$i = \text{average corporate-bond yield.}$

$\nu = \text{percentage of nonfarm housing units occupied at the end of the year.}$

$N^* = \text{millions of available nonfarm housing units at the end of the year.}$

$M_1 = \text{demand deposits adjusted and currency outside banks averaged during the year, measured in billions of current dollars.}$

$M_t = \text{time deposits, averaged during the year, measured in billions of current dollars.}$

$E_r = \text{excess reserves, averaged during the year, measured in millions of current dollars.}$

$T = \text{government revenues + corporate savings - transfer payments - government interest payments - inventory profits, all measured in billions of 1934 dollars.}$

* In what follows, we shall adopt the convention, $\Delta x = x - x_{-1}$ for any variable $x.$
\( G = \) government expenditures on goods and services—government interest payments + net exports + net investment of nonprofit institutions, all measured in billions of 1934 dollars.
\( D_t = \) gross construction expenditures on farm residences, measured in billions of 1934 dollars.
\( D'' = \) depreciation of all residences (farm and nonfarm) measured in billions of 1934 dollars.
\( W_t = \) government wage-salary bill, measured in billions of current dollars.
\( R_t = \) nonfarm rentals, paid and imputed, measured in billions of current dollars.
\( R_t = \) farm rentals, paid and imputed, measured in billions of current dollars.
\( u_t = \) estimate of the disturbances in the inventory-demand equation.

The equations are:
\[
I = 2.18 + 0.13 \frac{pX - \bar{E}}{q} + 0.04 \left( \frac{pX - \bar{E}}{q} \right)_{-1} - 0.09 K_{-1},
\]
(3.1) \( S = 0.49, \quad \delta^2 / S^2 = 1.50 \)
(demand for private producers' plant and equipment);
\[
H = 0.79 + 4.17p + 0.11(X - \Delta H) + 0.50H_{-1},
\]
(3.2) \( S = 0.61, \quad \delta^2 / S^2 = 2.17 \)
(demand for inventories);
\[
\Delta X = 2.99 - 4.25(u_t)_{-1} + 75.09\Delta p,
\]
(3.3) \( S = 5.98, \quad \delta^2 / S^2 = 1.69 \)
(output adjustment equation);
\[
W_t = 5.03 + 0.42(pX - \bar{E}) + 0.17(pX - \bar{E})_{-1} + 0.17(t - 1931),
\]
(3.4) \( S = 1.00, \quad \delta^2 / S^2 = 1.86 \)
(equation of the demand for labor);
\[
D_t = -9.03 + 3.74 \left( \frac{r}{q} \right)_{-1} + 0.02(Y + Y_{-1} + Y_{-2}) + 0.0043\Delta F
\]
(3.5) \( S = 0.21, \quad \delta^2 / S^2 = 2.26 \)
(demand for new owner-occupied nonfarm residences);
\[ D_t = -2.14 + 2.81 r_{t-1} + 0.02(q_t)_{t-1} - 0.44(q_t)_{t-2} + 0.0016(\Delta P)_{t-1} - 0.18 \delta; \]

(3.6)

\[ S = 0.26, \quad \delta^2/S^2 = 2.07 \]

(demand for new rented nonfarm residences);

\[ v = 178.01 + 0.29 Y - 2.62 r + 1.42(t - 1931) - 3.76 N; \]

(3.7)

\[ S = 0.79, \quad \delta^2/S^2 = 1.52 \]

(demand for nonfarm dwelling space);

\[ \Delta r = -2.15 + 0.02 v_{t-1} + 0.00071 Y + 0.17 \frac{1}{r_{t-1}} \]

(3.8)

\[ S = 0.03, \quad \delta^2/S^2 = 1.04 \]

(rent adjustment equation);

\[ C = 11.87 + 0.73 Y + 0.04(t - 1931), \]

(3.9)

\[ S = 1.36, \quad \delta^2/S^2 = 1.20 \]

(demand for consumer goods);

\[ \mathcal{M}_1 = 8.45 + 0.24 p(Y + T) + 0.03 p(Y + T)(t - 1931) \]

\[ - 1.43(t - 1931), \]

(3.10)

\[ S = 1.20, \quad \delta^2/S^2 = 1.33 \]

(demand for active cash balances);

\[ \mathcal{M}_2 = 15.37 + 0.28 i - 1.90 i_{t-1} + 0.74(\mathcal{M}_2)_{t-1} - 0.18(t - 1931), \]

(3.11)

\[ S = 0.67, \quad \delta^2/S^2 = 1.49 \]

(demand for idle cash balances);

\[ \Delta i = 2.00 - 0.17 \mathcal{E} - 0.37 i_{t-1} - 0.0052(t - 1931), \]

(3.12)

\[ S = 0.47, \quad \delta^2/S^2 = 1.77 \]

(interest-rate adjustment equation);

\[ Y + T = I + \Delta H + C + D_t + D_t + D_t - D'' + G \]

(definition of net national product);
\[ X = \frac{p(Y + T) - \mathcal{W}_1 - R_1 - R_3}{p} \]

(definition of privately produced output exclusive of housing services);

\[ \Delta K = I \]

(definition of net investment);

\[ R_3 = 0.278 \left( \frac{\nu N^*}{100} + \frac{v_{-1}N^*_{-1}}{100} \right) \frac{1}{2} \]

(definition of total nonfarm rent payments, paid and imputed).

All parameters have been estimated for the period 1922–1941, except for (3.6), (3.10), (3.11), (3.12), when the period is 1921–1941. The method of statistical estimation employed is called the “method of reduced forms” and is one of the methods suited for obtaining consistent estimates of parameters in systems of simultaneous economic equations. This method has been developed by T. W. Anderson, Jr., M. A. Girshick, and H. Rubin.\(^{16}\)

It should be pointed out that the parameter 0.278 in equation (3.16) is known a priori. This equation is a definition and is not a behavior equation with unknown parameters to be estimated from the observed data. Since \( r \) is an index number while \( R_3 \) is in billions of dollars, it is necessary to multiply the right-hand side of (3.16) by the base-year value of average rental payments.

The forecasts for fiscal 1947 will be made from equations (3.1), (3.2), (3.5), (3.6), (3.9), (3.13), and (3.14). Equation (3.3), if left in the system, would give us enough equations to determine the general level of prices, \( p \), but this equation is not very well established as can be seen by the very large estimate of the variance of the disturbance. We can probably do best to assign a particular value to the price level which will be determined by other considerations, as will be explained later. Equation (3.4) is omitted because we have no direct need for the de-

termination of $\mathcal{W}_1$ in the system. However, if it were required to forecast $\mathcal{W}_1$ as well as $p(Y+T)$, we should have to include (3.4). Implicitly, an equation like (3.4) will be used to determine the relation between $T$ and $Y$ for the step in which it is required to estimate corporate profits for each assumed level of $G\times P$. This estimate corresponds to step 3 on page 119 above. Equations (3.7) and (3.8) are suppressed for this calculation because the present level of rents is not determined in a free market as is assumed in these two equations. The specific assumptions pertaining to rent control will be stated below. Equations (3.10) and (3.11) are omitted because it is not desired to know the values of $M_1$ and $M_2$ for our forecasts. If we were interested in forecasting the demand for cash balances as well as the level of income and output we should need to make use of demand relations for cash, but for the present problem we can omit some of the equations referring to the money market. Equation (3.11), furthermore, does not fit the postwar data, and would be very poor for purposes of prediction. In the postwar situation, for observed values of $i$, $i_{-1}$, $(M_2)_{-1}$, and $t$, the demand for idle balances, as estimated by (3.11), is much less than the observed value. There are several possible explanations for this phenomenon, but one tempting explanation is the following: The Keynesian hypothesis that the demand for idle balances has very large (practically infinite) interest-elasticity at low interest rates requires a highly nonlinear form for the liquidity-preference equation (3.11). In the prewar years, the linear equation fits the data well, but now we are getting so close to "minimum" interest rates that the equation must be made very nonlinear in the region of present observations. There are several nonlinear functions that would fit the prewar data as well as (3.11) does and also fit the postwar observations very closely. Other explanations have to do with shifts (permanent or temporary) in the function, but, in any case, equation (3.11) is not satisfactory for prediction as it now stands. Fortunately, we are not attempting to estimate the demand for cash, so that (3.11) may be disregarded for the calculation in this paper. Finally, we shall also omit equation (3.12), which serves to determine the interest rate as a function of exogenous or predetermined variables. This equation also does not fit the postwar facts very closely and we can probably do better to assign the present rate of bond yields in equation (3.6), rather than attempt to estimate it from (3.12). During the war, the government and the central banking system made various manipulations in the money market to control the rate of interest, which may account for the fact that equation (3.12) appears not to hold very closely now.

Model III is an improvement over Models I and II in the sense that
more variables that are not known accurately in advance are made endogenous and can be estimated from the model, but Model III is still far from being complete. The most glaring deficiency is in equations to determine the various price levels. To a certain extent, this deficiency is a result of the fact that there are not sufficiently detailed data available to construct, from past observations, a model that contains certain market adjustment equations necessary for the determination of price levels. Equation (3.8) is an example of the type of equation that is needed to determine a specific price level.

Instead of using behavior equations of the mathematical model to determine the levels of various prices, we have made use of certain information outside the model in order to assign specific values to \( p, q, q_l, r \), etc. Our basic assumption in quantitative terms is that average prices (except rent) in fiscal 1947 will be about 10 per cent above those observed for fiscal 1946. Our reasoning is that prices will probably not fall by very much, if at all, in the latter part of the fiscal year because basic wage rates will not be permitted to fall. The latter observation follows from personal judgments (generally accepted) that collective bargaining in the early part of calendar 1947 will bring about some basic wage increases. Regardless of the level of prices assumed (within reason) the model leads to deflationary forecasts, i.e., falling production and employment, in the early part of calendar 1947; therefore we do not assume runaway price increases. However, the big increases of prices that occurred since the removal of OPA in the first half of fiscal 1947 will not be wiped out by the events of the second half of the fiscal year. There will probably be some price increase, on the average, for the fiscal year. In line with past experience since the end of the war when there have been similar rounds of wage and price increases, we assume a 10-per-cent change. In Tables I and II, it can be seen by how much small variations in the price index lead to variations in the forecasts of national product. This method of assuming a side relation between prices and wages, where the latter are estimated by guesses as to the outcome of collective bargaining, is not satisfactory, and work must be done in order to improve the model by the introduction of more price-formation equations.

In Table III are presented all the values assigned to predetermined or exogenous variables. In the appendix, the reader will find a detailed description of all the time series used in the estimation of the parameters of Model III and all the steps involved in the calculation of the values inserted in Table III. It is essential to study carefully the steps involved in the construction of the series in order to know how to calculate the values of Table III.
It is not possible to assign a specific value to the variable $T$ because this variable will change with changing income. From our knowledge of existing tax rates, unemployment compensation rates, etc., we obtain by the same methods that were used for (1.7):

$$T = -\frac{29.475}{p} + 0.385 (Y + T).$$

A word of explanation is required about (3.17). The variable $T$ is defined above as government revenues + corporate savings - transfer payments - government interest payments - inventory profits. Actually in the determination of the parameters from the data of the inter-
war period, inventory profits or losses were not subtracted from \( T \), although, perhaps, they should have been. For the prediction in fiscal 1947 it does not seem correct to neglect inventory profits because they are so large. The consumption function (3.9) probably understates rather than overstates the consumption corresponding to a particular level of disposable income. The inclusion of inventory profits in \( Y \) (their exclusion in \( T \)) helps to explain the high levels of consumption; therefore Model III is interpreted as including inventory profits in \( Y \), although this item was neglected in the past. Except for a couple of years in the interwar period, inventory profits were sufficiently small so that they would not have had an important influence on the numerical estimates of the parameters.\(^{11}\)

Equations (3.1), (3.2), (3.5), (3.6), (3.9), (3.13), (3.14), (3.17) and the assumptions in Table III, lead to the following results:

\[
Y = \$ 83.3 \text{ billion},
\]

\[
Y + T = 104.5,
\]

\[
p(Y + T) = 162.0,
\]

\[
G^\text{NP} = 177.5.\(^{12,13}\)
\]

The error to be attached to this forecast is not known exactly. In Models I and II, the statistical methods employed enabled us to determine the errors in forecasting that result from the variation of the disturbance and from the sampling fluctuations involved in the determination of the parameters of the forecasting equation, but we are not able to calculate both types of error in Model III. In the latter model we can calculate the error resulting from the variations in the disturbances but we do not have estimates of the other types of error,

\(^{11}\) There is no error in this procedure if the marginal propensity to consume out of inventory profits is the same as the marginal propensity to consume out of other disposable income. If the two marginal propensities differ, the correct estimate for the model should be a weighted average of the two. The weight attached to inventory profits for the past observation will be small if inventory profits were a small percentage of disposable income, as was the case. The error is undoubtedly negligible.

\(^{12}\) There are definite correction factors that lead from the variable \( p(Y + T) \) to the official government figures on \( G^\text{NP} \). The exact series used for \( p(Y + T) \) are given in the appendix so that the reader can determine exactly the relationship between this variable and the official figures of \( G^\text{NP} \). If we add government interest + depreciation and depletion − net imputed rents on owner-occupied nonfarm residences to \( p(Y + T) \) we get \( G^\text{NP} \).

\(^{13}\) It appears now (April, 1947) that the assumption of 10-per-cent price increases is a low estimate. If the assumptions are modified by allowing a greater price increase, say 15 or 20 per cent, the forecasted level of \( G^\text{NP} \) and the range of error will be raised.
i.e., that type which results from the fact that we have only a finite sample to determine the parameters of the model. The size of the first type of error, however, gives a rough guide as to the accuracy of the results. The estimated square root of the variance of the disturbances involved in forecasting disposable income (constant dollars) is about $3$ billion, the same as for Models I and II. If the sampling fluctuations lead to an error of the same relative size in all models, we can say that the estimates of $\mathcal{GNP}$ from Model III should lie in a range $\pm$ $13$ billion with a 70-per-cent probability.

It is instructive to examine the past record of Model III. In the spring of 1946, a list of exogenous and predetermined variables was prepared for the calendar year 1946 corresponding to the values given in Table III for fiscal 1946. At the time of the actual forecast, "reduced form" estimates of the parameters of Model III were not available; hence single-equation, least-squares estimates were used to make the forecasts. The estimated parameters obtained by the latter method are not appreciably different in this case from those obtained by the former method.

We shall now present a table of the assumptions actually used in the forecasts made in the spring of 1946 and substitute them into Model III. The comparison between actual and forecasted results will then show how well the prewar model fits the postwar data. The relation between $T$ and $Y$ that was used is given by

\begin{equation}
T = - \frac{42.4003}{p} + 0.440(Y + T). \tag{3.18}
\end{equation}

The values in Table IV and (3.18) may not be the same as observed values.

Equations (3.1), (3.2), (3.5), (3.6), (3.9), (3.13), (3.14), (3.18), and the assumptions lead to the following results

\begin{align*}
Y &= \$ 95.8 \text{ billion}, \\
Y + T &= 121.6, \\
p(Y + T) &= 186.0, \\
\mathcal{GNP} &= 201.0.
\end{align*}

If an error of $\pm 13$ billion is attached to this forecast of $\mathcal{GNP}$, the range will certainly include the observed value.\(^4\)

Equation (3.18) was used in the spring of 1946 as the best available estimate, but it may not agree with observations. If we substitute

\footnote{The Economic Report of the President to the Congress (Jan. 8, 1947) estimates $\mathcal{GNP}$ at $\$194$ billion for 1946.}
ECONOMETRIC MODELS AS A GUIDE TO POLICY

(3.17), which was computed more recently for fiscal 1947, for (3.18), the forecasted point estimate of $G_{NP}$ is reduced to $8191$ billion.

THE FORECAST

What is the interpretation of these numerical results? Models I and III, the most reliable systems, state that $G_{NP}$ will, at best, maintain its present level and is likely to fall during the remainder of fiscal 1947. Model I states that the current rate of capital formation and price level will just maintain $G_{NP}$ during the rest of the fiscal year, but if we try to estimate capital formation from the structure of the system,

Table IV

Assumptions and Initial Conditions for Model III (Calendar Years)

<table>
<thead>
<tr>
<th></th>
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<th>Calendar 1945</th>
<th>Calendar 1944</th>
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<td>7.0</td>
<td></td>
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<tr>
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<td></td>
</tr>
<tr>
<td>$p$</td>
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<tr>
<td>$H_{-1}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
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<td>1.15</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
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<td>105.</td>
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<td>$D''$</td>
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<td>$G$</td>
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<td></td>
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<td>$aF$</td>
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<td></td>
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<tr>
<td>$q(Y+T)$</td>
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</tr>
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<td>$q$</td>
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as in Model III, we find that $GNP$ is likely to fall. If there are any sizeable disturbances in these models, now, they are likely to be positive, i.e., on the side of abnormal spending. If our past estimates of the disturbances are used as a guide, we can say that $GNP$ is likely to be in the range $177.5$ billion to $190.5$ billion in the first half of 1947. The models point to a turning point for real income during fiscal 1947.

**NEGLIGENCE OF SUPPLY FACTORS?**

Models like those of this paper have often been criticized for including only the demand side of the national market to the neglect of the supply side. This type of criticism has been especially prevalent in the recent months because it is often stated that demand is temporarily insatiable and the only limitation on the expansion of income during the postwar transition is the ability to supply goods. How can models composed exclusively of demand factors be useful in making predictions during such a period?

This criticism appears wrong, immediately, to those actively engaged in the construction of econometric models because we know that it is exhaustive to say that the economy can be decomposed into three groups, say, households, business firms, and government, and then to include the behavior pattern of these three groups in our models. What can we have neglected? The very simplest models, such as Model I, are deceptive because all the variables of this model are classified as demand variables. We have the demand by households for consumer goods, the demand by business firms for producer goods, and the demand by government for its goods and services. Where is supply? The difficulty lies in the fact that there are some equations that are imbedded within the business demand for producer goods and cover up the supply side even though it is nonetheless present. Business firms demand capital in order to produce goods that they supply to the market. Once we know the demand by business firms for factors of production, we know how much business firms will supply to the market because there is a technical relation connecting the output of business firms to their input of productive factors. The technical relationship, the production function, has been, so to speak, solved out of the system in constructing the models, but it has not been neglected.

A very simple example will make these points clear. Let us use Model I with the additional variable $N_t =$ total number of employees, $w =$ wage rate, and $K_{-1} =$ total stock of fixed capital available at the beginning of the period. The complete set of equations is:

\[\text{\footnotesize\textsuperscript{18}}\text{ Or in a corresponding higher range for assumed price increases greater than 10 per cent.}\]
ECONOMETRIC MODELS AS A GUIDE TO POLICY

\[ \frac{C}{pN} = a_0 + a_1 \frac{\gamma}{pN} + a_2 \left( \frac{\gamma}{pN} \right)_{-1} + u_1, \]

(4.2) \[ GNP = C + \mathcal{I} + \mathcal{G}, \]

(4.3) \[ GNP = \gamma + \mathcal{C}, \]

(4.4) \[ \frac{\mathcal{I}}{pN} = \text{exogenous}, \]

(4.5) \[ \frac{G}{pN} = \text{exogenous}, \]

(4.6) \[ \frac{\mathcal{C}}{pN} = \text{exogenous}, \]

(4.7) \[ \frac{GNP}{p} = f \left( N_K, \frac{\mathcal{I}}{p}, K_{-1} \right) + u_2, \]

(4.8) \[ \frac{vN_K}{p} = \gamma_0 + \gamma_1 \frac{GNP}{p} + \gamma_2 \left( \frac{GNP}{p} \right)_{-1} + u_2, \]

(4.9) \[ \Delta K = \frac{\mathcal{I}}{p} \quad \text{-- depreciation}. \]

In this model, we can determine the demand for factors of production by business firms, their output or supply of goods, and real wages. This model might be called a model of effective demand, yet the production functions and demand for factors of production are part of the system. Equation (4.4) is the demand equation of business firms for producer goods; equation (4.7) is the technical input-output relation for business firms; and equation (4.8) is the demand equation of business firms for labor. It happens that the structure of the system is such that (4.1)–(4.6), as a unit, are sufficient to determine the level of output, but this does not mean that supply conditions are neglected.

There is a supply variable that is omitted from this model, namely, the supply of labor, or the labor force. The variable \( N_K \) represents the total demand for labor by business firms. Knowing \( N_K \) and the labor force, we can determine the amount of unemployment; however, the models have not been criticized for leaving the supply of labor out of account. The critics had the supply of commodities in mind rather than the supply of labor.

It is obvious that if the model forecast extraordinarily high levels of national product at present prices, say \( GNP = \$225 \) billion, this forecast, in real terms, must be discarded because it exceeds our present capacity to supply goods. In the event that the exogenous and lagged
variables are such that levels of national product exceeding our capacity are forecast, all that we can say is that real output will be at its maximum level and prices will rise. The exact rise in prices cannot be forecast with any confidence from the models of this paper, however. It is possible to construct models in which there are enough equations to determine prices as well as output, although satisfactory statistical estimates of such equations are still lacking.

THE USEFULNESS OF ECONOMETRIC MODELS

Those engaged in the construction of econometric models know only too well the limitations on these models. The ranges of error associated with forecasts at reasonable probability levels are larger than will be required for many problems. In several cases we shall find that the plus-minus bands of error include both inflation and deflation or yes and no. That part of the error associated with sampling fluctuations can be improved upon. We can get more data and better data, both of which give additional information and help to establish the parameters of the system with a greater degree of accuracy. For example, if we could get good quarterly observations for all series used in this paper over the entire interwar period we should have more information from which to estimate the parameters of the system. There would not be four times as much information, but there would be much more information.

It is, of course, important to know what we cannot do in order that we do not fool ourselves, but our results are not purely negative. They show clearly, in this paper, that the probability is high that national product will fall in the latter part of the present fiscal year. This forecast can be made in spite of the fact that the range of error is fairly wide, say $10 or $15 billion. From Model II, we deduce that deficit spending has a higher multiplier effect than is the case for an easy-money policy. This deduction follows in spite of the fact that the parameters of the model are subject to considerable error. In the recent past (fiscal 1946 and calendar 1946), Model III has shown that there would probably be an inflationary gap rather than a deflationary gap. Not only was this forecast definite, but it was also correct.

Even if a forecast from a model includes the joint possibilities of inflation and deflation, the forecast may be very useful in policy formation. Suppose that a particular value of $GNP$, call it $(GNP)^*$, represents neither inflation nor deflation. Suppose further that the forecast is that $GNP$ will be in the range $[-\varepsilon_1, (GNP)^*+\varepsilon_1]$ with (high) probability, $p^*$. If $0<\varepsilon_1<\varepsilon_2$, there is a greater chance for inflation than for deflation. Government agencies that are prepared to combat inflation should be given greater powers than the agencies that are prepared to combat deflation, although both agencies should be given
some powers because there is a possibility that either one may be needed. In the other case where \( 0 < \epsilon_2 < \epsilon_1 \), the greater powers should be given to the anti-deflationary agency. In both examples, the amounts to be given each type of agency will depend upon the exact sizes of \( \epsilon_1 \) and \( \epsilon_2 \). But the principle of action is obvious.

The above type of policy is applicable only in the case in which the costs of preparing for inflation, or for deflation, or for both, are negligible. If there is a cost attached to the carrying out of policies, such as the cost of diverting resources to precautionary government activities, a different type of calculation must be made. If the cost of preparing for inflation when the true situation is deflation, the costs of preparing for deflation when the true situation is inflation, and the costs of preparing for the correct situation are known in advance, it is also possible to advise the government on a correct choice of alternative policies even though the forecast interval covers both inflation and deflation simultaneously.

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The University of Chicago

(See Appendix beginning on following page.)
## APPENDIX ON DATA

### Models I and II: Time Series

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- **C** = Consumer expenditures on goods and services, measured in billions of current dollars, U. S. Department of Commerce.
- **T** = Private gross capital formation, measured in billions of current dollars, U. S. Department of Commerce.
- **G** = Government expenditures for goods and services, measured in billions of current dollars, U. S. Department of Commerce.
- **γ** = Disposable income, measured in billions of current dollars, U. S. Department of Commerce.
- **GNP** = Gross national product measured in billions of current dollars, U. S. Department of Commerce.
- **T** = GNP - γ.
- **N** = Population of the continental United States, measured in billions of persons, U. S. Bureau of the Census.
- **M** = Total deposits adjusted and currency outside banks, measured in billions of current dollars on June 30 of each year, Board of Governors of the Federal Reserve System.
### Model III: Time Series

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**C**: Consumption measured in billions of 1934 dollars:

\[ C = \frac{(1) + (3)}{(3)} \]


\(2\) = Imputed net rents on owner-occupied residences, S. Kuznets, National Income and Its Composition, Vol. II, p. 735. These data are given for 1910-1938. For 1939, 1940, 1941 the estimates for gross imputed rent have been extended by Kuznets' method (estimated number of owner-occupied units multiplied by average monthly rental; converted to net rents by use of free-market regression between net and gross imputed rents, 1933-1938):

1939: $1.5$ billion,
1940: $1.5$ billion,
1941: $1.0$ billion.

\(3\) = Price index implicit in the adjustment of consumers' outlay, 1934:1.00.
## Model III: Time Series (continued)

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<th></th>
<th>Y</th>
<th>p</th>
<th>W₂</th>
<th>W₁</th>
<th>R₁</th>
<th>R₂</th>
<th>r</th>
<th>ΔF</th>
<th>v*</th>
<th>N*</th>
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* A figure greater than 100 indicates that demand exceeds physical supply.

S. Kuznets, *op. cit.*, Vol. I, p. 145, col. 3, converted to 1934 base year. These data are given for 1919–1938. For 1939, 1940, 1941 the estimates have been extended by Kuznets' method (weighted average of Bureau of Labor Statistics cost-of-living index and U. S. Department of Agriculture index of prices paid by farmers for subsistence, weights being proportionate to ratio of urban and rural populations respectively):

1939: 1.026,
1940: 1.034,
1941: 1.093.

I: Net Investment in private producers' plant and equipment, measured in billions of 1934 dollars.

\[
I = \frac{(4)}{(5)} + \frac{(6)}{(7)} - \frac{(8)}{(9)} - \frac{(10)}{(7)}.
\]

### Model III: Time Series (concluded)

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<th>Year</th>
<th>$\mathcal{M}_2$</th>
<th>$\mathcal{E}_r$</th>
<th>$K$</th>
<th>$H$</th>
<th>$\mathcal{E}$</th>
<th>$v_2$</th>
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<td>0.68</td>
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</table>

(5) = Price index of business capital goods, 1934:1.00, S. Fabricant, *Capital Consumption and Adjustment*, pp. 178-197, converted to 1934 base year. These data are for 1919-1925. The figures for 1930-1941 have been supplied by Fabricant in private correspondence.

(6) = Expenditures on farm buildings (excluding dwellings) + expenditures on farm machinery + expenditures on farm trucks + 40 per cent of expenditures on farm automobiles other than trucks, *Net Farm Income and Parity Report*, U.S. Department of Agriculture, 1913, p. 27.

(7) = Price index of farm capital goods, 1934:1.00. Weighted average of indexes of prices paid for building materials, farm machinery, and motor vehicles. The weights are proportional to constant-dollar expenditures on farm buildings (excluding dwellings), on farm machinery, and on trucks +40 per cent of other motor vehicles respectively. *Agricultural Statistics*, 1943, p. 394, *Income Parity for Agriculture*, U.S. Department of Agriculture, Part III, Section 4, p. 11.

(9) = Price index underlying business depreciation charges, 1934 base, S. Fabricant, op. cit., p. 183, converted to 1934 base year. These data are for 1919-1935. The figures for 1936-1941 have been supplied by S. Fabricant in private correspondence.


q: Price index of private producers' plant and equipment. 1934:1.00.

\[ q = \frac{(4)}{(4) + (6)} + \frac{(5)}{(4) + (6)} \]

\[ \Delta H: \text{Net change in inventories, measured in billions of 1934 dollars.} \]

\[ \Delta H = \frac{(11)}{(12)} \]

(11) = Net inventory change, Mary S. Painter, loc. cit.


D1: Gross construction expenditures on owner-occupied, single-family, nonfarm residences, measured in billions of 1934 dollars.

\[ D1 = \frac{(13)-1.18-1.126\cdot(14)-0.63 + (15)\cdot0.23}{(16)} \]


1.18 = Correction for undervaluation of building permits, taken from D. Wickens, Residential Real Estate.


(14) = Number of single-family, nonfarm dwelling units constructed annually, Bureau of Labor Statistics, op. cit., table 18, p. 35.

0.63 = Fraction of single-family, nonfarm dwelling units constructed 1935-1940, that were owner-occupied in 1940, Census of Housing, loc. cit.

(15) = Expenditures for alterations, additions, and repairs, Twentieth Century Fund, American Housing, table 12, p. 367. These data are for 1919-1940. For 1941 it was assumed that maintenance was the same percentage of total housing expenditures as in 1940.

0.23 = 0.32 - 0.357/0.524, where 0.32 = the ratio of owner-occupied single-family, nonfarm dwelling units to total nonfarm dwelling units in 1940; 0.357 = the fraction of repairs estimated by the U.S. Department of Commerce to be attributable to owner-occupied, nonfarm residences (average of 1930 and 1940 figures); 0.524 = the ratio of owner-occupied nonfarm units to total nonfarm units, 1930.


\[ q1: \text{Index of construction costs, 1934:1.00.} \]

\[ q1 = (16) \]
$D_t$: Gross construction expenditures on rented, nonfarm residences, measured in billions of 1934 dollars.

$$D_t = \frac{(17)}{(16)} - D_b.$$  


$D_t$: Gross construction expenditures on farm residences, measured in billions of 1934 dollars.

$$D_t = \frac{(18)}{(19)}.$$  


(19) = Index of farm construction costs, 1934-1:00, Income Parity for Agriculture, Part II, Section 5, p. 27. These data are for 1919-1940. The 1941 figure was taken from a freehand regression between (19) and the index of the cost of building materials taken from Agricultural Prices, Feb. 29, 1944, col. 7, p. 25. All indexes are converted to 1934 base year.

$D''$: Depreciation of all residences (farm and nonfarm), measured in billions of 1934 dollars.

$$D'' = \left[67.6 - \sum_{t=1}^{1934} (D_1 + D_2 + D_3 + \sum_{t=1}^{1934} D_{i''}) \right] \frac{0.03}{0.97}$$

$$-(D_1 + D_2 + D_3 + \sum_{t=1}^{1934} D_{i''})_t \frac{0.015}{0.97}, \quad t < 1933,$$

$$D'' = 67.6 \frac{0.03}{0.97} - (D_1 + D_2 + D_3)_t \frac{0.015}{0.97}, \quad t = 1933,$$

$$D'' = 67.6(0.03) + (D_1 + D_2 + D_3)_t (0.015), \quad t = 1934,$$

$$D'' = (67.6)(0.03)(1934)(0.03)$$

$$+ \sum_{t=1934}^{\infty} (D_1 + D_2 + D_3)_t (0.985)(0.97, 1 - 1934) (0.03)$$

$$+ (D_1 + D_2 + D_3)_t (0.015), \quad t > 1934.$$  

67.6 = The estimated value, Jan. 1, 1934, of the stock of residential dwellings in the U.S. It is 80 per cent of Wickens’ (Residential Real Estate, table A8, p. 93) figure for the value of nonfarm land and dwellings plus 14.3 per cent of the value of total farm real estate, Net Farm Income and Parity Report, U.S. Department of Agriculture, 1943, table 18, p. 29. The figures of 80 per cent and 14.3 per cent are taken as the proportions of nonfarm residential and farm real estate respectively that are represented by dwellings alone.

$G$: Government expenditures for goods and services (exclusive of government interest payments) plus net exports plus investment of nonprofit institutions, all measured in billions of 1934 dollars.

$$G = \frac{(20) - (21) - (22)}{(23)} + \frac{(22)}{(16)} + \frac{(12)}{(16)} + \frac{(25) - 0.1}{(16)}.$$  

(20) = Government expenditures for goods and services, Mary S. Painter, loc. cit.


These data for 1919-1928. J. Mosak, op. cit., p. 51. These data are for 1929-1941.

\( (23) = \) Index of wholesale prices of nonfarm products, 1934:1.00, Agricultural Statistics, 1942, p. 649, converted to 1934 base year.

\( (24) = \) Net exports and monetary use of gold and silver, Mary S. Painter, loc. cit.

\( (25) = \) Gross construction expenditures by nonprofit institutions, G. Terbovitch, loc. cit., and F. Dirks, loc. cit.

\( 0.1 = \) Annual depreciation charges attributed to the plant of nonprofit institutions. This figure is about 3 per cent of the 1934, end-of-year, stock of capital owned by these institutions.

\( Y + T = \) Net national product, measured in billions of 1934 dollars.

\[ Y + T = C + I + \Delta H + D_s + D_t + D_c - D'' + G. \]

\( Y = \) Disposable income, measured in billions of 1934 dollars.

\[ Y = \frac{(1) + (2) + (4) + (6) - (8) - (10) + (11) + (17) + (18) - (16)D''}{(9)} + (20) + (24) + (26) + 0.1 - (20) - (27) - (28) + (29). \]

\( (26) = \) Federal government receipts, Annual Report of the Secretary of the Treasury.

\( (27) = \) State and local government receipts, National Industrial Conference Board, Economic Almanac, 1944-1945, p. 102. These data are given for fiscal years and are converted to a calendar-year basis by taking a two-year moving average of a three-year moving average.


\( (29) = \) Transfer payments, Survey of Current Business, Vol. 22, May, 1942, p. 12, and August, 1943, p. 13. These data are for 1929-1941. The data for 1919-1928 are set at a constant rate of $0.7 billion per annum. This constant figure is justified on the grounds that veterans' payments by the federal government were stable at $0.4 billion for the entire period of the 1920's and that state relief expenditures were also stable at $0.1 billion for the same period. The remaining transfer payments, those of local governments, were assumed to be stable also. Hence the figure of $0.7 billion, observed for 1929 and 1930, was extended to the previous years.

\( p = \) Price index of output as a whole, 1934:1.00

\[ p = \frac{(1) + (2) + (4) + (6) - (8)}{(9)} + (10) + (11) + (17) + (18) - (16)D'' + (20) - (21) + (24) + (26) - 0.1. \]

\( \bar{W}_1 = \) Private wage-salary bill, measured in billions of current dollars.

\[ \bar{W}_1 = (30) - (31). \]

\( (30) = \) Total employee compensation, M. Hoffenberg, loc. cit. (Commerce estimates).

\( (31) = \) Government wages and salaries, including work-relief wages, Survey of Current Business, March, 1943, p. 22. These data are for 1929-1941.
The figures for 1919–1928 are estimated from a free-hand regression between the U.S. Department of Commerce estimates (1929–1938) and Kuznets' estimates (1929–1938). See S. Kuznets, op. cit., Vol. II, p. 811. Kuznets' figures are known for 1919–1928; hence the corresponding commerce figures can be estimated from the regression.

\( W_t \): Government wage-salary bill, measured in billions of current dollars.

\[ W_t = (31). \]

\( R_t \): Nonfarm rentals, paid and imputed, measured in billions of current dollars.

\[ R_t = (32). \]


\[ R_t = 0.278 \left( \frac{vN_t}{100} + \frac{v_{-1}N_{-1}}{100} \right)^{1/2}. \]

See below for definitions of \( v \) and \( N_t \).

\( R_t \): Farm rentals, paid and imputed, measured in billions of current dollars.

\[ R_t = (33). \]

(33) = Gross rentals, paid and imputed, for farm dwellings, Agricultural Statistics, 1942, p. 660.

\( r \): Index of rents, 1934:1.00.

\[ r = (34). \]


\( \Delta F \): Thousands of new nonfarm families.

\[ \Delta F = (35). \]

(35) = Annual number of families added in nonfarm areas, Twentieth Century Fund, op. cit., table 40, p. 418. These data are for 1919–1939. For 1940 and 1941, the data are the first differences in the beginning-of-the-year number of families given in the Statistical Abstract of the United States, 1943, table 40, p. 46, multiplied by 0.795, the ratio of nonfarm families to all families in 1940.

\( v \): Percentage of nonfarm housing units occupied at the end of the year.

\[ v = (36), \quad 1919–1927, \]

\[ v = (37), \quad 1928–1941. \]

(36) = Ratio of nonfarm families to available nonfarm dwelling units. L. Chawner, Residential Building, Housing Monograph Series, No. 1, table VI, p. 16.

$N^* = \text{Millions of available nonfarm dwelling units at the end of the year.}$

$$N^* = 24.6 - \sum_{t=1919}^{t=1927} \Delta(38)_t, \quad t = 1919-1927,$$

$$N^* = (39), \quad 1928-1941.$$  

(38) = Available nonfarm dwelling units, at the end of the year, L. Chawner, *op. cit.*, Table VI, p. 16.

24.6 = Available nonfarm dwelling units on January 1, 1929, "Dwelling Units in the United States, 1929-1942," special release of the United States Department of Commerce.

(39) = Available nonfarm dwelling units, at the end of the year, "Dwelling Units in the United States, 1929-1942."

$i$: Average corporate-bond yield.

$$i = (40).$$


$M_1$: Demand deposits adjusted plus currency outside banks, averaged during the year, measured in billions of current dollars.

$$M_1 = (41).$$

(41) = Demand deposits adjusted plus currency outside banks, average of beginning-, middle-, and end-of-the-year figures (except 1919-1923, for which middle-of-year figures are used), Board of Governors of the Federal Reserve System, *op. cit.*, p. 34.

$M_2$: Time deposits, averaged during the year, measured in billions of current dollars.

$$M_2 = (42).$$

(42) = Time deposits, average of beginning-, middle-, and end-of-year figures (except 1919-1923, for which middle-of-year figures are used), Board of Governors of the Federal Reserve System, *op. cit.*, p. 34.

$E_r$: Excess reserves, averaged during the year, measured in millions of current dollars.

$$E_r = (43), \quad 1919-1928,$$

$$E_r = (44), \quad 1929-1941.$$  


$K$: End-of-year stock of private producers' plant and equipment measured in billions of 1934 dollars.

$$K = 107.8 - \sum_{t=1919}^{t=1934} I_t, \quad t < 1934,$$

$$K = 107.8, \quad 1934,$$

$$K = 107.8 + \sum_{t=1935}^{t}, \quad t > 1934.$$  

107.8 = End-of-1934 stock of private producers' plant and equipment. Net capital assets, excluding land, for corporations, end-of-1934, are taken from S. Fabricant, *op. cit.*, p. 271. These data are converted to estimates
for both corporate and noncorporate enterprises by dividing the figure for each industry by the ratio of corporate output to total output. These ratios are:

- Mining and Quarrying: 0.96
- Manufacturing: 0.92
- Construction: 0.60
- Transportation and Public Utilities: 1.00
- Trade: 0.63
- Service: 0.30
- Finance and Real Estate: 0.84

These ratios are found in *The Structure of the American Economy*, National Resources Planning Board, pp. 375-376. The capital for the service industries is corrected by subtracting the capital of nonprofit institutions. Agricultural capital, exclusive of livestock, land, and dwellings is taken from *The Structure of the American Economy*, p. 377.

**H**: End-of-year stock of inventories, measured in billions of 1934 dollars.

\[
H = 21.8 - \sum_{t=1}^{133} (\Delta H)_t, \quad t < 1934, \\
H = 21.8, \quad 1934, \\
H = 21.8 + \sum_{t=1934}^{t} (\Delta H)_t, \quad t > 1934.
\]

21.8 = End-of-1934 stock of inventories.
Business inventories, end of 1934: $17,913 million
Less: Inventories of Agricultural Corporations: 196

---

|---|
| Excise taxes, measured in billions of current dollars. 
\[
\text{\(\mathcal{E}\)} = (45).
\]


**Computations of Predetermined and Exogenous Variables Used in Model III**

\[
\mathcal{E}:
\]

The components of \(\mathcal{E}\) that represent federal taxes can be obtained from the *Treasury Bulletin* for past months, and those components that represent state and local taxes can be obtained from the *Statistical Abstract* for recent years. There is little variation in the state and local taxes so they have been set arbitrarily at the levels given by the
latest observations in the Statistical Abstract. The sum of the components of \( E = 7.3 \text{ billion} \) for fiscal 1946. A figure this large for fiscal 1947 is not consistent with the levels of \( GNP \) generated by the model for that period; i.e., present tax rates would not yield taxes as high as those of last year if \( GNP \) falls from last year's level. By successive approximations, we obtained assumptions of falling excise taxes and falling \( GNP \) that are consistent with each other at current tax rates. The variable \( E \) does not, by itself, play an important role in the model, however, and a slight error in the assumed value of \( E \) will have a negligible result on the forecast.

\( K_{-1} \): Expenditures on private producers' plant and equipment during the war years (since Pearl Harbor) approximately, though not quite, balanced depreciation charges. It was assumed that the stock of capital in plant and equipment could be set at $111.0 billion for the beginning of fiscal 1946. This figure is in 1934 dollars. Net investment in 1934 dollars for fiscal 1946 has been estimated at $23.3 billion, which brings the stock of private capital up to $133.3 billion for the beginning of fiscal 1947. To this figure we add approximately $1 billion to account for the transfer of surplus property, in the form of producers' plant, from the public to the private sector of the economy. (See 8th Report by the Director of War Mobilization and Reconversion, Oct. 1, 1946, p. 69.)

\( p \): From the published observations for fiscal 1946, we calculated, according to the definitions used in Model III, \( p(Y+T) \) and \( Y+T \). The ratio of these two quantities gives \( p \). We have assumed approximately a 10-per-cent rise from fiscal 1946 to fiscal 1947 for this variable.

\( H_{-1} \): To our figure for year-end inventories in 1941, we added the net annual change, in 1934 dollars, up to the end of 1945. There are also data for the first half of 1946, which were added in 1934 dollars. The result was the stock of inventories existing at the beginning of fiscal 1947, $27.9 billion.

\( r \): It was assumed that strict rent controls would be maintained throughout the first half of the fiscal year and would be relaxed to permit an increase in the ceilings by 10 to 15 per cent during the second half of the fiscal year. We placed the average increase for the year at 5 points in the index.

\( Y_{-1}, Y_{-2} \): The difference between the Commerce Department definition of disposable income and the definition used in Model III is equal to net imputed rents on owner-occupied residences. The sum of $2 billion was added to the Commerce estimates for fiscal 1945 and for fiscal 1946 to get disposable income in current dollars. The price index of consumer goods was then used as a deflator to get disposable income in 1934 dollars. The price index is a weighted average of the cost-of-living index and the index of prices paid by farmers, the weights being 0.77 and 0.23 respectively. These weights are the percentages of the population in nonfarm and farm areas respectively.

\( q_t \): For fiscal 1945 and fiscal 1946, the index of construction costs was taken from the observations (American Appraisal Co., national average), transformed to a 1934 base. A 10-per-cent increase was assumed for fiscal 1947 as compared with fiscal 1946.

\( i \): The average corporate-bond yield, which changes very slowly, was kept at its present level.
\( D_1 \): Farm residential construction was put at its peacetime maximum. This figure is very small and can have little influence on the final result.

\( D' \): Depreciation of residences has been at a nearly constant rate over the last two decades. Small changes in the stock of housing will cause very small changes in depreciation. Maximum building rates will not change this figure (in tenths of billions) for a few years. The latest prewar rates were extended to fiscal 1946 and fiscal 1947.

\( G \): For fiscal 1946, it is possible to use observed values of government spending, (excluding interest), foreign trade, and net investment of nonprofit institutions, all properly deflated. For fiscal 1947 we used official government forecasts of expenditures (excluding interest) and the foreign balance. The assumed wholesale price index (10 per cent above fiscal 1946) was used as a deflator. The net investment of nonprofit institutions was put at its prewar maximum; however, this figure has a negligible effect on the final result.

\( W_2 \): Government wages and salaries are known for fiscal 1946. Official forecasts were used for fiscal 1947.

\( R_1 + R_2 \): Assuming an increase in the number of housing units and in the level of rents, we should find an increase in total rents. A 5-per cent increase from the observations of fiscal 1946 was assumed for fiscal 1947.

\( \Delta F \): The net increase in the number of families (farm and nonfarm) has been forecast for calendar years in the Statistical Abstract of the United States, 1944-45, p. 49. The percentage of all families that will be in nonfarm areas is set at 82.7 per cent (see National Housing Agency, Housing Needs, November, 1944). The 1946 forecast was used unchanged (except for rounding off) for fiscal 1946. The forecast for calendar 1947 was revised upward in order to apply to fiscal 1947 since the latter year will catch some of the demobilization while the former will not. The upward revision was from about 416 thousand families to 500 thousand families.

\( p(Y + T) \): The observed value of net national product in current dollars for fiscal 1946 was formed from the sum:

- Consumer expenditures
- $2 billion imputed net rents
- net change in inventories
- construction
- private producers' equipment
- government expenditures
- net exports
- $11 billion depreciation
- $5.3 billion government interest payments.

\( q \): For fiscal 1946 we calculated \( q \) from the ratio of expenditures on producers' plant and equipment in current dollars to expenditures in 1934 dollars. The latter figure was obtained by deflating equipment expenditures by the wholesale price index of machinery and plant expenditures by the construction cost index. We assumed a 10-per cent increase in \( q \) from fiscal 1946 to fiscal 1947.