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Contributions to the Study of Oscillatory Time-Series. *M. G. Kendall* (Statistician, Chamber of Shipping of the United Kingdom, Bury Court, St. Mary Axe, London, E.C.2). National Institute of Economic and Social Research, Occasional Papers, IX. London: Cambridge University Press, 1946. Pp. viii, 76. 7s. 6d. Paper. (New York 11: Macmillan Co. [60 Fifth Ave.], 1946. \$1.75.)

REVIEW* BY T. W. ANDERSON

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THE main purpose of this book is to study the results of applying certain methods of times series analysis to series generated by an "autoregressive" scheme. An autoregressive scheme is defined by a stochastic difference equation

$$u_{t+2} + au_{t+1} + bu_t = \epsilon_t,$$

where u_t is a quantity observed at time t , the coefficients a and b are given numbers, and ϵ_t is a random term or "disturbance" with a given probability distribution. The author chooses four pairs of numbers a and b and, using these as coefficients, constructs four artificial times series of u_t by taking sequences of ϵ_t from random number tables. The "period" of the autoregressive scheme is taken to be the period of the solution of the difference equation, which is

$$\frac{2\pi}{\cos^{-1}\left(\frac{-a}{2\sqrt{b}}\right)}.$$

This period is estimated from each series by periodogram analysis, correlogram analysis, counting peaks, and counting "upcrosses" (a negative observation followed by a positive one). The principal conclusion is that periodogram analysis applied to an autoregressive series gives a poor estimate of the

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period and furthermore gives misleading and confusing results. The last two methods do not give good estimates. In fact, in the population (i.e., infinite series) the period may be quite different from the mean distance between peaks or mean distance between upcrosses. The correlogram provides a fairly good estimate of the period, but the correlations do not damp out in an autoregressive series according to the mathematical expectation; hence, the correlogram does not necessarily indicate that the series is autoregressive. The variate difference method does not give a good estimate of the variance of the random element. Some economic time series are considered as well as the four artificial series.

The inferences drawn in the book are very interesting, but unfortunately, because of the empirical method used, are not very reliable. The four artificial series can be thought of as four observations, each from a statistical population of all series for a given a , b , and distribution of ϵ_t . One cannot have much confidence in conclusions drawn on the basis of only four observations, particularly when the problems are highly complicated. In studying some of the statistical techniques, the author does obtain the expected values of the statistics involved (or the expected values of the reciprocals), but he does not obtain indications of the sampling errors. The most significant part of the study, the effect of applying periodogram analysis of autoregressive series, needs a mathematical analysis to give definite conclusions.

In the comparison of methods it would have been instructive to consider a method designed specifically for the autoregressive model. From the maximum likelihood estimates (assuming normality of ϵ_t) of the coefficients a and b one can calculate an estimate of the period. This estimate has certain desirable statistical properties such as consistency.

The book should be educational to investigators who have applied methods such as periodogram analysis and have obtained results difficult to interpret. The author points out that the mathematical model of a finite series of trigonometric functions with a superimposed random term does not fit in economic theory well. The stochastic difference equation gives a better description of the economic process; the parameters of this model may have economic meaning. It is generally recognized that in a large number of economic time series there is no period in the sense of recurrence of phenomena after a certain exact time unit; the autoregressive model does not impose a strict period. In terms of stochastic difference equations the "period" is of secondary interest; the coefficients of the equations are the primary consideration. As the author suggests, much more mathematical statistical research is needed along this line.