TECHNOLOGICAL CHANGE AND AGGREGATION

By Kenneth May

Recent discussion of the aggregation problem indicates that agreement has not yet been reached on objectives and methods. Nevertheless, some definite results have been obtained. Additional ones may be valuable not only for their own sake, but also for the light they might cast on fundamental questions. A method of aggregation should provide us with a way of studying the effect on a macroeconomic model of changes in a micromodel with which it is connected. In particular, it would be interesting to know how technological changes in the micromodel are reflected in the macromodel.

In the first section, technological change is discussed in a simplified one-industry model. Increments of the variables of the model are expressed in terms of a general functional change in the production function. In the second section, a formula is established giving the change in the aggregate production function in terms of changes in the production functions of the two-industry model from which it is derived. Although very simple models and mathematical tools are utilized in these sections, the method and point of view are quite general and could be applied to study the effect of functional changes on the variables of any model or on the functions involved in any other model derivable from it. In the last section, there is a brief review of the aggregation problem to date.

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This paper, together with the papers by Kenneth May, Shuo Shan Pu, and Lawrence R. Klein (items, C, D, E in footnote 1), will be reprinted as Cowles Commission Papers, New Series, No. 19.

1. TECHNOLOGICAL CHANGE IN A ONE-INDUSTRY MODEL

A production function stands for a schedule which gives the outputs corresponding to different factor inputs. Technological change is here considered as synonymous with modifications of this schedule, i.e., changes in the production function. The change may be expressible in terms of changes in the values of one or more parameters of the function, or it may be expressed more generally as a functional increment not explicitly associated with particular parameters.

In order to fix ideas in a simple context, we consider such general functional changes in the one-industry model which was derived in a previous article. This model connects total employment $N$, aggregate output $U$, the aggregate price $P$, and the wage rate $w$ by the following two relations:

\begin{align}
U &= \psi(N), \\
\psi'(N) &= \frac{w}{P} = \rho.
\end{align}

If we consider the variables as $U$, $N$, and $\rho$, the model evidently has one degree of freedom. We prefer to leave unspecified the relation chosen to make the model completely determinate in order to concentrate attention on changes resulting from modifications in the function $\psi$.

Suppose that the function $\psi$ is replaced by $\psi^*$, where

\begin{equation}
\Delta\psi(N) = \psi^*(N) - \psi(N)
\end{equation}

defines the change or functional increment in $\psi$. We now have a new
set (1)–(2). Writing the new values of the variables as the old values satisfying (1)–(2) plus increments, we have

\[ U + \Delta U = \psi(N + \Delta N) + \Delta \psi(N + \Delta N), \]

\[ p + \Delta p = \psi'(N + \Delta N) + \frac{d\Delta \psi}{dN}_{N, \Delta N}, \]

which together with (1)–(2) determine any two increments as functions of the third. In particular, if the increments are all small,\(^8\) we can write approximations,

\[ \delta U = \psi'(N)\delta N + \delta \psi(N), \]

\[ \delta p = \psi''(N)\delta N + \frac{d\delta \psi(N)}{dN}, \]

that establish two linear relations among the three differentials. A third relation, derived from whatever equation was chosen to complete the model, would enable us to solve for the differentials as functions of \(N\). Alternatively, we may consider the implications of some special hypothesis, e.g., that employment is maintained constant during the technological change, or \(\delta N = 0\).

Since the characteristic feature of technological change in our society is an increase in productivity, we consider two special cases of this type. First we take \(\delta \psi = \lambda \psi\), or

\[ \psi^*(N) = (1 + \lambda)\psi(N), \]

where \(\lambda\) is small. This means that the same employment produces \((1 + \lambda)\) as much as before, the percentage increase being independent of employment. In this case we find for the hypothesis \(\delta N = 0\) that

\[ \delta U = \lambda \psi(N) > 0, \]

\[ \delta p = \lambda \psi'(N) > 0. \]

In economic terms, these last equations mean that maintained employment with this type of technological change implies an increase in both output and the real wage rate.

As a second type of modification in \(\psi(N)\), we replace it by \(\psi[(1 + \epsilon)N]\).

\(^8\) By a small functional increment we mean a function that takes small values for all values of the variables in the ranges considered. It is sufficient for what follows that \(\delta \psi\) be expressible as \(\delta f(N)\); where \(\epsilon\) is an infinitesimal and \(f(N)\) is an arbitrary function satisfying the same conditions as to continuity, differentiability, etc., that are assumed for production functions in order to allow marginal discussions. Although \(\delta \psi\) could be replaced by \(\delta f(N)\) throughout the discussion, we prefer to use \(\delta \psi\) in order to emphasize that we are dealing with a change in the function \(\psi\).
This change is specifically associated with labor and means that one worker now does the work formerly done by \((1 + \varepsilon)\) workers. In this case,

\[(11) \quad \delta \psi(N) = \psi[(1 + \varepsilon)N] - \psi(N) = \varepsilon N \psi'(N),\]

since \(\varepsilon\) is assumed small. Similarly

\[(12) \quad \frac{d\delta \psi(N)}{dN} = \varepsilon \psi'(N) + N \psi''(N),\]

so that (6) and (7) become

\[(6') \quad \delta U = \psi'(N)\delta N + \varepsilon N \psi'(N),\]
\[(7') \quad \delta p = \psi''(N)\delta N + \varepsilon \psi'(N) + N \psi''(N).\]

For the hypothesis \(\delta N = 0\), these reduce to

\[(9') \quad \delta U = \varepsilon N \psi'(N) > 0,\]
\[(10') \quad \delta p = \varepsilon \psi'(N) + N \psi''(N).\]

The sign of \(\delta U\) is evidently positive, but since \(\psi'' < 0\), the sign of \(\delta p\) depends on that of \(\psi' + N \psi''\).9

Suppose that instead of maintained employment, we assume constant output. Then \(\delta U = 0\), and from (6') we have a decrease in employment equal to \(\delta N = -\varepsilon N\), an increase in the real wage rate given by (7') as \(\delta p = \varepsilon p\), and an unchanged total real wage since \(\delta(pN) = p\delta N + N \delta p\). The net effect of such a technological change with constant output is thus an unchanged total real wage income divided among fewer workers.10

2. THE AGGREGATION OF TECHNOLOGICAL CHANGE11

In the previous section, we discussed the effect on a one-industry model of changes in its production function. It has been shown that this aggregate production function is uniquely determined by the func-

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9 This quantity has an interesting economic interpretation. Since \(d(pN)/dn = \psi' + N \psi''\), the condition that a technological change of this type with fixed employment will increase the real wage rate is the same as the condition that an increase in employment with fixed technology will increase the total real wage.


11 The approach here adopted to technological change was formulated in "Distribution and Unit Productivity," a MS privately circulated in 1939.
tions of the micromodel from which the one-industry model is derived.\textsuperscript{12} We are interested here in discussing the effect on the aggregate production function of changes in the production functions of the micromodel. Since the method of aggregation and the resulting macromodel were found to be the same whether we considered a general equilibrium involving \(n\) capital goods, \(m\) consumption goods, and \(s\) types of labor, or some particular case, we will work in the simple context where \(n = 1\), \(m = 1\), \(s = 1\). We take as our micromodel the following system of one degree of freedom:

\begin{align}
U_1(1) + U_1(3) &= \phi(U_1(1), N_1), \\
U_2 &= \theta(U_1(3), N_3), \\
\phi_1(U_1(1), N_1) &= 1, \\
\phi_3(U_1(1), N_3) &= p_2/p_1, \\
\theta_1(U_1(3), N_3) &= p_1, \\
\theta_3(U_1(3), N_3) &= p_2,
\end{align}

in which \(U_2\) is the output of consumption goods, \(U_1(1)\) and \(U_1(3)\) the amounts of capital goods used in the two industries, \(N_1\) and \(N_3\) the employments, \(P_1\) and \(P_3\) the prices of capital and consumption goods, \(w\) the wage rate, and \(p_1\) and \(p_2\) real prices defined by \(p_1 = P_1/P_3\) and \(p_2 = w/P_3\). Subscripts on functions indicate partial derivatives.

If we define total employment by

\begin{equation}
N = N_1 + N_3,
\end{equation}

aggregate output by \(U = U_3\), and corresponding prices by \(P = P_3\) and \(p = p_3\), the model (21)-(27) determines the macromodel

\begin{align}
U &= \psi(N), \\
p &= \psi'(N),
\end{align}

where the function \(\psi\) is given by

\begin{equation}
\psi(N) = \theta[U_1(1)(N), N_3(N)].
\end{equation}

\textsuperscript{12} \textit{Loc. cit.}, (C). In fact, the aggregation process here outlined sets up a transformation whose elements are functions (see footnote 4) in which the functions of the micromodel determine those of the macromodel. The resulting aggregate production function thus depends upon all the functions involved in the macromodel, not merely the production functions. Here we will concern ourselves only with the effect on the macromodel of changes in the production functions of the micromodel. However, the same methods could be applied to studying the effects of changes in other parameters or relations. See section 3 of this paper.
in which $U_l^{(0)}(N)$ and $N_2(N)$ stand for the expressions found by solving (21)–(27) for $U_l^{(0)}$ and $N_3$ in terms of $N$.\(^{11}\)

Suppose that the functions $\phi$ and $\theta$ are replaced by $\phi^*$ and $\theta^*$, so that

\begin{align}
\Delta \phi(U_l^{(1)} N_1) &= \phi^*(U_l^{(1)} N_1) - \phi(U_l^{(1)} N_1), \\
\Delta \theta(U_l^{(1)} N_2) &= \theta^*(U_l^{(1)} N_2) - \theta(U_l^{(1)} N_2).
\end{align}

We should now have a new set like (21)–(27) in which the values of the variables in terms of $N$ would be different from what they were before. In fact, expressing the new values as the old plus increments, we have

\begin{align}
U_l^{(1)} + \Delta U_l^{(1)} + U_l^{(0)} + \Delta U_l^{(0)}
&= \phi(U_l^{(1)} + \Delta U_l^{(1)}, N_1 + \Delta N_1) \\
&\quad + \Delta \phi(U_l^{(1)} + \Delta U_l^{(1)}, N_1 + \Delta N_1), \\
U_3 + \Delta U_3 &= \theta(U_l^{(3)} + \Delta U_l^{(3)}, N_2 + \Delta N_2) \\
&\quad + \Delta \theta(U_l^{(3)} + \Delta U_l^{(3)}, N_2 + \Delta N_2), \\
\frac{\partial}{\partial U_l^{(0)}} \left[ \phi + \Delta \phi \right]_{N_1 + \Delta N_1, U_l^{(0)}} &= 1, \\
\frac{\partial}{\partial N_1} \left[ \phi + \Delta \phi \right]_{N_1 + \Delta N_1, U_l^{(0)}} &= \frac{\rho_1 + \Delta \rho_1}{\rho_1 + \Delta \rho_1}, \\
\frac{\partial}{\partial U_l^{(3)}} \left[ \theta + \Delta \theta \right]_{N_2 + \Delta N_2, U_l^{(3)}} &= \rho_2 + \Delta \rho_2, \\
\frac{\partial}{\partial N_2} \left[ \theta + \Delta \theta \right]_{N_2 + \Delta N_2, U_l^{(3)}} &= \rho_3 + \Delta \rho_3, \\
\Delta N_1 + \Delta N_2 &= 0,
\end{align}

which, together with (21)–(27) determine the increments as functions of $N$ depending on $\Delta \phi$ and $\Delta \theta$. Equation (27\(^\prime\)) means merely that we are interested in the changes in the variables as functions of $N$.

But from (22) and (30), the change in the function $\psi(N)$ is given by

\begin{align}
\Delta \psi(N) &= \Delta U_3(N),
\end{align}

in which $\Delta U_3(N)$ is found by solving (21\(^\prime\))–(27\(^\prime\)) for $\Delta U_3$ as a function of $N$.

\(^{11}\) *Loc. cit.* (C), secs. 2, 3, and 7. For more general micromodels, the aggregate definitions are more complex, but the method and result are essentially the same, as indicated in secs. 4–8 of the cited article. Klein [*loc. cit.* (E)] proposes studying technological change in a model which is essentially (21)–(28) with the addition of certain other variables. The method he proposes, if carried through, would be essentially the same as that indicated here. In fact his equation (37) could be derived from his model (22)–(36) as we derived (28) above.
If the functional changes are small, we write \( \delta \phi \) and \( \delta \theta \), and (21')–(27') can be replaced by linear approximations.\(^{14}\) In particular, (21'), (22'), and (27') become

\[
\delta U_1^{(3)} = \phi_1 \delta N_1 + \delta \phi,
\]

\[
\delta U_2 = \theta_1 \delta U_1^{(3)} + \theta_2 \delta N_2 + \delta \theta,
\]

\[
\delta N_1 + \delta N_2 = 0.
\]

Replacing \( \delta N_1 \) by \( -\delta N_2 \) in the expression for \( \delta U_1^{(3)} \) and substituting in (34), we have

\[
\delta U_3 = \delta \theta + \theta_1 \delta \phi + [\theta_2 - \theta_1 \phi_2] \delta N_3.
\]

But from (23)–(26), the coefficient of \( \delta N_2 \) is zero. Hence, utilizing (30'), we find the desired result is

\[
\delta \psi(N) = \delta \theta [U_1^{(3)}(N), N_2(N)]
+ \theta_1 [U_1^{(3)}(N), N_2(N)] \delta \phi [U_1^{(1)}(N), N_1(N)],
\]

which expresses the functional change in \( \psi(N) \) as a function of \( N \) given in terms of the functional changes \( \delta \phi \) and \( \delta \theta \) with their arguments replaced by the expression in terms of \( N \) found by solving (21)–(27).

The equation (39) is a relation between functions holding for all values of \( N \) in the range considered. For any particular value of \( N \), it gives the numerical increment in aggregate output in terms of the increments in the two industries caused by the technological changes. But, more generally, it enables us to find the change in the production function and hence the new function resulting from functional changes \( \delta \phi \) and \( \delta \theta \) without solving the new micromodel.\(^{15}\) It also enables us to

\(^{14}\) See footnotes 4, 7, and 8. Because of the separation of the variables we do not need to utilize all our equations to find \( \delta U_3 \). We assume here, as elsewhere in this paper, that the functions involved are such as to make these operations legitimate.

\(^{15}\) Suppose for example that \( \psi \) is quadratic and \( \theta \) is linear, so that we have the model

\[
U_1^{(1)} + U_1^{(3)} = AN_1 + BN_1^{(1)}N_1 + CN_1^{(1)}N_1 + \epsilon N_1 + K,
\]

\[
U_1^{(1)} = \epsilon N_1 + \epsilon N_1 + k, \quad \epsilon = p_1,
\]

\[
BN_1 + C = 1, \quad \epsilon = p_2,
\]

\[
2AN_1 + BN_1^{(1)} + E = p_2/p_1, \quad N_1 + N_1 = N.
\]

Solving the above for \( U_1^{(3)} \) and \( N_1 \) in terms of \( N \) and substituting in the formula (30) we have

\[
\psi(N) = \epsilon N + \tau,
\]

where

\[
\tau = \frac{1}{B_1} \left[ \epsilon (AC + BK + BCE - 2AC + A + BE) + B_1k - \epsilon B(1 - C) \right].
\]

Suppose now that \( \theta \) is modified by the addition of quadratic terms so that
draw general qualitative conclusions about the aggregate effect of technological changes.

For example, suppose a technological change occurs so that output increases in both industries by the same percentage \( \lambda \), the percentage being independent of employment. Then \( \delta \phi = \lambda \phi \) and \( \delta \theta = \lambda \theta \). The formula (39) gives

\[
(40) \quad \delta \psi = \lambda [\theta + \theta \phi].
\]

Dividing both sides by \( \psi \) and noting (30) we have

\[
(41) \quad \frac{\delta \psi}{\psi} = \lambda \left[ 1 + \frac{\theta \phi}{\theta} \right],
\]

which shows that the percentage increase in aggregate output is greater than \( \lambda \) by an amount dependent on employment and the functions involved. Thus, even in this simplified example, it appears that technological change has a cumulative effect in the aggregate and that extreme care must be exercised in drawing conclusions about over-all effects from data on particular firms.\(^{16}\)

3. THE AGGREGATION PROBLEM

Until recent years economic theory has dealt primarily with the individual firm or household, treating the economy as a whole only in terms of general-equilibrium systems of such microeconomic units. In the last decade, increasing attention has been given to theories couched directly in terms of relations among economic aggregates. Such macro-models have been constructed on an empirical or intuitive basis, often by analogy with microeconomic behavior. It is inevitable that efforts

\[
\theta^* = \alpha N^2 + \epsilon b N \psi + cU_1 n + e N_1 + k.
\]

Utilizing (39) and the expressions for \( U_1 n \) and \( N_h \), we have immediately

\[
\frac{\delta \psi}{\psi} = g N^2 + h N + l,
\]

and hence the new production function

\[
\psi^* = g N^2 + (e + h)N + (r + l),
\]

where

\[
g = \alpha,
\]

\[
h = \frac{e}{B^2} \left( b(AC + B^2 K - BCE - 2AC + A + BE) - 2aB(1 - C) \right),
\]

and

\[
l = \frac{e(1 - C)}{B^2} \left[ aB(1 - C) - b(AC + B^2 K - BCE + 2AC + A + BE) \right].
\]

\(^{16}\) In the discussion of possible economic effects of atomic energy there has been a tendency to ignore the interrelations of industries and hence to underestimate aggregate effects.
be made to establish a more rigorous connection between the two types of theory, both in the interest of scientific consistency and because it appears that results in this field may clarify fundamental issues involved in both types of theory.

The theory of gases offers a suggestive analogy from physics. The laws of behavior of a gaseous mass were formulated in terms of observed relations among aggregates such as temperature, pressure, and volume. Then it was shown that the theory of mass behavior (macrotheory) could be deduced from the laws of behavior of gaseous particles (microtheory). On the one hand, the theory of behavior of the particles was confirmed because its aggregate implications corresponded with observation. On the other hand, the derivation explained certain observed deviations from simplified aggregate relations such as Boyle's Law. It is unsafe to carry analogies between sciences too far, but there are certain features independent of the particular subject matter. In the first place, the macrotheory was derived from the microtheory, each theory taking the form of systems of equations. Secondly, the aggregates were not available for arbitrary definition for the purpose of facilitating the derivation, but their definitions had to conform to the meaning of the aggregates observed and considered in connection with existing macrotheory. Finally, the macrolaws followed from the microlaws, but they were not the same. Because of the interrelations of the particles and the averaging of random movements, the macrolaws had a distinct character.17

It is certainly reasonable to pose a similar aggregation problem in economics. Of course, the situation is complicated by the multiplicity of models, both micro- and macroeconomic, and by the frequent ambiguity in the meaning of the variables considered, particularly in macromodels. But provided we know what variables are being considered in a macromodel, it is reasonable to ask whether that macromodel follows from a particular micromodel. And given a microtheory and certain aggregates it is reasonable to ask what laws among the aggregates are implied. In any case, since the economy is hardly less complicated than a gas, it seems rash to insist in advance on the exact character of the aggregate implications of macromodels. The proposition that "the behavior of a group of individuals, or a group of firms, obeys the same laws as the behavior of a single unit"18 is evidently a heuristic guide rather than an answer to the aggregation problem.

Francis W. Dresch's initial attack on the aggregation problem was

17 See Sir James Jeans, An Introduction to the Kinetic Theory of Gases, New York, 1940. The above approach is of course taken for granted in any work on theoretical physics.

18 J. R. Hicks, Value and Capital, Oxford, 1939, p. 245.
made along the above classical lines. He considered a general equilibrium of the traditional type and defined aggregates so that their economic significance was that of the variables of G. C. Evans' two-industry simplified economic system. The exact indices chosen were adaptations of the Divisia indices, originally constructed to satisfy certain rigorous theoretical criteria and bearing about the same relation to the numerous, approximately equal, and well-known indices as does the "force of interest" to periodically compounded interest rates. Dresch then showed that these aggregates are related in ways very similar to the relations of the simplified model. The macromodel was not actually derived, because the existence of the aggregate production functions was not established, but it may be remarked that no one has yet bettered this effort to connect a two-industry model with a general equilibrium.

Lawrence R. Klein, in his first contribution to the aggregation problem, considered taking the micromodel and the definitions of the indices as given and deriving the macromodel. However, he decided instead to "assume the theory of micro- and macroeconomics, and then construct aggregates (usually in the form of index numbers) which are consistent with these two theories." In addition he set the condition that the aggregates of factors and output should be functionally related independently of the relations of the micromodel other than the firms' production functions. Since these latter make up a system of \( m + r + s - A \) degrees of freedom, while the desired production function gives a system of two degrees of freedom, it is evident that a set of aggregate definitions will not in general imply an aggregate production function. However, since the two systems are taken as given and the definitions are to be made consistent with them, the problem is indeterminate in the sense that we can construct as many such aggregates as we please simply by defining one as a function of the other two and then determining parameters so that the macromodel has whatever other characters have been assumed. Klein facilitates this by assuming special forms for the production functions.

Klein's problem is interesting, but it is not the same as the problem of deriving one model from another, and his macromodel does not follow from his micromodel. On the contrary, the macro- and micro-

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19 Evans' model is essentially that used in section 2 above.
21 Loc. cit., (B).
22 Ibid., p. 94.
23 Ibid., p. 94.
models have both been taken as given, and it is the indices which are derived, though not uniquely. Essentially, Klein has proposed new criteria for index construction which appear to be not always consistent with accepted criteria. In addition it should be pointed out that the meaning of indices defined in this way depend upon the production functions of the micromodel. This means that with technological change, the definitions of the aggregates may change. And it must not be forgotten that different definitions mean that we are talking about different quantities unless they give approximately the same numerical values.

Shou Shan Pu’s contribution indicates that he views the aggregation problem as one of deriving one model from another. When he says that “all we have to require is that there exists a definite pattern of distribution . . .,” this appears to be equivalent to saying that we must have a determinate micromodel in order to be able to derive a determinate macromodel, or more generally that the degrees of freedom of the two must be the same. In the same note, Pu adds a definite result to the work on aggregation by deriving a one-industry macromodel of a general equilibrium in which labor, capital, and consumption goods are homogeneous.

The author’s approach has been indicated sufficiently above. It is consistent with those of Dresch and Pu, and may be formulated in general terms as follows: Given a microsystem involving \( \alpha \) economic variables and \( \beta \) independent equations

\[
(50) \quad f_i(x_1, x_2, \ldots, x_\alpha) = 0, \quad i = 1, \ldots, \beta,
\]

and hence having \( \rho = \alpha - \beta \) degrees of freedom, and given \( \gamma \) independent aggregates defined by

\[
(51) \quad X_j = G_j(x_1, x_2, \ldots, x_\alpha) \quad j = 1, \ldots, \gamma,
\]

then we may derive \( \gamma - \rho \) independent equations among the aggregates alone:

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28 The statement that “it has been shown that a well-defined macroeconomic system can be established from a knowledge of the microeconomic system” is thus misleading [loc. cit., (E), p. 310].


30 This is a property of the Divisia index and various other theoretically acceptable indices which all give approximately the same results. They are all functions of prices and quantities, but their definitions are independent of the functions and parameters of the micromodel. On the other hand, the indices derived by Klein in loc. cit., (B), are dependent in their definitions on the functions of micromodel. Hence if the character of the production functions change, the meaning of the aggregates may be modified.

31 Loc. cit., (D).
\[(52) \quad F_i(X_1, X_2, \ldots, X_\gamma) = 0, \quad i = 1, 2, \ldots, (\gamma - \rho)\]

This set of equations constitutes a macromodel of (50). Its equations may be found by choosing any \(\rho + 1\) of the equations (51) and eliminating the microvariables from these and (50). Of all possible resulting equations, \(\gamma - \rho\) will be independent so that the macromodel has the same number of degrees of freedom as the micromodel.

In the above we make no restrictions on the variables that occur in (50), which may be a static or dynamic system. Exogenous variables and parameters are of course implicit in (50). Evidently the functions of the macrosystem are dependent on all the functions of (50) unless some separation of the system is possible. We assume that the definitions of the aggregates are chosen so as to be theoretically acceptable measures of the macroquantities in which we are interested. This being the case, the properties of the aggregates \(X_j\) as given by (52) are derived from the micromodel (50). In this way we can work out the macroimplications of any micromodel with respect to whatever aggregates we chose, provided we take \(\gamma > \rho\).\footnote{Evidently the elimination is impossible if \(\gamma \leq \rho\). This is the case in the problem posed by Klein and hence additional conditions have to be imposed on the functions involved.}

The above method of aggregation is not, of course, suggested as the method of finding empirically the parameters of a macromodel, i.e., by determining the functions of (50) empirically and then deriving those of (52). The derivation method is suggested as a way of explaining and deriving the general form of a macromodel, the parameters of which are then to be determined directly by statistical methods. The same remarks apply to the aggregation of technological change. Actually, since statistical data on aggregates is easier to obtain than data on individual economic units, by connecting micro- and macromodels we have a method of testing the former by seeing whether their macroeconomic implications are verified.

It would appear from the above that there should be no controversy over the approach of Klein and that of Dresch and Pu, since it seems to be a matter of posing two different problems. However, the former objects to utilizing the entire micromodel in deriving the production functions of the macromodel because “there are certain equations in microeconomics that are independent of the equilibrium conditions and we should expect that the corresponding equations of macroeconomics will also be independent of the equilibrium conditions.”\footnote{\textit{Loc. cit.}, (E), p. 303.} But if one examines the one-industry models derived in \textit{loc. cit.}, (C) and (D), one notices that their production functions are independent of
the equilibrium conditions involved in the macromodel, just as the production functions of the micromodel are independent of the equilibrium conditions of the micromodel. Hence the production function in our one-industry model plays the same role in that model as do the firms’ production functions in the micromodel. On the other hand the aggregate production function is dependent on all the functions of the micromodel, including the behavior equations such as profit-maximization conditions, as well as upon all exogenous variables and parameters. This is the mathematical expression of the fact that the productive possibilities of an economy are dependent not only upon the productive possibilities of the individual firms (reflected in production functions) but on the manner in which these technological possibilities are utilized, as determined by the socio-economic framework (reflected in behavior equations and institutional parameters). Thus the fact that our aggregate production function is not purely technological corresponds to the social character of aggregate production.

Moreover, if we examine the production function of a particular firm, it appears that it, too, is an aggregate relation dependent upon nontechnical as well as technical facts. It tells us what output corresponds to total inputs to the firm of the factors of production, but it does not indicate what goes on within the firm. In order to get a single-valued production function we have to assume some decision as to how each combination of the factors is to be utilized, and this implies matters of organization as well as the technological characteristics of fixed capital equipment, raw materials, etc. Furthermore, the input-output relation depends upon other factors such as worker morale, management methods, and the time schedule of production. In short, the firm’s production function cannot be derived alone from a knowledge of the production functions of its subunits, any more than the whole economy’s production function can be derived solely from the firms’ production functions. Both involve relations that are nontechnological and even noneconomic in character.

The above discussion suggests that the terms “micro” and “macro” are relative, and that what is economic and what is “purely” technological depend upon the point of view. We shall have to go to units much smaller than the firm to isolate the latter. Finally, it appears that the problem of deriving a macromodel involving given aggregates of a micromodel leads to aggregate production functions whose character is determined by the entire microeconomic structure.

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