Cowles Foundation Paper 19b
Reprinted from
ECONOMETRICA, Journal of the Econometric Society, Vol. 14, No. 4, October, 1946
The University of Chicago, Chicago 37, Illinois, U.S.A.

A NOTE ON MACROECONOMICS

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IN an article published in the April, 1946, issue of this JOURNAL, Dr. Klein presents a new approach to the problem of the construction of macroeconomic values,[1] a problem which has not yet been given the wide attention it deserves. Two criteria for aggregates are proposed in Dr. Klein's article: (1) if there exist functional relations that connect output and input for the individual firm, there should also exist functional relations that connect aggregate output and aggregate input for the economy as a whole or an appropriate subsection, and (2) if profits are maximized by the individual firms so that the marginal-productivity equations hold under perfect competition, then the aggregative marginal-productivity equations must also hold.[2] In this note, we shall first show that these criteria are too restrictive and unnecessarily so. On the basis of our criticisms, we shall then attempt to establish different criteria for the construction of macroeconomic values in the general case.

I. Let us first consider, in nonmathematical terms, what the first criterion really means. It means, as one can easily see after following carefully Dr. Klein's arguments, that *the aggregate output must be independent of the distribution of the various inputs.* If X represents the aggregate output and N and Z represent two aggregate inputs, labor and capital, the first criterion requires that X depends only on the magnitudes of N and Z, and not on the way in which N and Z are distributed among different individual firms, nor on the way in which N and Z are distributed among the different types of labor and capital within any individual firm. In other words, as long as N and Z are kept constant, the way in which they are distributed must be of no significance. It is obvious that this criterion is most unlikely to be satisfied in any practical case. If this criterion is strictly adhered to, one will find in most cases that either no aggregates can be found to fulfil this criterion, or, in the case where this criterion is satisfied by the manipulation of the construction of aggregates, the aggregates will become such monsters that they are completely void of any economic significance.[3] The field in which macroeconomics may apply would then indeed be extremely limited.

[1] Lawrence R. Klein, "Macroeconomics and the Theory of Rational Behavior," ECONOMETRICA, Vol. 14, April, 1946, pp. 93–108.

[2] *Ibid.*, pp. 94–95.

[3] In the case of the extension of the Cobb-Douglas production function, in which Dr. Klein finds that the first criterion is satisfied (*ibid.*, pp. 102–103), the aggregate labor N and the aggregate capital Z as constructed certainly have no

The second criterion, which is relevant only for the construction of an aggregate production function, seems also unnecessary and arbitrary. If a unique aggregate production function exists, it would be useful in a macroeconomic system even though the second criterion is not satisfied. The conditions for the maximization of profits can be represented as well by some form of equations other than the usual marginal-productivity equations.

II. Fortunately, the requirement that the aggregate output must be independent of the distribution of the inputs, though a sufficient condition, is not a necessary condition for the existence of a unique aggregate production function. Instead of requiring that the distribution of inputs must be totally irrelevant to the determination of the aggregate output, all we have to require is that there exists a definite *pattern of distribution, i.e. a definite way in which inputs must be distributed both as among different individual firms and as among different types of inputs*. As long as there are any definite relations that determine this pattern of distribution, a unique aggregate production function can be formulated.

Consider first the simplest case where output, labor, and capital are all perfectly homogeneous. The individual production functions are then of the form of

$$x_\alpha = f_\alpha(n_\alpha, z_\alpha), \qquad \alpha = 1, 2, \cdots, A, \tag{1}$$

where x_α, n_α, and z_α are respectively the output, the labor input, and the capital input of the αth firm. The natural type of aggregation is obviously simple summation. We have, therefore,

$$N = \sum_{\alpha=1}^{A} n_\alpha, \qquad Z = \sum_{\alpha=1}^{A} z_\alpha, \qquad \alpha = 1, 2, \cdots, A. \tag{2}$$

Now, if we assume that the pattern of distribution of inputs among different individual firms is determined by the marginal-productivity equations of the individual firms, we have

$$\frac{\partial x_\alpha}{\partial n_\alpha} = \frac{\partial x_\beta}{\partial n_\beta}, \qquad \frac{\partial x_\alpha}{\partial z_\alpha} = \frac{\partial x_\beta}{\partial z_\beta}, \qquad \begin{array}{l} \alpha = 1, 2, \cdots, A, \\ \beta = 1, 2, \cdots, A. \end{array} \tag{3}$$

In (1), (2), and (3), we have $3A$ equations and $3A$ variables, not

connection whatsoever with the economic variables which we are actually interested in, the total volume of employment and the total quantity of capital. Even though we may solve for N and Z in a macroeconomic system which contains such an aggregate production function as one of the relations, we shall still be at a complete loss as to the actual magnitudes of the total volume of employment and of the total quantity of capital.

counting N and Z as variables. A transformation,[4] therefore, can be made such that all the n_α and z_α are expressed in terms of N and Z. Substituting N and Z for n_α and z_α in (1) and summing up all the individual production functions, we obtain a unique aggregate production function,

$$X = F(N, Z),$$

where

$$X = \sum_{\alpha=1}^{A} x_\alpha, \qquad \alpha = 1, 2, \cdots, A. \tag{4}$$

It can be easily proved[5] that

$$\frac{\partial X}{\partial N} = \frac{\partial x_\alpha}{\partial n_\alpha}, \qquad \alpha = 1, 2, \cdots, A,$$

and

$$\frac{\partial X}{\partial Z} = \frac{\partial x_\alpha}{\partial z_\alpha}, \qquad \alpha = 1, 2, \cdots, A.$$

Hence, in this special case, Dr. Klein's second criterion is also fulfilled.

Since the marginal-productivity equations are used as determining the patterns of distribution of inputs among individual firms, the aggregate production function holds only when the relative positions of the individual firms are at equilibrium. This limitation confines our aggregate production function to a macrostatic system. The pattern

[4] This transformation depends, of course, on the nonvanishing of the Jacobian. This condition, however, is practically always fulfilled, provided that the marginal-productivity equations of the individual firms can be satisfied.

[5] From (4), (1), and (3), we have

$$\begin{aligned}\frac{\partial X}{\partial N} &= \sum_{\alpha=1}^{A} \frac{\partial x_\alpha}{\partial N} \\ &= \sum_{\alpha=1}^{A} \frac{\partial x_\alpha}{\partial n_\alpha}\frac{\partial n_\alpha}{\partial N} + \sum_{\alpha=1}^{A} \frac{\partial x_\alpha}{\partial z_\alpha}\frac{\partial z_\alpha}{\partial N} \\ &= \frac{\partial x_\alpha}{\partial n_\alpha}\sum_{\alpha=1}^{A} \frac{\partial n_\alpha}{\partial N} + \frac{\partial x_\alpha}{\partial z_\alpha}\sum_{\alpha=1}^{A} \frac{\partial z_\alpha}{\partial N} \\ &= \frac{\partial x_\alpha}{\partial n_\alpha} + \frac{\partial x_\alpha}{\partial z_\alpha}\frac{\partial Z}{\partial N}, \qquad \alpha = 1, 2, \cdots, A.\end{aligned}$$

Since Z is held constant,

$$\frac{\partial X}{\partial N} = \frac{\partial x_\alpha}{\partial n_\alpha},$$

Similarly,

$$\frac{\partial X}{\partial Z} = \frac{\partial x_\alpha}{\partial z_\alpha}.$$

of distribution, however, does not have to be determined by the equilibrium marginal-productivity equations. The distribution of inputs may be determined by such relations that the resulting aggregate production function is also useful for a dynamic analysis.

In the general case where x_α, n_α, and z_α are not all homogeneous, we must have, in addition to the pattern of distribution of inputs as among the different individual firms, a definite pattern of distribution of X, N, and Z as among the different types of outputs and inputs. If such a definite distribution pattern exists, each individual production function,

$$f_\alpha(x_{1\alpha}, \cdots, x_{m\alpha}; n_{1\alpha}, \cdots, n_{\gamma\alpha}; z_{1\alpha}, \cdots, z_{s\alpha}) = 0, \qquad \alpha = 1, 2, \cdots, A,$$

can be transformed into the form of (1), which, as has been shown, can then be transformed into a unique aggregate production function.[6]

III. The criteria for the construction of macroeconomic values in the general case may now be formulated as follows:

(1) For the construction of an individual macroeconomic value, there must exist a definite pattern of distribution of that macroeconomic value as among the different elements of which it is the composite.

(2) For the construction of a macroeconomic relation, there must also exist a definite pattern of distribution of all the macroeconomic values involved as among the different units of economic decision, such as firms or households.

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[6] The simplest example of this distribution pattern is the case where various outputs are produced in given proportions and various inputs are employed also in fixed proportions. In such a case, all the x's, n's, and z's can be transformed respectively into the single variables X, N, and Z.