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A POST-MORTEM ON TRANSITION PREDICTIONS
OF NATIONAL PRODUCT¹

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WE ALL recall clearly the headlines in last autumn's press, declaring that "Government economists predict 8 million unemployed by 1946." At the same time, the newspapers were telling us of congressional action to lower corporate and personal income taxes, administrative action to relax ceiling prices on construction materials, lobbyists' pressure to weaken the strength of price controls, and many other inflationary policies. We now find ourselves in the first half of 1946 with about three million unemployed and facing one of the greatest inflationary pressures that we have ever experienced. The economists who were warning us of a deflationary danger during the early months of the postwar transition period should have been stressing precisely opposite economic policy.

There is no question that they made a mistake, but there are many questions

concerning why they made the mistake. Was it a result of their methodology? Was it a result of their intuitive judgments? Was it a result of their theoretical economic systems? It is very important to answer these questions because many economists would discredit much more than the wrong estimates themselves. Some economists claim that the wrong predictions show that the entire theoretical economic model and the methodology of forecasting are wrong. We shall attempt to show that such claims are unfounded and that errors in forecasting may have nothing to do with the validity of many of the underlying theories. There are some economists who made more accurate predictions with other methods, but we shall also attempt to show that these more accurate predictions do not prove that their methods are superior to those that failed.

The current controversy contains several "red herrings," which should be disposed of before we can get at the truth of the matter. In the first place, many of the estimates for the posttransition period are loosely quoted in reference to the transition period. The estimates of J.

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Mosak² for 1950 have frequently been mentioned, along with those pertaining to 1945, 1946, and 1947. While the estimates for the latter years are proving to be wrong, the former may still be correct, unfortunately. Second, the estimates of gross national product and of unemployment should be separated. It is possible for economists to predict output more closely than the difference between labor input and the labor force. Their models actually are intended to forecast gross national product, and their estimates of unemployment are a by-product. The validity of their models must be judged on the basis of the accuracy of their estimates of output rather than of unemployment.

It is not implied that unemployment estimates are unimportant. Our objective should be to forecast unemployment, but some of the steps in reaching that objective may be independent of others. A complete model and a good model will enable us to forecast both the level of output and the level of unemployment. In the abnormal conditions of the transition period, we may have to be satisfied with partial results.

THE METHOD

The customary model for prediction can be called the simplest Keynesian model: Savings as a function of income equals autonomous investment.³ This

² "Forecasting Postwar Demand," *Economic Reconstruction*, ed. S. Harris (New York: McGraw-Hill Book Co., Inc., 1945).

³ The terms "autonomous" and "exogenous," and their respective opposites, "induced" and "endogenous," will be used synonymously in this paper. They are all commonly found in the recent economic literature. Autonomous variables are those economic quantities that are determined outside the system of narrow economic forces. They are the result of political, sociological, institutional, technological, or natural forces only. Eventually we may hope to develop a complete social theory in which there are no autonomous variables except those like weather.

model has been slightly modified for the particular characteristics of the transition period, but the modifications are not basic. The model can be written as

$$GNP = C(GNP) + \bar{I},$$

where GNP = gross national product; the function $C(GNP)$ is the consumption schedule; and \bar{I} = autonomous investment.

On the basis of a priori information, economists attempt to estimate \bar{I} , and from numerical data over a long-time period they attempt to get a statistical approximation of the function $C(GNP)$. For the transition estimates the common practice is to include in C only nondurables and services exclusive of paid rentals. Durable consumer goods and rents are classed with \bar{I} .

The function $C(GNP)$ is estimated in two steps. First, statisticians calculate the relation between consumption and disposable income, $C = C^*(Y)$. The difference between GNP and Y is governmental revenue + business reserves⁴ + corporate savings - transfer payments. The function $C = C^*(Y)$ is a structural behavior equation which can be derived, in a general form, from rational economic behavior patterns. The function $C(GNP)$ depends upon the relation between Y and GNP , a relation which is based, not on economic behavior patterns, but on autonomous governmental action with respect to the setting of tax rates, unemployment compensation rates, etc. From their knowledge of the autonomous parameters, such as tax rates, technicians of the Bureau of the Budget have very carefully computed $Y = Y(GNP)$ for different periods of the transition. For each tax system they obtain a different relation between Y and GNP . The statistician then substitutes the appropriate au-

⁴ Mainly depreciation charges.

onomous function $Y = Y(GNP)$ into the statistical behavior equation $C = C^*(Y)$ to get $C = C(GNP)$. The parameters of the Y -function, and hence of the C -function, will be different for each period of the transition, depending upon the autonomous factors.

Our Washington forecaster then computes, item by item, the components of \bar{I} . For example, he gets information from the Bureau of the Budget on governmental expenditures, from Commerce Department questionnaire surveys on capital formation, etc. He makes allowances for production bottlenecks in supplying a virtually unlimited demand for some components, and he takes into account the quantity of surplus government property that will be used to satisfy some of the autonomous demand in the transition period. The latter correction should be subtracted from capital formation because no income is created by transferring ownership on already existing property.

This is the method. Let us see how it works.

ACCURACY OF THE ESTIMATES

Some of the better-known estimates among economists are those of E. E. Hagen, assisted by Mrs. Nora Kirkpatrick, both of the Office of War Mobilization and Reconversion. These estimates were presented in a paper before the Conference on Research in Income and Wealth in November, 1945. The author saw them at an earlier date and hazards the guess that they had some influence on governmental policy declarations in the days just following the end of the war. In Table 1 the Hagen-Kirkpatrick estimates are compared with the official observations for the fourth quarter of 1945, published in the *Survey of Current Business*, February, 1946, page 7.

It is immediately obvious where this forecast failed—in the prediction of consumer expenditures, particularly expenditures on nondurable goods. The forecasts of most of the autonomous items of government expenditures or capital formation are only slightly below the observations, and these errors together do not contribute so much to the total error as does the error in consumption alone.

The order of magnitude of the error involved is great, and, what is more serious, it is great enough to lead to disastrous policy recommendations. The predicted *GNP* of \$164.5 billion should call for an inflationary policy, but this is just the opposite of the policy that was needed.

The forecasts for the first quarter of 1946 are as wide of the mark as are those for the fourth quarter of 1945, and it is generally conceded that these forecasts will continue to be wrong for some time in the future. For example, Hagen and Kirkpatrick predicted *GNP* of \$161.8 billion and unemployment of 8.1 million for the first quarter of 1946, while the *Sixth Report by the Director of War Mobilization and Reconversion* estimates *GNP* at approximately \$180 billion and unemployment at approximately 3 million. A very recent (May, 1946) press release of the Commerce Department puts *GNP* at \$183 billion for this period.

EXPLANATIONS OF THE FORECASTING ERROR

Errors may arise in the forecasting procedures of the Washington economists through three sources: (1) the autonomous variables may be set at wrong levels; (2) variables may be improperly classified, according as they are autonomous or induced; and (3) the estimates of the structural behavior equations involving the nonautonomous variables

may be incorrect from an economic theoretical point of view and from a statistical point of view.

The first type of error does not appear to be serious from the data presented above. Furthermore, any method of forming economic policy judgments is

error. If induced variables are set at autonomous levels that agree with the observed facts, this error will not, by itself, lead to an error in the estimate of *GNP*, but there will be an error in the evaluation of various multipliers in the system. The second source of error will

TABLE 1
GROSS NATIONAL PRODUCT: SEASONALLY ADJUSTED ANNUAL RATE

	Hagen-Kirkpatrick Estimate (In Billions)	U.S. Dept. Commerce Actual Observations (In Billions)
Gross National Product	\$164.5	\$182.8
Government expenditures	55.5	57.7
Federal war	39.7	43.3
Federal nonwar	8.0	6.5
State and local	7.8	8.0
Private gross capital formation	12.8	14.2
Construction	2.8	3.7
Producers' durable equipment	6.0	7.2
Net inventory change	4.0	1.6
Net exports	0.0	1.8
Consumer Expenditures	96.2	110.9
Durable goods	9.0	8.4
Nondurable goods	57.7	69.5
Services (excluding rent)	23.5	33.0
Rent	6.0	
Disposable income	119.9	137.3
	(In Millions)	(In Millions)
Civilian employment	46.1	51.7
Armed forces	10.8	9.2
Unemployment	6.3	1.8
Labor force	63.2	62.7

subject to this error; consequently, the econometric method should not be discredited for wrong forecasts of autonomous variables. In any case, there is no serious problem in connection with this source of error.

Where induced variables were classified as autonomous in the transition forecasts, no serious harm resulted. It is often possible to obtain some of the correct results and yet commit the second type of

lead us to estimate incorrectly the *increase* in *GNP* resulting from a given *change* in government spending. We may claim that the Hagen-Kirkpatrick estimate committed an error in classifying construction, inventories, and producers' equipment as autonomous variables; however, this error did not account principally for their low forecasts, since their autonomous variables were nearly equal to the observations.

Most critics will agree that the consumption function is incorrect,⁵ i.e., that the third source of error is prominent. But the critics are strongly divided on the reasons why the consumption function is incorrect. The consumption function may be incorrect because it was determined statistically in isolation from the rest of the system. The statistical parameters were calculated from least-squares regressions of nondurable consumption on disposable income and of service consumption on disposable income. No account was taken in these regressions of the interrelationships between these variables and other equations of the system.⁶ It is not known whether this type of error will lead to an overestimate or an underestimate of consumption.⁷ At least as important as the statistical errors, however, are the errors of economic theory. It is a real problem to construct a satisfactory economic model, and the pages

immediately following will deal with this problem more carefully.

There are undoubtedly many arguments advanced to explain why the consumption of nondurable goods is high, relative to the pre-war regression between this type of consumption and disposable income. We shall discuss arguments that are either plausible or commonly made.

THE FACTS

In order to make clear the various reasons advocated for changing the Hagen-Kirkpatrick model, it will be illuminating to consider, in detail, the reasons for the failure of their method. In the *Survey of Current Business* (February, 1945, p. 5) there are three graphs which shed much light on the mystery. These three diagrams show the scatter plots between disposable income and three types of consumer expenditures—durables, nondurables, and services. All the variables are aggregates measured in current dollars. It will be noticed that during the war years the observations for nondurable expenditures were on the 1929-40 regression, while the observations for durables and services were, for obvious reasons, far below the regression. The forecasters reasoned that nondurables, which had maintained their peacetime relation to income during the war, would maintain that relation during the postwar transition and that services would return to the peacetime relation for all categories except paid rentals, while durables would remain below the corresponding regression line. Thus they made durables plus paid rentals autonomous, and other services plus nondurables induced, according to the pre-war regressions in relation to disposable income.

⁵ Some, however, will claim that it was only temporarily incorrect during the special case of the post-war transition period and that it will be correct when we settle down again to a peacetime economy.

⁶ For a popular exposition of the ideas underlying statistical estimation in economic systems, see the article by T. Koopmans, "Statistical Estimation of Simultaneous Economic Relations," *Journal of the American Statistical Association*, XL (December, 1945), 448-66.

⁷ The author's colleague, T. Haavelmo, has shown in a forthcoming article that the estimate of the marginal propensity to consume calculated as the least-squares regression of total consumption on disposable income is greater than the estimate obtained when all the equations of the simple Keynesian model used by the forecasters are taken into account simultaneously. The estimate of the constant term in the linear consumption function is greater by Haavelmo's method than by the customary least-squares method. He did not deal with the problem of estimating the parameters of the demand functions for nondurables, durables, and services separately. His results are very interesting, especially because the pessimistic forecasts of transition *GNP* have often been criticized for being based on a model with a marginal propensity to consume that is too low.

As matters developed, nondurable consumption expenditures were far above the regression for the fourth quarter of 1945, while services and durables were still below. These are the facts; the correct explanation of the error must show why the consumption of nondurables was so far above the pre-war regression line.

DEMobilIZATION

It must be remembered that consumer expenditures cover only civilian expenditures for consumer goods and services. We may have experienced a temporary deviation from established patterns by the sudden injection of demobilized servicemen into our civilian population over a very short period of time. While it may seem possible that the 8.5 million men restored to civilian life during the period between V-J Day and March 31, 1946, would have caused an abnormally high demand for nondurable consumer goods, the argument is not unambiguously established. It is true that a sudden increase of 8.5 million civilian mouths to feed and backs to clothe may very well have led to a temporary increase in the civilian consumption of nondurables, but their disappearance from civilian life during the pre-war and the war years did not cause an abnormally low demand for nondurables. The demobilization process was strongly concentrated in a few months and thus may account for asymmetric behavior patterns, as compared with the period of induction.

Demobilization is a special factor. If it has accounted for a large underestimate of nondurable consumption, we should not be led to alter the statistical models for the post transition period on account of their failures during the transition.

INSTABILITY OF THE CONSUMPTION FUNCTION

Some economists claim that the consumption function changes position radically in different economic situations, making it very difficult to forecast future consumption on the basis of past relationships between consumption and other economic variables. A recent exposition of this hypothesis is that of W. S. Woytinsky,⁸ who claims that there is a different consumption function during depression and during prosperity. With our previous notation, this may be expressed, in a simple example:

$$C = a_0(Y) + a_1(Y)Y,$$

where

$$a_0 = b_0 + b_1Y,$$

$$a_1 = c_0 + c_1Y;$$

or

$$C = d_0 + d_1Y + d_2Y^2.$$

This function⁹ has in simple form the properties stressed by Woytinsky, but there is so little curvature in the past relationship that there is very little gain in introducing the quadratic term. Either by the above simple scheme or by introducing additional variables (including time) into the consumption function, we can test the hypothesis that the consumption-income relation remains substantially valid over long periods of time. As yet, no econometrician has ever refuted the hypothesis. We find much evidence of this stability in both time-series and family-budget data.

⁸ "Relationship between Consumers' Expenditures, Savings, and Disposable Income," *Review of Economic Statistics*, XXVIII (February, 1946), 1-12.

⁹ In a more general case, $C = f(Y, a_0[Y], a_1[Y], \dots, a_n[Y])$, where a_0, \dots, a_n are certain parameters of the consumption function which fluctuate cyclically. If $f, a_0,$ and a_1 are linear and all other a 's vanish, we get the simplest representation of this hypothesis, given in the text.

Woytinsky has another method of proving his claims. A complete discussion of his methods involves some digression, but the establishment of the true consumption function is so important in forecasting that we must rid the field of false alternatives. Woytinsky claims that we get one savings function if we use the data for years that he defines as "depression years" and another savings function if we use the data for years that he defines as "prosperity years."¹⁰ This is a scientifically unacceptable attitude because it makes the results depend entirely on personal judgment. If we define prosperity as high national income and depression as low national income, then national income should be a variable in the consumption function. If we think that the parameters vary over the business cycle, then we should make the parameters functions of income. But no econometrician is allowed to choose those observations which yield predetermined results. If the savings-income relationship has a *systematic* error when fitted to the data of the entire period covering both prosperity and depression (Woytinsky's point), then the residual variation not explained by the calculated savings function will have a systematic pattern which is not random. The duty of the econometrician is then to introduce other *objective* variables until the residual variation does become random. Woytinsky never examined his residual variation. It is possible to construct savings functions which have random residual variation over a period which covers all phases of the business cycle. These are

¹⁰ It is completely trivial to distinguish between the consumption function and the savings function since the two are related by a simple definition. Some writers seem to think that they get more information from the savings-income data than from the consumption-income data.

the functions which must be considered as candidates for the true savings function and not those fitted to Woytinsky's personally selected points.

It should be pointed out that there are sometimes objective criteria for rejecting observations. We may agree beforehand that economic behavior patterns are distorted in wartime; hence we may cast out all observations pertaining to war years. There is, however, little room for personal judgment in this case because we usually know when we have a war economy and when we have a peacetime economy.

Woytinsky has another tool with which he tries to demonstrate the instability of the consumption or savings function. He correlates the ratio of savings to income with income and concludes that different regressions hold for different periods. If we write

$$\frac{S}{Y} = e_0 + e_1 Y,$$

then

$$S = e_0 Y + e_1 Y^2;$$

thus we find that there is no constant term in the savings function. The extremely poor correlations that Woytinsky finds between S/Y and Y should actually lead him to consider the alternative hypothesis that there is a constant term in the savings function instead of his conclusion that the savings function is not stable. He bases his argument on mechanical least-squares fitting to observations, which gives empirical equations that are not acceptable on any statistical grounds.

There is a good reason for believing that there is a significant constant term in the savings function. The aggregate savings function is obtained by summing all the individual savings functions. Sup-

pose an individual has zero income. Will he consume anything? Of course, he will have positive consumption (negative savings), which represents his minimum subsistence level. Similarly, the community, as a whole, will show a positive expenditure for zero income. Woytinsky's saving-rate functions fail to show this constant term. A glance at family-budget data on consumption by income class shows very clearly that people spend more than their incomes in the low-income classes and that the consumption function has a positive intercept for zero income.

We must look further for a satisfactory explanation of the failure of the Hagen-Kirkpatrick consumption estimates.

LIQUIDITY

It has been suggested that the Keynesian models of macroeconomics be amended by introducing cash balances or, more generally, liquid assets as a variable into the savings (consumption) function.¹¹ The arguments about the relation of cash balances to savings are being revived again today, although for slightly different reasons than were put forth originally. Several economists claim that consumption is being maintained at an abnormally high level now because people have accumulated huge amounts of cash and liquid securities (in the form of war bonds). It is true that there is an observed correlation today between high levels of consumption and high levels of liquid assets, but it has not been proved that the structural savings function contains liquid assets as an important variable.

The relationship between consumption and liquid assets is a hypothesis which must be tested from the available

data. To test the existence of the consumption-income relationship, we examine the time-series data and the family-budget data. In the case at hand, we are very unfortunate in having no family-budget data and must rely on time-series data plus some recent surveys that have been conducted to discover what people intend to do with their war bonds and cash balances.

First, we must establish the theoretical principles from which we construct our consumption functions. A simple theory of consumer behavior is that the individual maximizes some preference function which depends on his consumption of all types of present and future goods, subject to the constraint that current spending and saving (spending on future goods) just exhaust individual income. It is well known that this maximization procedure leads to the individual demand equations for each commodity as a function of the prices of all goods and the budget constraint, which is income. The prices of future goods are related to present prices by the interest rate, so that the independent variables in the demand functions are present prices, the interest rate, and income. If these individual demand functions are approximately linear, we may aggregate them over all commodities and all individuals to find that total consumption of present goods depends upon the price level of consumer goods, the interest rate, and the level of the community's disposable income.

The consumption functions that are calculated today are based on this principle, except for the fact that they allow the income term to dominate the price level and the interest rate. They use only the expenditure-income correlation. In some cases prices are introduced as a deflator, i.e., they correlate real consumption and real income. This procedure is

¹¹ See, e.g., A. C. Pigou, "The Classical Stationary State," *Economic Journal*, LIII (December, 1943), 343-51.

correct if the demand equations derived from the basic theory of consumer behavior are homogeneous of order zero in prices, as is commonly assumed.¹²

One obvious way to introduce liquid assets into the theory of the consumption function is to modify the budget constraint. Instead of maximizing satisfactions, subject to the condition that spending plus savings exhausts income, we can maximize satisfactions subject to the condition that spending plus savings exhausts some function of income and accumulated liquid assets. Alternatively, we may define the utility function to depend upon the consumption of present and future goods, the holding of securities, and the holding of cash. We may now proceed to maximize this function, subject to two constraints: (1) spending plus savings equals income, and (2) the value of securities plus cash equals historically accumulated savings plus current savings.

In these cases the maximization process leads to demand equations with the new constraining factors as variables. In either case, both income and historically accumulated savings enter as independent variables.

This is the theoretical foundation of our hypothesis. Let us now examine the data to see whether or not it is true.

Correlations between consumption and cash balances from time-series data of the interwar period are nil, if the relationship between consumption, income, and time trend is taken into account. This same result was obtained independently by J. Mosak.¹³ It is true that cash

balances were much smaller in the interwar period, but so was the whole scale of operations in the economic system. We must admit, however, that liquid assets are extremely large today even in relation to the other variables of the system. It is also true that, in the past, assets were highly concentrated in a few hands. It is not known whether or not they are more democratically held today, but it is known that there is still much concentration. For example, governmental surveys^{13a} show that the upper 10 per cent of owners of liquid assets hold 60 per cent of the value of all liquid assets. The upper 20 per cent hold 77 per cent, and the lower 50 per cent hold only 3 per cent. It is also shown in these reports that the distribution of ownership of liquid assets was much less equitable than was the case for 1945 incomes, if we measure the degree of equitability by the area under the Lorenz curve. The people who own these assets are not those whose spending habits are likely to be influenced greatly by these holdings. When asked about their intentions with regard to the use of E bonds, only 5 per cent of all holders answered that they would use the proceeds for consumer expenditures. It is also estimated that these 5 per cent would spend only about \$1.5 billion. Furthermore, only 5 per cent of all holders intend to use their E bonds for any purpose in 1946. Asset holders appear from these questions to be much more willing to spend bank deposits than E bonds in the near future. However, the anticipated expenditures of all types of assets on consumer goods should not be large enough to cause much of an error in

¹² This homogeneity property means that, if prices and incomes are all changed in the same proportion, the demand for consumer goods will remain invariant.

¹³ See his article, "National Budgets and National Policy," *American Economic Review*, March, 1946, pp. 20-43.

^{13a} Some of the results of the surveys are found in "A National Survey of Liquid Assets," *Federal Reserve Bulletin*, June, 1946, pp. 574-80. The full report can be obtained from the Division of Program Surveys, Bureau of Agricultural Economics, U.S. Department of Agriculture.

the aggregate consumption function, relating consumption to income alone.¹⁴ A future study of the influence of asset holdings on spending out of *current income* has been promised by those who conduct these surveys, and these results may be very important in testing the hypothesis that consumption depends upon the holding of assets as well as income. There is much more information in these reports, but it all points, thus far, to the same conclusion, namely, that the influence of liquid assets on consumption will be small. The hypothesis of the influence of liquidity is plausible and worthy of consideration, but it has not yet been shown to be correct.

SAVING VERSUS SPENDING

One of the simplest theories of consumer behavior is the following: Individuals decide on the basis of their income how much they will spend and how much they will save (not spend). This behavior pattern means that, in a period when there are shortages of certain types of consumer goods, people will spend on available goods that which they are unable to spend on unavailable goods. Thus, today, according to this theory, we may argue that people spend on nondurables because they cannot buy durables or services. Furthermore, we say that the "spillover" from unavailable goods to available goods is just enough to put total expenditures on the normal expenditure-income regression. During the war there were shortages, but civilians were not free to compensate by spending extra sums on available nondurables because of price control, rationing, and patriotic saving drives. Instead of arguing that the wartime experience of spending on nondurables according to the pre-war

¹⁴ The demand for residential housing (investment) may be very differently affected.

regression between these expenditures and income gives us confidence in extrapolating this regression into the transition period (the method of Hagen and Kirkpatrick), we should argue that this wartime phenomenon was accidental. If there had not been artificial restraining factors, consumers would have been spending on nondurables above this regression, as was the case in the last quarter of 1945. In this latter period we find that consumers spent funds on nondurables above the regression line by just that amount which made total expenditures lie almost on the pre-war regression between total consumption and income.

It is one thing to say that there is a stable peacetime relationship between total consumption and income, and something else again to say that there is a stable relationship between each of several categories of expenditures and income. The forecasters adopted the latter formulation and ended up with poor forecasts. *If the Washington economists had used the total consumption function fitted to the data for 1929-41, they would have been nearly correct in their forecasts for the fourth quarter of 1945.*¹⁵

¹⁵ Mosak's aggregate consumption function and Hagen's estimates of autonomous investment and governmental expenditures generate a *GNP* of \$178 billion for the fourth quarter of 1945, very close to the observed figure. There are some compensating errors in this calculation, however, because the autonomous items are slightly below the observed figures, while Mosak's consumption function lies slightly above the observed point. If we use the correct values of the autonomous variables and Mosak's consumption function, the calculated value of *GNP* is above the observed value. The exact figure is not given here because it is necessary to know the true values of the autonomous parameters in the function $Y = Y(GNP)$, which have not yet been estimated. It is worth noting that Haavelmo's estimate of the consumption function, mentioned above, and the true values of the autonomous variables generate a *GNP* of \$187 billion for the fourth quarter of 1945, which is closer than the results obtained by any other method yet tested.

The saving-spending hypothesis of consumer behavior is plausible and is not refuted by the factual data, although we need more observations before we can be certain of its correctness. This hypothesis is certainly confirmed for the pre-war data and also for the first transition observations, but this is not sufficient to establish its validity.

RELATIVE PRICES

There is yet another explanation of why expenditure-income relationships for various types of commodities cannot be used independently for projection into the future, i.e., why the regression between nondurable consumption expenditures and disposable income does not hold in the transition period when there are bottlenecks in the durable consumer-goods market.

The theory of consumer behavior tells us that the demand for each good in the system is a function of all relative prices, interest rates, disposable income, and possibly cash balances or general liquid wealth. Let us neglect, as a first approximation, the influence of interest rates and liquid assets. An aggregative approximation to our demand functions for durable goods, nondurable goods, and services is

$$\frac{C_1}{p_1} = a_{01} + a_{11} \frac{p_1}{p} + a_{21} \frac{Y}{p},$$

$$\frac{C_2}{p_2} = a_{02} + a_{12} \frac{p_2}{p} + a_{22} \frac{Y}{p},$$

$$\frac{C_3}{p_3} = a_{03} + a_{13} \frac{p_3}{p} + a_{23} \frac{Y}{p},$$

where C_1 = per capita expenditures on durables, p_1 = price of durables, C_2 = per capita expenditures on nondurables, p_2 = price of nondurables, C_3 = per capita expenditures on services, p_3 = price of services, p = price of consumption as

a whole, and Y = per capita disposable income.

Suppose that there are shortages of durable goods so that we assign the value $(C_1/p_1) = (\bar{C}_1/\bar{p}_1)$ to this category. The estimates of C_2/p_2 and C_3/p_3 from the last two equations cannot be made from the income term alone but must take into account p_2/p and p_3/p , the relative prices, which will, in turn, be influenced by the fact that durables are scarce. The coefficients of the relative-price terms are all negative. If, in the transition period, the prices of services and durables are extremely high relative to nondurables, there will tend to be an excess demand for nondurables above the consumption-income regression. We can hire domestic servants today, but only at a price. The durable goods which are almost nonexistent have virtually infinite prices. We should expect the relative-price term to favor the purchase of nondurables, as has been the case to date.

The general proposition that is to be made in this section is the following: We cannot estimate the consumption of a particular class of commodities from the relationship to the income variable alone but must consider relative prices as well. The Hagen-Kirkpatrick estimates are derived from the simple income regressions by type of commodity, which are not correct demand equations.

For the period 1919-39, there are time series¹⁶ on all the variables in the preceding equations which enable us to estimate the coefficients numerically. The actual numerical size of the coefficients need not be taken too seriously, since they are known, in general, to be biased¹⁷ estimates, calculated without a knowl-

¹⁶ See the appendix below.

¹⁷ An estimate of a parameter is biased if its mathematical expectation does not equal the parameter.

edge of the entire system. They are single-equation, least-squares estimates which have systematic bias no matter how large the sample used. However, they do show that there is a strong correlation between consumption and relative prices when the consumption-income correlation is taken into account. This is the main point. In the future we can hope to use methods which produce estimates that are unbiased for sufficiently large samples, but the present figures indicate roughly the orders of magnitude of the parameters involved.

The least-squares equations based on the observations 1919-39 are

$$\begin{aligned} \frac{C_1}{p_1} &= 118.53 - 125.96 \frac{p_1}{p} + .1151 \frac{Y}{p} - .6468 (t-1929), & \bar{R} &= .98, & \frac{\delta^2}{S^2} &= 2.08, \\ & (15.26) & & \bar{S} &= \$2.97, \\ \frac{C_2}{p_2} &= 204.43 - 96.70 \frac{p_2}{p} + .2843 \frac{Y}{p}, & \bar{R} &= .94, & \frac{\delta^2}{S^2} &= 1.87, \\ & (14.50) & & \bar{S} &= \$6.60, \\ \frac{C_3}{p_3} &= 68.32 - 46.35 \frac{p_3}{p} + .2551 \frac{Y}{p} + .8165 (t-1929), & \bar{R} &= .73, & \frac{\delta^2}{S^2} &= 1.63. \\ & (19.30) & & \bar{S} &= \$11.15, \end{aligned}$$

Consumption and income are measured in dollars per capita per year. The prices are index numbers with the base-year values equal to unity for 1934. The statistics \bar{R} and \bar{S} are corrected for degrees of freedom, and the statistic δ^2/S^2 is the ratio of the mean square successive difference to the variance of the residuals. The distribution of this ratio has been tabulated,¹⁸ and we conclude from its size that we cannot reject the hypothesis that our residual variation is random.¹⁹

¹⁸ J. von Neumann "Distribution of the Ratio of the Mean Square Successive Difference to the Variance," *Annals of Mathematical Statistics*, XII, 367-95; B. I. Hart and J. von Neumann, "Tabulation of the Probabilities for the Ratio of the Mean Square Successive Difference to the Variance," *Annals of Mathematical Statistics*, XIII, 207-14.

¹⁹ The statistic used to test the randomness of the residual variation is not entirely satisfactory, on theoretical grounds, for our purposes, but it is the best indicator known to the author.

The figures in parentheses below the regression coefficients are standard errors.

The principal thing to note is that a low relative price of nondurable goods in the transition period could easily account for the observed excess purchases above the consumption-income regression. Furthermore, the least-squares estimate of the marginal propensity to consume, obtained by adding a_{21} , a_{22} , and a_{23} , is much smaller than the estimate obtained from the simple correlation between total consumption and total income.

The relative-price theory of this section need not be inconsistent with the saving-spending theory of the preceding

section. We may reconcile them as follows: The three equations

$$\begin{aligned} \frac{C_1}{p_1} &= a_{01} + a_{11} \frac{p_1}{p} + a_{21} \frac{Y}{p}, \\ \frac{C_2}{p_2} &= a_{02} + a_{12} \frac{p_2}{p} + a_{22} \frac{Y}{p}, \\ \frac{C_3}{p_3} &= a_{03} + a_{13} \frac{p_3}{p} + a_{23} \frac{Y}{p}, \end{aligned}$$

can be written as

$$p \left(\frac{C_1}{p_1} + \frac{C_2}{p_2} + \frac{C_3}{p_3} \right) = (a_{01} + a_{02} + a_{03}) p + a_{11} p_1 + a_{12} p_2 + a_{13} p_3 + (a_{21} + a_{22} + a_{23}) Y.$$

Our method of construction of price and consumption aggregates leads to the following definition:

Total consumer expenditures

$$= C = p \left(\frac{C_1}{p_1} + \frac{C_2}{p_2} + \frac{C_3}{p_3} \right).$$

Hence we have

$$\frac{C}{p} = a_0 + \frac{a_{11}p_1 + a_{12}p_2 + a_{13}p_3}{p} + a_2 \frac{Y}{p}.$$

If the data show that

$$a_{11}p_1 + a_{12}p_2 + a_{13}p_3 = \lambda p$$

then our equation will be

$$\frac{C}{p} = a_1 + a_2 \frac{Y}{p},$$

which is the usual form of the consumption function. The test of whether the general price index is approximately proportional to the above specified linear combination can be carried out only after we have obtained unbiased estimates of the relevant parameters.²⁰ In any case, it is not evident that the saving-spending theory and the relative-price theory are contradictory.

Although the relative-price theory may indicate some of the reasons why the forecasts of national product were below the observed levels, it is hardly possible to make use of this theory for quantitative estimates. In order to estimate the different types of consumption during a period in which prices are autonomously fixed, we must have exact data on these fixed levels of prices, which are to be substituted into the demand equations. We must be able to estimate how much the consumption of nondurables will be increased as a result of the fact that the prices of nondurables are favorable, relative to other prices. Unfortunately, none of the published price indexes is satisfactory for this purpose at the present time. The published indexes were satisfactory in the pre-war years in order to

²⁰ The least-squares estimates of a_{11} , a_{12} , a_{13} , when used as weights in the above formula for the price index, lead to a fairly stable ratio (λ) between the derived and the actual index. The annual figures for this ratio vary between -248 and -284 , with a mean at -263 .

establish the regressions, but they are completely inadequate now. In the first place, the practice among those who construct index numbers is to drop scarce goods from the index in such a way that there will presumably be no effect on the index. It is, however, the scarce durable goods and services which are extremely costly today. Weights may be changed because the quantities are completely different from the base-year quantities, but there is a big difference between an index with high prices at a low weight and an index which ignores these prices. The relative-price theory, as an explanation of high nondurable consumption, depends on the changed price relationship between the goods that are now scarce and those that are now available—changes that are not reflected in the published data.

There are numerous other grounds on which the official indexes are an understatement of the actual price levels. The agencies which compute the index numbers admit the existence of quality deterioration but, at the same time, classify this phenomenon as unmeasurable; consequently, there is a serious downward bias in the price index.²¹ Other events, such as uptrading, the growth of black markets, population redistribution, etc., have led to an added bias in the price indexes.

The saving-spending approach is perhaps our best alternative at this time. It was shown above that the saving-spending theory, in constant dollars (real terms), may be equivalent to the relative-price theory for estimates of total national product. But it is difficult to use the saving-spending theory in terms of constant dollars because the general

²¹ This was one of the major points stressed by the C.I.O. in its criticism of the Bureau of Labor Statistics cost-of-living index.

price indexes are faulty. An empirical observation may be our salvation, namely, that statistical models couched in terms of constant dollars do not give very different results from models couched in terms of current dollars when there is no hyperinflation or hyperdeflation. If this empirical fact holds in the future, we can proceed, as was suggested above, by combining the aggregate consumption function in current dollars with the Hagen-Kirkpatrick estimates of the autonomous variables.

A TECHNICALITY

Recall that an important step in the Hagen-Kirkpatrick model is the estimation of the autonomous function $Y = Y(GNP)$, which enables us to pass from disposable income (the relevant variable of the consumption function) to gross national product (the relevant variable for the estimation of employment). This function was constructed by the Bureau of the Budget from information based on tax rates, unemployment compensation rates, and guesses as to the level of depreciation and corporate savings. If the function were correct, the substitution of the observed *GNP* for the fourth quarter of 1945 should give the observed Y . But we get

$$Y = 41.8 + .475 (GNP),$$

$$Y = 41.8 + .475 (182.8),$$

$$Y = 128.63.$$

The observed disposable income was 137.3. This error led to a downward bias in the final result for the estimate of *GNP*, but this error was less serious than the error of the underestimate of the level of the consumption function.

Hagen, in private correspondence, has pointed out the reason for this technical error and shows that it is something that

could not possibly have been avoided. According to his explanation, the function $Y = Y(GNP)$ is seasonally adjusted. Consequently, in the construction of the function, it was assumed that the quarterly instalment on personal income taxes, due on December 15, should be subtracted from the fourth-quarter income payments. But Congress later postponed this quarterly instalment to January 15. This fact, alone, accounts for most of the difference between the observed disposable income and the calculated disposable income.

There was also a smaller error in the autonomous function because demobilization of the armed forces proceeded at a much faster rate than had been anticipated and therefore led to an extraordinarily high rate of mustering-out pay.

A POSITIVE SUGGESTION

Two possible reactions to the recent failures are the following: (1) We may now discard these new-fangled and difficult econometric methods which have been proved wrong and relax again into armchair comments about the future course of economic events. This is certainly the line of least resistance. (2) We may tackle the forecasting problem with renewed vigor, making use of the valuable information that we have gained from this trial. We shall dwell upon the second reaction exclusively.

A principal failure of the customary models is that they are not sufficiently detailed. There are too many variables which are classified as autonomous when they are actually induced, and there are no dynamic elements injected into the system. The surplus of autonomous variables results from a failure to discover all the appropriate relationships constituting the system. In addition to the consumption function, we should have the

investment function, the inventory function, the housing function, the price-formation equations, etc. There exist good theories from which these additional functions can be constructed, and one should not prematurely despair of determining these functions from the observed data. All the details of the construction of these functions must be the material of subsequent papers, but at this stage the statement can be made that many preliminary investigations²² have shown these functions to exist.

The dynamical aspects of statistical models are also very important. While the consumption function has not been found to depend heavily upon lags or other dynamical elements, those functions which are erroneously assumed to be autonomous have been found to be very greatly influenced by such variables. For example, there are very strong correlations between current purchases of new plant, equipment, dwellings, and the holding of stocks of goods, on the one hand, and the past levels of prices, output, or income, on the other.

The existence of lags means that there are definite links between national income yesterday and today and tomorrow. Optimum forecasts of tomorrow's income should take into account the recently realized levels of income. Rather than make wild guesses, we should do better to forecast tomorrow's income as dependent on today's income. The extremely high wartime incomes made it unlikely (though not impossible) that the transition levels of income would be very low.

There are two ways of using the complicated dynamical systems for economic

forecasting and policy recommendations. We may attempt to estimate all the relevant structural characteristics of the system, such as the various multipliers, elasticities, marginal propensities, marginal productivities, etc. The other method of procedure is that of pure prediction, where we do not try to estimate all the structural characteristics of the system, but only those combinations that are necessary for prediction. The latter method cannot be used for all types of policy.

A simple example will make these ideas more specific. Suppose that we have a system of linear difference equations which describe the operation of the economy

$$L_i(y_{1t}, \dots, y_{nt}, y_{1, t-1}, \dots, y_{n, t-1}, \dots, y_{1, t-p}, \dots, y_{n, t-p}, z_1, \dots, z_m) = u_{it} \quad (i = 1, 2, \dots, n),$$

where L_i is a linear operator, $y_{j, t-k}$ = the j th endogenous variable of the k th preceding time period, z_j = j th exogenous variable (current or lagged), u_{it} = i th random disturbance. If we know the initial conditions and the stochastic properties of the u 's, we may study the entire time path of the system of endogenous variables, in terms of the exogenous variables.

The statistical treatment of the above system of equations involves the computation of "optimal" estimates of all the parameters of the L_i -functions, that is, estimates that have properties such as "lack of bias," "consistency," etc.²³ We shall then be able to do two things: (1) We shall be able to forecast, for short periods in the future, the course of any endogenous variable in the system by as-

²² Presented by the author in a paper read before the Econometric Society in January, 1946. An abstract appears in *Econometrica*, April, 1946, pp. 159-62.

²³ An estimate of a parameter is said to be "unbiased" if its mathematical expectation equals the parameter. A consistent estimate can be roughly defined as unbiased in large samples.

suming that the estimated parameters remain unchanged and solving the statistical equations in terms of the exogenous variables. (2) If the parameters are supposed to change in a known way because of government policy or for other reasons, we can estimate the effect of these changes upon the endogenous variables.

The alternative method of procedure is to solve the system of equations for any particular endogenous variable in terms of the exogenous variables and the lagged values of all the endogenous variables.

$$y_{it} = L_i^*(y_{1,t-1}, \dots, y_{n,t-1}, \dots, y_{1,t-p}, \dots, y_{n,t-p}, z_1, \dots, z_m) + v_i \quad (i = 1, 2, \dots, n),$$

where the v 's are linear combinations of the u 's. If the u 's are nonautocorrelated random time series, then the v 's will also be nonautocorrelated random time series. The L_i^* -functions are called the "reduced forms," as distinct from the L_i -functions, which are called "structural behavior equations." We may estimate the parameters of the reduced forms and then have enough information to forecast the time path of any single variable of the system, say *GNP*, in terms of the exogenous variables. There are great simplifying advantages in working with the L_i^* -functions rather than the L_i -functions because much more elementary statistical methods may be used to estimate the parameters of the former functions than may be used to estimate the parameters of the latter. In fact, the obvious procedure, in order to obtain the coefficients of the L_i^* -functions, is to treat y_{it} as the dependent variable and to determine all the least-squares regression coefficients of this variable on the lagged variables and exogenous variables as independent variables. However, if we aim at estimating

structural behavior equations, we cannot, in general, treat a single equation independently of the rest of the system and must use other methods of estimation. Obviously, it is preferable to work with the structural equations since we can do everything with them that we can do with the reduced forms, and, in addition, we can make certain types of policy judgments that we cannot make on the basis of the reduced forms.

A disadvantage of the reduced-form method of estimation is that there are usually not many degrees of freedom available for the estimation of the parameters, because of the short period of observation available in most economic series. There are usually so many exogenous and lagged endogenous variables in the system that twenty or thirty annual observations may be insufficient for statistical estimation.

Another type of reduced form is the following:

$$y_{it} = L_i^{**}(y_{i,t-1}, \dots, y_{i,t-q}, z_1, \dots, z_r) + w_i \quad (i = 1, 2, \dots, n),$$

where q and r depend upon the order of lags in the entire system. The L_i^{**} -functions are obtained from the L_i -functions by solving for any single endogenous variable in terms of its own lagged values and the exogenous variables. These reduced forms show the time path of any endogenous variable in the system in terms of exogenous variables and are very useful in studying the cyclical behavior of various endogenous variables. The primary disadvantage of this system is that the disturbances, the w 's, will not, in general, be nonautocorrelated even though the u 's are nonautocorrelated in the original system. It can easily be shown that the process of reduction from the system involving the L_i -functions to the system involving the L_i^{**} -functions

usually will introduce autocorrelation in the disturbances. We must have resort to more complicated statistical methods in the case when the disturbances are autocorrelated.

The author has attempted to get first approximations to the L_T -functions by the single-equation, least-squares techniques, although it is known that the estimated values of the parameters might be changed considerably when the equation-system methods are applied. Bearing in mind the tentative nature of the conclusions, the author used these first approximations in September, 1945, to forecast the level of *GNP* for the fiscal year ending June, 1946. The usual assumptions were made about price control, government spending, tax receipts, etc. These autonomous variables in connection with the behavior equations of the system led to a forecast of *GNP* of about \$190 billion for the fiscal year 1946. This forecast will be slightly above the true value, but it could have served as an intelligent guide to policy. It showed definitely that the danger for this fiscal year was one of inflation rather than of deflation.

There are some basic reasons why this model gave different results from the Hagen-Kirkpatrick model: (1) There were many lags in the system which tied the future levels of many economic variables to the past values, especially in capital formation. (2) The total consumption function, rather than the component regressions on disposable income alone, was used. The total consumption function held true, while the individual regressions did not, as was discussed in previous sections.

AN APOLOGY?

Econometric models must necessarily be based on the long series of records

that we have compiled from the workings of a normal peacetime economy. The models are meant to describe behavior patterns in a normal peacetime economy, not in a period of the aftermath of the world's greatest war, when there is a huge number of serious economic dislocations. We can look forward to much better results in the post-transition period, when there will be no production bottlenecks, European famine, price control, stocks of surplus war materials, readjustments to civilian life, etc.

The alternatives to econometric methods of forecasting do not appear to be promising. We must form judgments in order to execute policies. Suppose these judgments are formed by the unsystematic methods of guessing which, in the period 1939-41, formed the basis of estimates of our capacity to produce or of the taxes necessary to prevent inflation. We cannot be very hopeful about solving our economic problems if we have to rely on such methods in the future. Most of the pre-war guessers seriously underestimated the levels which national product would reach during the war, and they incorrectly predicted that, in an economy where the government spends nearly 50 per cent of the *GNP* on war goods, the mild tax rates voted by Congress in conjunction with direct price controls would not be able to prevent a disastrous inflation. Perhaps careful econometric studies of productivity and demand could have demonstrated, in advance, our potential level of output and the size of the inflationary gap to be closed by direct controls. Econometric methods certainly could not have been worse than any other methods that were used.

Economic policy during the 1930's was also based on incorrect judgments. We

experimented with government deficits of \$2-\$3 billion, but large-scale unemployment persisted. During the war the deficit grew manifold, and full employment followed immediately. We never realized the true size of our deflationary gap during the great depression because we did not have an accurate knowledge,

then, of full-employment output. Again, econometric studies might have been able to show the necessary amount of government spending in order to generate a full-employment level of output. Certainly, alternative methods did not answer the appropriate questions of policy.

APPENDIX

THE TIME SERIES AND THEIR CONSTRUCTION

Year	C ₁	C ₂	C ₃	p ₁	p ₂	p ₃	p	Y
1919.....	\$62.2	\$360.7	\$ 85.2	1.365	1.828	0.695	1.391	\$541.6
1920.....	66.7	377.6	138.1	1.602	2.042	0.936	1.559	619.0
1921.....	52.5	295.7	161.2	1.419	1.384	1.190	1.320	538.0
1922.....	59.1	297.1	154.5	1.151	1.330	1.149	1.248	532.5
1923.....	73.2	318.9	166.1	1.099	1.410	1.134	1.271	609.2
1924.....	69.2	304.1	186.7	1.102	1.346	1.210	1.262	594.1
1925.....	82.9	336.7	168.3	1.049	1.408	1.242	1.294	632.8
1926.....	81.8	340.7	185.7	1.003	1.382	1.313	1.296	654.2
1927.....	77.3	336.9	183.1	1.056	1.299	1.313	1.266	630.3
1928.....	77.2	330.3	195.8	1.070	1.301	1.311	1.269	626.6
1929.....	81.3	321.9	197.1	1.080	1.283	1.333	1.264	643.0
1930.....	65.8	293.3	186.9	1.059	1.185	1.361	1.221	596.4
1931.....	50.8	251.5	148.3	1.013	1.017	1.314	1.098	501.5
1932.....	33.6	196.3	125.0	0.995	0.866	1.191	0.972	382.9
1933.....	27.1	192.7	127.4	0.983	0.879	1.032	0.937	344.8
1934.....	38.0	225.5	121.9	1.000	1.000	1.000	1.000	398.8
1935.....	44.8	238.9	135.2	0.950	1.097	0.977	1.038	440.9
1936.....	52.3	264.7	152.1	0.922	1.102	1.000	1.044	517.8
1937.....	59.0	277.1	159.1	0.933	1.154	1.041	1.087	527.8
1938.....	46.2	261.9	153.3	0.968	1.044	1.064	1.042	483.0
1939.....	48.9	249.1	184.9	0.985	1.013	1.275	1.026	530.3

C₁: Per capita expenditures on durable consumer goods.

$$C_1 = \frac{(1)[(2) + (3)]}{(4)(5)} \quad (1919-28),$$

$$C_1 = \frac{(6)}{(5)} \quad (1929-38),$$

$$C_1 = \frac{(7)}{(5)} \quad (1939).$$

(1) = Consumer expenditures on durable consumer goods (S. Kuznets, *National Income and Capital Formation, 1919-1935*, p. 85).

(2) = Total consumer expenditures, exclusive of net imputed rents on owner-occupied, nonfarm dwellings (Mary S. Painter, "Estimates of Gross National Product,

1919-1928," *Federal Reserve Bulletin*, September, 1945, pp. 872-73).

(3) = Net imputed rents on owner-occupied, nonfarm dwellings (S. Kuznets, *National Income and Its Composition, 1919-1938*, II, 735).

(4) = Total consumer expenditures (S. Kuznets, *National Income and Capital Formation, 1919-1935*, p. 85).

(5) = Population of continental United States (*Statistical Abstract of the United States* [1943], p. 3).

(6) = Consumer expenditures on durable goods (*Survey of Current Business*, April, 1942, p. 15).

(7) = Consumer expenditures on durable goods (*Survey of Current Business*, April, 1944, p. 13).

C_2 : Per capita expenditures on nondurable consumer goods.

$$C_2 = \frac{(8) [(2) + (3)]}{(4)(5)} \quad (1919-28),$$

$$C_2 = \frac{(9)}{(5)} \quad (1929-38),$$

$$C_2 = \frac{(10)}{(5)} \quad (1939).$$

(8) = Consumer expenditures on perishable and semidurable commodities (S. Kuznets, *National Income and Capital Formation, 1919-1935*, p. 85).

(9) = Consumer expenditures on perishable and semidurable goods (*Survey of Current Business*, April, 1942, p. 15).

(10) = Consumer expenditures on nondurable goods (*Survey of Current Business*, April, 1944, p. 13).

C_3 : Per capita expenditures on consumers' services.

$$C_3 = \frac{(11) [(2) + (3)]}{(4)(5)} \quad (1919-28),$$

$$C_3 = \frac{(12) - (9) + (3)}{(5)} \quad (1929-38)$$

$$C_3 = \frac{(13) + (14)}{(5)} \quad (1939).$$

(11) = Consumer expenditures on services not embodied in finished commodities (S. Kuznets, *National Income and Capital Formation, 1919-1935*, p. 85).

(12) = Consumer expenditures on nondurable goods and services (*Survey of Current Business*, May, 1942, p. 12).

(13) = Consumer expenditures on services (*Survey of Current Business*, April, 1944, p. 13).

(14) = Net imputed rents on owner-occupied, nonfarm dwellings, estimated by the present writer according to Kuznets' methods (see *National Income and Its Composition, 1919-1938*, II, 753). The figure for 1939 is \$1.5 billion.

p_1 : Price index of durable consumer goods.

$$p_1 = (15) \div 98.5$$

(15) = Price index of durable consumer goods, 1913: 100 (William H. Shaw, *Finished Commodities since 1879* ["National Bureau of Economic Research, Occasional Papers," No. 3], pp. 7-8). 98.5 = Price index of durable consumer goods for 1934, with 1913: 100.

p_2 : Price index of nondurable consumer goods.

$$p_2 = \frac{(16)(17) + (18)(19)}{(17) + (19)} \div 110.8.$$

(16) = Price index of perishable consumer goods, 1913: 100 (*loc. cit.*).

(17) = Output of perishable consumer goods measured in 1913 prices (*ibid.*).

(18) = Price index of semidurable consumer goods, 1913: 100 (*ibid.*).

(19) = Output of semidurable consumer goods measured in 1913 prices (*ibid.*).

$$110.8 = \left[\frac{(16)(17) + (18)(19)}{(17) + (19)} \right]_{1934},$$

where 1934 indicates the year for which the values are computed.

p : Price index of consumption as a whole.

$$p = (20) \div 79.3 \quad (1919-38),$$

$$p = 1.026 \quad (1939).$$

(20) = Price index implicit in the adjustment of total consumers' outlay, 1929: 100 (S. Kuznets, *National Income and Its Composition, 1919-1938*, I, 145. Table 4, col. 3).

79.3 = Price index for 1934.

1.026 = Weighted average of the Bureau of Labor Statistics cost-of-living index and the index of prices paid by farmers for subsistence. The weights are proportional to the nonfarm and farm populations, respectively, and the index is converted to 1934 as a base year.

p_1 : Price index of consumers' services.

$$p_3 = \frac{C_3}{\frac{(2) + (3)}{p} \frac{C_1}{p_1} \frac{C_2}{p_2}} \quad (1919-38)$$

$$p_3 = \frac{C_3}{\frac{(2) + (14)}{p} \frac{C_1}{p_1} \frac{C_2}{p_2}} \quad (1939).$$

Y : Per capita disposable income.

$$Y = \frac{(21)}{(5)}.$$

(21) = Disposable income measured in billions of current dollars. This figure is built up from various component series and

does not agree exactly with the published series of the Commerce Department. This particular series has been used because the consumption equations calculated here will become part of a more complete econometric study in which the author's own figures on disposable income will be used. Space here does not permit a detailed explanation of each of the steps used in the preparation of the series on disposable income; however, this series does not differ very much from the other available estimates of disposable income.

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