

MULTIPLIER EFFECTS OF A BALANCED BUDGET:
SOME MONETARY IMPLICATIONS OF
MR. HAAVELMO'S PAPER*

By G. HABERLER

It may be worth while to point out the monetary implications of Mr. Haavelmo's interesting paper in the October, 1945, number of this journal.¹ It will be sufficient to refer to the simple case of a linear consumption function with which he starts his discussion. The following argument seems to hold, however, for his more general case also.

In monetary terms, what Mr. Haavelmo assumes to happen is this: The sum T is taxed away from employed members of the community and handed over to unemployed workers who are supposed to perform some useful services and thus to add to "gross" real national income. (I use this and other terms in Haavelmo's sense.)

The taxpayers cut their expenditures on consumption by aT and the newly employed increase theirs by aT , so that total private expenditure on consumption (and investment) remains unchanged, which leaves a net addition to gross income of T , which is the assumed value to the community of the services of the newly employed individuals.

Now the question I want to raise is this: How is this possible without a time lag? If there were such a lag, that is to say, if it took the money some time to travel from the taxpayer to the government, thence to the unemployed, and on to the market for consumption goods, there would be a drop in private expenditure on consumption, and, assuming (with Haavelmo) unchanged wages and prices, employment would fall. Mr. Haavelmo evidently assumes that the reduction in expenditure by the taxpayer and the equal expansion in expenditure by the newly employed occur simultaneously. How is that possible?

Since we must assume that the collection and final disbursement of taxes takes time, the time lag in expenditure can be avoided only (a) if the taxpayers dishoard, that is, spend out of idle funds until the money appears again as demand for consumption, spent by the newly employed; or (b) if the government finances its expenditures during the critical interval by borrowing. Assumption (a) implies dissaving by the taxpayers; assumption (b) deficit financing by the government. In Keynesian language we would speak of a change in the liquidity preference (demand for M_1 , transaction money) with respect to net in-

* This note by Professor Haberler, the following ones by Dr. Goodwin and Mr. Hagen, and the reply by Mr. Haavelmo, will be reprinted as Cowles Commission Papers, New Series, No. 15.

¹ Trygve Haavelmo, "Multiplier Effects of a Balanced Budget," *ECONOMETRICA*, Vol. 13, October, 1945, pp. 311-318.

come. In terms of the quantity theory, assumption (a), as well as the unrealistic assumption that no time lag is involved in collecting and disbursing money, implies an increase in the velocity of circulation of money.

Let us ask next what happens if the collection and disbursement of money by the government takes time, in other words if we apply a time-sequence analysis. Assuming the time lag to be one unit period, i.e., that the aT dollars are spent by the newly employed individuals one period after they are withdrawn from expenditure by the taxpayers, a dwindling series of deficiencies of private expenditures is set up: these expenditures are reduced by aT dollars in the first period. In the second period the government collects again T dollars, taxpayers decrease their expenditures by aT dollars but that decrease is now offset by an equal increase on the part of the newly employed workers. However the decrease of the first period has reduced incomes by aT dollars elsewhere, which in turn results in a reduction of expenditures of a^2T dollars in the next period. Hence in the second period total private expenditure falls by a^2T dollars. Similarly in the third period by a^3T dollars and so on. It follows that gross income increases by $T(1-a)$ in the first period, $T(1-a^2)$ in the second period, and so on. Hence Haavelmo's solution is approached only asymptotically (provided $a < 1$).²

This case, too, implies an increase in the velocity of circulation of money and a decrease in the liquidity preference. Moreover, the aggregate propensity to consume (with respect to gross income) has gone up, because it is assumed that the government spends all T dollars it collects, i.e., the government has a marginal propensity to consume of one, while the taxpayer has a marginal propensity to consume of less than one, i.e., reduces his expenditures by aT dollars when T dollars are taxed away.

Putting things this way will (a) make the matter less paradoxical to many people and (b) make the difference between Haavelmo and the writers whom he criticizes appear smaller.

May I sound a note of warning which is supported by the preceding argument and could be strengthened by other considerations? The formal precision of Haavelmo's argument should not deceive us into believing that tax-financed expenditures are as powerful an anti-depression measure as deficit-financed expenditures.

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² See Dr. R. M. Goodwin's general solution in the following paper.

MULTIPLIER EFFECTS OF A BALANCED BUDGET:
THE IMPLICATION OF A LAG FOR
MR. HAAVELMO'S ANALYSIS

By R. M. GOODWIN

Taking consumption, income, etc., to be continuous functions of time and measuring time in unit lags, we might restate Mr. Haavelmo's¹ problem as follows:

$$(1) \quad U(t) = a\{R(t-1) - T(t-1)\} + Nb,,$$

$$(2) \quad R(t) = U(t) + V(t) + T(t-1),$$

where V is constant, and T has two values, zero for $t < 0$, T for $t > 0$. This says expenditure lags behind income. We might say (I think with greater significance) that income lags behind expenditure. Then the equations would read

$$(3) \quad U(t) = a\{R(t) - T(t)\} + Nb,$$

$$(4) \quad R(t) = U(t-1) + V(t-1) + T(t-1).$$

Formally it makes no difference to the result which assumption we choose. In both cases

$$(5) \quad R(t) - aR(t-1) = Nb + V + (1-a)T(t-1).$$

One possible solution of this, as can be verified by substituting back, is

$$(6) \quad R = \frac{Nb + V}{1 - a} + T.$$

It is identical with Mr. Haavelmo's result. Another solution, the general solution of the homogeneous equation, is obtained by setting the constants equal to zero.

$$(7) \quad R(t) = A\lambda^t = Ae^{(t\ln\lambda)t}$$

satisfies it if

$$(8) \quad \lambda = a.$$

A is an arbitrary constant (in general an arbitrary periodic but here a simple constant) to be determined from the behavior over any one initial lag period. The complete solution is the sum of the two:

$$R(t) = Ae^{(t\ln a)t} + \frac{Nb + V}{1 - a} + T.$$

¹ See Trygve Haavelmo, "Multiplier Effects of a Balanced Budget," *ECONOMETRICA*, Vol. 14, October, 1945, pp. 311-318.

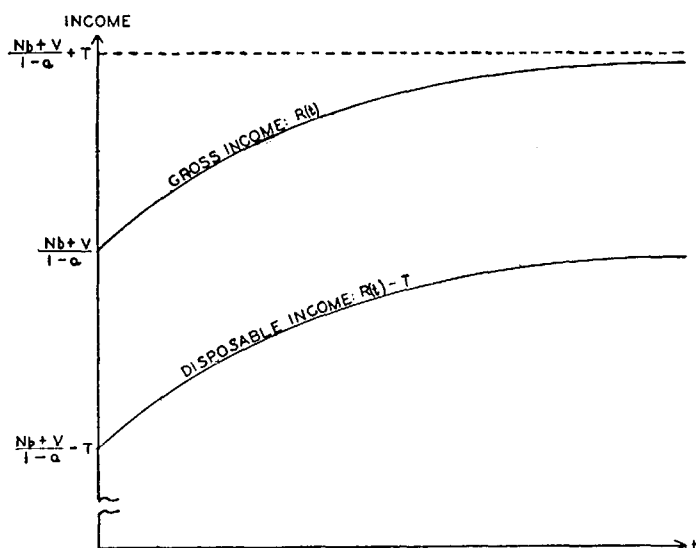


FIGURE 1

Before the tax,

$$R(t) = \frac{Nb + V}{1 - a}, \quad t < 0.$$

Therefore

$$A = -T,$$

so that finally

$$R(t) = -Te^{(1-a)t} + T + \frac{Nb + V}{1 - a}.$$

If $a > 1$, this is unstable as with all multiplier mechanisms. On the other hand if $a < 1$, it is stable and is plotted roughly in Figure 1. It is evident that Haavelmo's solution is approached asymptotically. With a constant amount of money, velocity will rise proportionately with income.

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MULTIPLIER EFFECTS OF A BALANCED BUDGET:
FURTHER ANALYSIS

By EVERETT E. HAGEN

1. INTRODUCTION

Mr. Haavelmo's useful analysis of the multiplier effects of a balanced budget¹ can be generalized so as to take account of time lags and of the possible effect of government spending balanced by an equal amount of taxes upon investment.

Consider the case of a linear consumption function. Using Haavelmo's notation, we have

$$(1.1) \quad U(R_0) = aR_0 + Nb,$$

where R_0 = total income before any tax is imposed, $U(R_0)$ = total private consumption expenditure, a and b are positive constants ($0 < a < 1$) and N is the number of individuals.

Then with V denoting total private investment, assumed constant, total national income R_0 is defined implicitly by

$$(1.2) \quad R_0 = aR_0 + Nb + V,$$

$$(1.3) \quad = \frac{Nb + V}{1 - a}.$$

It may be noted that this formulation may also be used to state the case in which part of investment is autonomous, and is denoted by V , and the rest depends linearly upon income after taxes² (net income, in Haavelmo's terminology). Let total investment I be defined as

$$(1.4) \quad I = cR_0 + V,$$

and total consumption as

$$(1.5) \quad U(R_0) = dR_0 + Nb,$$

and let $a = c + d$. Then, as in (1.2) above, national income is defined by

$$R_0 = aR_0 + Nb + V.$$

From (1.3), Haavelmo shows that when a tax T is imposed and the

¹ Trygve Haavelmo, "Multiplier Effects of a Balanced Budget," *ECONOMETRICA*, Vol. 13, October, 1945, pp. 311-318.

² It is probably preferable, even in a simple model, to make a more complex assumption, namely that the induced portion of investment depends upon the total market for private output, which will equal net income plus that share of T which is used to purchase goods and services from private enterprises.

proceeds fully spent by the government, the resulting gross national income R is:³

$$(1.6, 7) \quad R = \frac{Nb + V}{1 - a} + T = R_0 + T.$$

This conclusion holds regardless of the numerical value of the marginal propensity to consume. It depends however upon the assumption that the tax money is spent by the government without the elapsing of any time interval whatever—i.e., that at that point the velocity of circulation = infinity. For otherwise the imposition of taxes would reduce consumer expenditures below the level $(aR_0 + Nb)$ and the (later) addition of government expenditures T would not at once raise gross national income to $(R_0 + T)$. Alternatively, and preferably, (1.7) may be regarded as indicating the situation when equilibrium is again reached.

2. PERIOD ANALYSIS: INVESTMENT CONSTANT

It will be useful to consider the case in which a time lag is allowed for, i.e., in which consumer spending depends upon the net income of the preceding period, and government spending equals the taxes of the previous period.

In this case, using subscripts to denote time periods,

$$(2.1) \quad R_1 = a(R_0 - T) + Nb + V + T$$

$$(2.2) \quad = R_0 + (1 - a)T,$$

and

$$(2.3) \quad R_2 = a(R_1 - T) + Nb + V + T \\ = a[R_0 + (1 - a)T - T] + Nb + V + T,$$

which gives

$$(2.4) \quad R_2 = R_0 + (1 - a^2)T.$$

Similarly

$$(2.5) \quad R_n = R_0 + (1 - a^n)T.$$

The limiting case is:

$$(2.6) \quad \lim_{n \rightarrow \infty} R_n = R_0 + T.$$

Haavelmo's equation (1.7) above may thus be regarded as a special case, the limiting case, of (2.5).

This formulation makes it easier to understand the quotations from

³ The elasticity of money supply is implicitly assumed = ∞ .

Kaldor, Hansen and Perloff, and Wallich, cited by Haavelmo. For example, the meaning of the proviso by Hansen and Perloff that the initial expenditure be financed by borrowing becomes clear. Only if it is so financed will the gross national product be raised *at once* by the full amount of the increase in government expenditures.

The value of $(1 - a^n)$ is of course larger, for any given value of n , the smaller the value of a . Thus we reach the interesting conclusion that the increase in national income will be greater at any given length of time after an increase in taxes and government expenditure, the *lower* the marginal propensity to consume. The initial depressing effect of taxes upon consumption will be greater, the higher the marginal propensity to consume. And the initial disadvantage will never be overcome, even though R will asymptotically approach $R_0 + T$, whatever the value of a .

3. LINEAR FUNCTIONS: THE MOST GENERAL CASE

It is sometimes asserted that an increase in government expenditures balanced by taxes diminishes private investment. Assume that the government expenditures are of a sort that affects some part of private investment, and that the relationship is linear, so that total investment is implicitly defined as

$$(3.1) \quad I = V + cT \quad (c \leq 0).$$

Then

$$(3.2) \quad R_1 = a(R_0 - T) + Nb + V + T + cT$$

$$(3.3) \quad = R_0 + (1 - a + c)T.$$

This formulation assumes that private investment of a given period is affected by government expenditures of that period, financed by taxes of the previous period. Since investment decisions are presumably affected primarily by judgments concerning the total effect of government action upon future profits, it seems reasonable that the taxes and government expenditures would be considered as a unit by businessmen.

For the period R_2 ,

$$(3.4) \quad R_2 = a(R_1 - T) + Nb + V + T + cT \\ = a[R_0 + (1 - a + c)T - T] + Nb + V + T + cT$$

$$(3.5) \quad = R_0 + (1 + a)(1 - a + c)T.$$

It may be shown that

$$(3.6) \quad R_n = R_0 + (1 + a + a^2 + \cdots + a^{n-1})(1 - a + c)T,$$

which gives

$$(3.7) \quad \lim_{n \rightarrow \infty} R_n = R_0 + \left(\frac{1}{1-a} \right) (1-a+c)T$$

$$(3.8) \quad = R_0 + \left(1 + \frac{c}{1-a} \right) T.$$

This is the general statement of the relationships here discussed when the consumption and investment functions are linear. The change in national income will equal T plus (the increase in investment times the consumption multiplier). Thus, by a familiar application of "multiplier analysis," it is obvious that the deviation (positive or negative) of the new level of national income from $R_0 + T$ will be the greater, the greater the marginal propensity to consume.

Several special cases may be noted. In each, T refers to an increase in government expenditures balanced by taxes.

(i) $c > 0$. If private investment is stimulated by an increase in government expenditures balanced by taxes, the increase in national income (when equilibrium is again reached) will be greater than T by an amount equal to the increase in investment times the consumption multiplier.

(ii) $c = 0$. If private investment is not a function of T , (3.8) reduces to the special case expressed in (2.6) and the increase in national income will equal T .

(iii) $0 > c > a - 1$. If private investment is adversely affected by an increase in government expenditures balanced by taxes, and if the ratio of the decline in private investment to T is less than $1 - a$, the marginal propensity to save, then the increase in national income will be less than T by an amount equal to the decrease in investment times the consumption multiplier.

(iv) $c = a - 1$ (or, $-c = 1 - a$). If the ratio of the decline in private investment to T equals the marginal propensity to save, the increase in government expenditures balanced by taxes will cause no change in national income.

(v) $c > a - 1$. If the ratio of the decline in private investment to T is greater than the marginal propensity to save, an increase in government expenditures balanced by taxes will reduce national income. The reduction will equal [the excess of c over $(1 - a)$] times T times the consumption multiplier.

While the effect of T upon investment might be similarly allowed for in the cases in which the consumption function is not linear, the more complicated analysis will not be attempted here. The nature of the effect may be seen by analogy with the cases discussed above in which the function is linear.

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MULTIPLIER EFFECTS OF A BALANCED BUDGET:
REPLY

By TRYGVE HAAVELMO

I am grateful to Professor Haberler¹ for his comments, which add a certain amount of economic realism to the dry theorems that were submitted in my article. If my simplified analysis should have some effect in the direction of inducing others to work out solutions under more complicated and realistic assumptions the article would have accomplished more than was expected.²

It might be worth while to point out that the solutions suggested by Haberler not only are in full conformity with those in my article but could be used to derive even stronger theorems than those I obtained. My results (in particular Theorem I, p. 315), were established under the assumption that the government is sufficiently rational and intelligent in its planning of expenditures to avoid temporary hoarding of tax money, either by the taxpayers or by the government itself. Under this and other assumptions I compared the stationary *level* of income before and after imposing an additional income tax (= additional public spending). I verified the conjecture already made by many others: that the tax leads to an increase in the level of gross income by an amount exactly equal to the tax. Haberler's analysis as well as Dr. R. M. Goodwin's,³ indicates that this will be the final result even if the government's expenditures lag behind tax payments. Their results may be derived from the following model.

Let R_τ be gross income (as defined in my article) during the period τ . Further let T_τ be the tax on R_τ , collected during period τ , and let S_τ be government spending during the period τ . Total gross income during each period will then be equal to consumer expenditures plus expenditures for investment goods plus government expenditures during that period. If we assume that consumers' expenditures depend on income (after taxes) during the preceding period and that government expenditures are equal to tax receipts during the preceding period, we may write

¹ "Multiplier Effects of a Balanced Budget: Some Monetary Implications of Mr. Haavelmo's Paper," above, pp. 148-149.

² Interesting examples of possible generalizations are already contained in Mr. Everett E. Hagen's paper: "Multiplier Effects of a Balanced Budget: Further Analysis," above, pp. 152-155. The reader might, perhaps, find that there is some degree of unnecessary overlapping between the first part of Hagen's analysis and my comments below. This is due, however, to the fact that I did not have access to his manuscript until after my "Reply" had gone to the printer.

³ "Multiplier Effects of a Balanced Budget: The Implication of a Lag for Mr. Haavelmo's Analysis," above, pp. 150-151.

$$(1) \quad R_\tau = a(R_{\tau-1} - T_{\tau-1}) + S_\tau + C, \quad \tau = 1, 2, \dots,$$

$$(2) \quad S_\tau = T_{\tau-1} = \begin{cases} 0 & \text{for } \tau < 1, \\ T & \text{for } \tau \geq 1, \end{cases}$$

where a is the (constant) marginal propensity to consume, while C is a constant equal to investment (assumed to be constant) plus a constant level of minimum consumers' expenditures. If the system was in the stationary state before the tax was imposed, i.e., if

$$(3) \quad R_0 = \frac{C}{1-a},$$

the solution of the system (1), (2) is

$$(4) \quad R_\tau = R_0 + T - a^\tau T, \quad \tau = 1, 2, \dots$$

(A more detailed explanation of this result is given in Mr. R. M. Goodwin's note in this issue.) $(R_\tau - T)$, or "disposable income," is, therefore, smaller than R_0 by $a^\tau T$ dollars in the first period, by $a^2 T$ dollars in the second period, and so on. The stationary level of R_τ , provided $0 < a < 1$, is

$$(5) \quad \bar{R} = R_0 + T.$$

If this system is allowed to continue undisturbed, the total loss of "disposable income" to taxpayers will be

$$(6) \quad \sum_{\tau=1}^{\infty} [R_0 - (R_\tau - T)] = \sum_{\tau=1}^{\infty} a^\tau \cdot T = \frac{a}{1-a} T.$$

If the variables R and T are measured as annual rates while the unit of τ is only a fraction, say q , of a year, the total loss of disposable income will be only $q[a/(1-a)]T$, which is the smaller the shorter the actual lag "1" used as the time unit above.

It might be observed that if there is no lag between income receipts and consumers' expenditures, while the government is one period behind in spending the tax money, the entire loss of disposable income as given by (6) will be incurred in the first period. For then, instead of (1) and (2), we have

$$(1') \quad R_\tau = a(R_\tau - T_\tau) + S_\tau + C, \quad \tau = 1, 2, \dots,$$

$$(2') \quad S_\tau = T_{\tau-1} = \begin{cases} 0 & \text{for } \tau < 2, \\ T & \text{for } \tau \geq 2. \end{cases}$$

From this system we obtain

$$(4') \quad R_1 = R_0 - \frac{a}{1-a} T,$$

$$(4'') \quad R_\tau = R_0 + T, \quad \tau = 2, 3, \dots$$

Going back to the system (1), (2) it is interesting to observe what happens if the tax is abandoned after n time periods, i.e., if $T_\tau = 0$ for $\tau > n-1$. The initial loss in disposable income of taxpayers during the first n time periods will then gradually be regained by the taxpayers. This is seen as follows.

$$(7) \quad R_{n+s} = aR_{n+s-1} + C = a^s R_n + (1-a^s)R_0, \quad s = 1, 2, \dots,$$

where R_n follows from (4) by setting $\tau = n$. The sum of all the differences $(R_\tau - R_0)$, where τ runs from 1 to n in (4) and then from $\tau = n+1$ to infinity in (7), will then be

$$(8) \quad \sum_1^\infty (R_\tau - R_0) = \left[nT - a \frac{1-a^n}{1-a} T \right] + \left[a \frac{1-a^n}{1-a} T \right] = nT.$$

Thus the total gain in gross income from a tax T (=government expenditures) lasting for n time periods is exactly equal to nT . The system (1'), (2') gives the same result.

It is seen from the analysis above that, while a lag in government expenditures may cause delay in reaching the full multiplier effect of the tax, the final result will be the same as if there were no lag. We may, therefore, also say that initial deficit spending by the government is not an essential economic force in the type of income expansion that was discussed in my article. The force that drives gross income up when a tax=government expenditures is imposed, is the over-all increase in the marginal propensity to spend, resulting from the government's propensity to spend being equal to unity.

Since the multiplier effect of each dollar of tax is only equal to 1, there can, of course, be no disagreement with Haberler on the point that deficit spending, usually, would be much more effective in increasing total income.

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