MULTIPLIER EFFECTS OF A BALANCED BUDGET*†

By Trygve Haavelmo

1. INTRODUCTION

It has commonly been argued that public spending, to be a remedy against unemployment, must be deficit spending and not spending balanced by an equal amount of taxes, since, in the latter case, the government would only be taking back with one hand what it gives with the other. One necessary qualification of this statement is, of course, well known, namely, that taxes corresponding to an equal amount of public spending may lead to a redistribution of incomes which, in turn, may lead to a higher level of national consumption at a given level of private investment. The effect of such redistribution, however, depends essentially on whether or not there is any substantial difference in the marginal propensities to consume, as between the various income groups. If, for example, the propensity-to-consume function of the individual is a linear function of personal income the marginal propensity to consume will be constant for all levels of income, and there could be no redistribution effect (unless the redistribution had an effect on private investment).

In this latter case it might then be thought that public spending balanced by an equal amount of taxes would have no effect upon total income and employment in the society (apart from a possible effect, indirectly, on the propensity to invest). This commonly made conjecture is, however, false, as has already been pointed out by several writers on the subject.† In a situation with unemployment and idle resources there is a definite employment-creating effect of public outlays even when they are fully covered by tax revenues. And this is true quite

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† I wish to express my sincere thanks to Professor J. Marschak for many helpful suggestions.

1 See, e.g., P. A. Samuelson, “Full Employment After the War" in Postwar Economic Problems, edited by S. E. Harris, New York, 1943, p. 44.
Henry C. Wallich, "Income-generating Effects of a Balanced Budget," Quarterly Journal of Economics, Vol. 59, November, 1944, pp. 78–91. (My attention has been drawn to this important article which I had not heard of at the time when my manuscript was submitted for publication. Mr. Wallich’s paper, I am sure, deserves more extensive comments than those I had occasion to add to the present article).
apart from whatever other effects the taxes and the expenditures might have on the distribution of income or on the behavior of consumers and investors.

Although this idea is not a new one, there still seems to be much need for a rigorous theoretical analysis of the whole subject. The existing literature is not altogether clear on the matter. Mr. Kaldor, for example, in discussing the possibility of full employment under a balanced budget, explains the matter as follows:

Full employment could be secured, however, by means of increased public outlay, even if the State expenditure is fully covered by taxation—for the reason that an increase in taxation is not likely to reduce private outlay by the full amount of the taxes paid. It may be assumed that all taxes have some influence on the savings of the individuals on whom they fall; taxes which fall on the poor have a relatively large effect on consumption and a relatively small effect on savings; with taxes paid by the rich it is probably the other way round. Hence an increase in public expenditure will cause a net addition to the total outlay of the community even if it is covered by taxation; . . . .

This statement would seem to convey the idea that taxes equal to public expenditure can create employment only to the extent that they cut down on people's savings. This is not correct. We shall show below that public expenditures covered by taxes have an employment-generating effect which is independent of the numerical value of the propensity to consume.

Hansen and Perloff, in their comments on the same subject, write as follows:

. . . Moreover, an increase in useful governmental expenditures (the initial expenditure being financed by borrowing) will tend to raise the national income even though subsequently financed from consumption taxes. Thus, when additional government expenditures are paid out to the public, the income receipts of individuals are increased. If, now, subsequently a consumption tax is imposed equivalent to the enlarged income, it follows that private expenditures after taxes remain as before. The Gross National Product is increased by the amount of the new government spending while private expenditures remain the same. Thus in this case the total Gross National Product (governmental expenditures plus private expenditures) is enlarged roughly by the amount of the new expenditure but not by a magnified amount.

Here the final conclusion, namely that expenditures covered by taxes will raise income (and employment) by the amount of the tax, is correct. The assumption of the initial expenditure's being financed by borrowing is, however, unnecessary. Indeed, if this assumption were necessary, the conclusion would not hold in the second year, the third year, etc., since then current expenditure would equal current taxes.

* Kaldor, op. cit., p. 346.

Mr. Wallich, in his recent article dealing directly with the subject discussed here, has reached the same conclusion as Hansen and Perloff. He has a clear illustration in terms of a numerical example. His more general discussion of the "reason why," however, might perhaps give rise to misunderstandings. He writes, in part:

The reason why national income can increase in this instance, without an increase in investment and without a redistribution of income from the higher to the lower income groups, is that the additional income financed by the Government does not give rise to new net saving. It is true that the previously unemployed will save part of their new income, but an equal volume of savings of the initially employed is absorbed by the additional tax. Since the two groups are assumed to be similar, the savings of one are offset by the reduction in the savings of the other. [Footnote omitted] By absorbing part of originally existing income and respending it in its entirety, the Government prevents some fraction of this amount from being saved, as it otherwise would be. . . .

If investment is assumed to remain constant it seems unnecessary to prove that saving remains constant. What is needed is a rigorous proof that under these circumstances total income will actually rise as a result of the taxation and spending. For this purpose the argument that the government spends income that otherwise would have been partly saved is dangerous as it might lead to the false belief that the higher the propensity to save for the public the larger the effect of the fiscal policy discussed. As already mentioned we shall see that this is not the case.

The whole matter may in fact be stated much more simply as follows: Let us use the words "net income" to designate the sum of incomes at the individuals' disposal after they had paid the taxes. The words "gross income" will mean the sum of individuals' incomes before taxes. Gross income is thus the sum of earnings made by individuals in producing goods and services: it is equal to the money value of goods and services produced, either for private or for public needs. Thus while the demand of private people for goods and services depends on net income, their employment depends on gross income. Extra public expenditure covered, simultaneously, by taxes can obviously be added to the existing gross income in such a way that it will leave the people with exactly the same amount of net income, and hence will leave the private demand at exactly the same level as before the tax was imposed (provided the tax policy does not lead to a change in the distribution of net incomes and, thereby, to a change in the marginal propensity to consume of the society as a whole). But, while the government collects the tax money without any direct compensation to the individual taxpayer, the government requires goods and services from

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* Wallich, op. cit., p. 80.
the public in return for money expenditures. Now, if there were already full employment before the tax was imposed, the result would be that the public as a whole would have to work partly for the government instead of working for their own direct benefit. Then they could not pay the taxes by working more. If, however, there is a sufficient amount of idle manpower and resources the amount of employment and productive services required by the government will come forth in addition to what is wanted by the private sector of the economy. The gross income, i.e., the money value of all goods and services produced (for private as well as public needs) will have increased, although the net income has remained unchanged. In fact, from an employment point of view, the result for the society as a whole will be exactly the same as if the government had ordered idle manpower and resources to work without any direct compensation.

In the following we propose to give a more accurate demonstration of this simple conclusion.

2. THE SIMPLEST CASE: A LINEAR CONSUMPTION FUNCTION

The assumption of a linear consumption function is of particular interest here, since, in that case, as already mentioned, no multiplier effect can result from a redistribution of incomes. This simplifying assumption, therefore, allows us to isolate whatever “pure” multiplier effects might be generated by public spending balanced by taxes.

We shall use the symbol \( r \) to denote gross individual money income, while \( \bar{r} \) will denote gross average individual money income, and \( R \) gross total national money income. (Throughout this study we shall assume that there are sufficient unused manpower and resources available to justify the assumption of a constant level of prices. We shall further assume that we are dealing with a “closed economy.”) Since we shall be interested in comparing incomes before and after imposing a certain income tax we shall indicate by \( r_0, \bar{r}_0, \) and \( R_0 \) the individual, average, and total income, respectively, before any tax is imposed, while \( r, \bar{r}, \) and \( R \) will be used to denote the same income concepts (gross, i.e., including taxes paid) after a certain income tax is imposed. The total number of individuals, \( N \), is assumed to remain constant.

We assume that the private consumption expenditure, \( u(r_0) \), of an individual having the net income \( r_0 \) is given by

\[
(2.1) \quad u(r_0) = ar_0 + b,
\]

where \( a \) and \( b \) are positive constants \((0 < a < 1)\). Then, whatever be the income distribution, the average consumer expenditure \( \bar{u} \) is given by

\[
(2.2) \quad \bar{u} = u(\bar{r}_0) = a\bar{r}_0 + b
\]
and the total consumer expenditure of all the individuals, $U(R_0)$, is given by

$$U(R_0) = aR_0 + Nb.$$  

Let $V$ denote total private investment. In all that follows we shall assume $V$ to remain constant. The average investment $V/N$, then also a constant, we shall denote by $v$. Total national income $R_0$ (= total consumer and investment expenditures) is then defined implicitly by

$$R_0 = aR_0 + Nb + V,$$

which gives

$$R_0 = \frac{Nb + V}{1 - a}.$$  

If now a tax totalling $T$ dollars is imposed on incomes, and the tax money is fully spent by the government, the resulting total gross national income, earned in the production of goods and services for both private and government use (= consumer expenditure + private investment + government spending) is defined implicitly by

$$R = a(R - T) + Nb + V + T,$$

which gives

$$R = \frac{Nb + V}{1 - a} + T.$$  

Comparing (2.5) and (2.7) we have the following

**Theorem I:** If the consumption function is linear, and total private investment is a constant, a tax $T$, that is fully spent will raise total gross national income by an amount $T$ and leave total private net income and consumption unchanged. And this holds regardless of the numerical value of the marginal propensity to consume, $a$.

The result obtained may also be expressed as follows: If the government spends $T$ dollars and at the same time covers this expenditure by taxes, the multiplier effect, per dollar spent, will be equal to 1.

This, of course, does not mean that the net income and consumption of every single individual necessarily remain the same after the tax has been imposed. For the tax is a certain loss of net income to every individual while the gain from the government expenditure is only an average gain. The individual gains might differ widely.

From (2.7) it follows that by making $T$ sufficiently large one can reach a full-employment level of $R$. It is interesting to consider the rate of taxation that such a full-employment policy might require. Let $\lambda$ be
the tax rate imposed on $R$. [The distribution of the taxes as between
the various individual incomes is here irrelevant, owing to the assumption (2.1).] Then we have

\[
R = aR(1 - \lambda) + Nb + V + \lambda R
\]

or

\[
R = \frac{Nb + V}{(1 - a)(1 - \lambda)} = \frac{R_0}{(1 - \lambda)}
\]

In other words, a tax rate of, say, 50 per cent will double the total gross
income that existed before the tax was imposed.

In the preceding analysis it has of course been assumed that the vari-
ous services and benefits which the government is able to provide
through the spending of the tax money are not counted by the individu-
als as a part of their consumption or their savings. This assumption is
necessary in order to consider $a$ and $b$ as independent of the tax. The
government might no doubt provide such services and benefits in re-
turn for the taxes that, in particular, the demand for private savings
would be reduced. The government might, on the other hand, provide
goods and services that would cover a certain part of regular consumer
needs. If, as a result of the tax and spending policy, the propensity to
consume were changed from $a$ to, say, $a(\lambda)$ then, instead of (2.9), we
would have

\[
R_1 = \frac{R_0(1 - a)}{(1 - \lambda)(1 - a(\lambda))}
\]

If $a(\lambda) > a$, $R_1 > R$; if $a(\lambda) < a$, $R_1 < R$.

It would no doubt take a considerable amount of research to obtain
actual information on the influence on consumers’ behavior of the vari-
ous types of services and benefits provided directly or indirectly by the
government. But such a study might be well worth while.

3. MORE GENERAL CASE: NONLINEAR CONSUMPTION FUNCTION

We shall first study the effect of a proportional income tax imposed
on all individual incomes, the total tax revenue being spent by the
government. Since we assume that the consumption function might be
nonlinear we shall have to make some additional assumptions about the
behavior of the income distribution through this process of taxation and
public spending. The usual simplifying assumption is that “the income
distribution remains unchanged.” Taken literally, this assumption
makes little sense, since a “constant income distribution” would mean
that the total (or average) income as well as all other parameters of the
distribution would have to remain constant at one level. Usually, what
is meant is that, if the average income varies, the income distribution will be subject only to a proportional stretch or squeeze.

Let $\Phi(x)$ denote a certain relative frequency distribution where the average of $x$ is equal to 1, and let us assume that the ratio $r/w$ is distributed as $x$, for all values of $w$. The distribution of $r$ will then be

$$\Phi\left(\frac{r}{w}\right) \frac{1}{w},$$

i.e., the distribution will belong to a parametric class defined by the form $\Phi$, and the parameter $w$. We shall assume here that the structure of the economy is such that the income distribution always must belong to this class.

Let $r_0$ denote individual incomes before the tax is imposed, and let us assume as before that the average investment per individual, $v$, is a given constant. Further, let the consumption function be $u(r_0)$. Then the average income, $\bar{r}_0$, is given by

$$\bar{r}_0 = \int_0^\infty u(r_0) \Phi\left(\frac{r_0}{\bar{r}_0}\right) \frac{1}{\bar{r}_0} \, dr_0 + v.$$

If now a proportional tax rate $\lambda$ is imposed, the resulting gross average income, $\bar{\bar{r}}$, is defined by

$$\bar{\bar{r}} = \int_0^\infty u[r(1 - \lambda)] \Phi\left(\frac{r}{\bar{\bar{r}}}\right) \frac{1}{\bar{\bar{r}}} \, dr + \lambda \bar{r} + v.$$

If we denote the net income, namely $(1 - \lambda)r$, by $r_\lambda$, and the average of $r_\lambda$ by $\bar{r}_\lambda$, this relation may be written as

$$\bar{r}_\lambda = \int_0^\infty u(r_\lambda) \Phi\left(\frac{r_\lambda}{\bar{r}_\lambda}\right) \frac{1}{\bar{r}_\lambda} \, dr_\lambda + v.$$

Comparing (3.2) and (3.4) we see that the implicit definition of $\bar{r}_0$ by (3.2) is identical with the implicit definition of $\bar{r}_\lambda$ by (3.4). Hence, if this definition is unique, we have

$$\bar{r}_\lambda = \bar{r}_0, \quad \bar{r} = \frac{\bar{r}_0}{1 - \lambda}.$$

We therefore have

Theorem II: If the income distribution has the property of always remaining within the class defined by (3.1) the effect of a proportional tax, fully spent, will be exactly the same as if the propensity-to-consume function had been linear, i.e., private net income and consumption remain unchanged while gross national income rises by the total amount of the tax, and this result is independent of the form of $u$. 
We have studied the particular case of a proportional tax rate because this seemed the most reasonable assumption in connection with the assumption that the income distribution always must belong to the one-parameter class (3.1). But the particular assumption about a proportional tax rate is not essential. If the assumption is made that also the distribution of net income, \( r_n \), always must belong to the same parametric family of the type (3.1) then our results will follow without making any separate restrictions upon the manner in which the taxes are collected and spent. For we can then write down the equation (3.4) directly. This equation defines \( \bar{r} \) as a function of \( v \) independently of the size and the distribution of the tax. We must, therefore, have \( \bar{r} = \bar{r}_n \), and \( \bar{r} = \bar{r}_n + \) the average amount of tax. This gives us:

**Theorem III:** If the structure of the economy is such that it maintains a constant "relative" distribution of net incomes whatever the tax is, then the average net income will be the same as before the tax was imposed, while total gross income will increase by the total amount of the tax.

It might perhaps be worth while in this connection to point out that the assumption we have made about the income distribution is not exactly equivalent to saying that "all incomes change in the same proportion." For let \( \Phi(r_n/\bar{r}_n) \) and \( \Phi(r_i/\bar{r}_n) \) be the income distribution before and after imposing the tax, respectively. Then these two distributions are only the marginal distributions of the two variables \( r_n \) and \( r_i \). The knowledge of these two distributions does not uniquely determine the joint distribution of \( r_n \) and \( r_i \). If, for example—and only as an illustration—the two distributions were normal distributions, the correlation coefficient would still be free to take any value from \(-1\) to \(+1\). The practical meaning of this remark is that a study of the income distribution before and after the introduction of a tax will not fully reveal the eventual "reshuffling" that the individual income receivers might have been subject to as a result of the tax and spending policy of the government.

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It would of course be interesting to study the effects of more general forms of tax rates and more general forms of respending the tax money. Such an analysis, however, would take us into a general discussion of the effects of a redistribution of incomes, and that was not our objective. We only wanted to demonstrate that a "balanced budget" has a direct multiplier effect, with a multiplier equal to 1, in addition to whatever (positive or negative) effects there might be from a redistribution of income.

Washington, D. C.