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Efficient Estimation
with A Priori Information

Thomas J. Rothenberg

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Preface

Over the years econometricians have developed special statistical methods for handling the problems that arise in analyzing economic data. These special problems and methods are the topic of numerous textbooks and are studied by nearly every economics graduate student. Although econometric techniques do have features which distinguish them from the techniques used in other fields of application, there is a danger that these special features are being overemphasized. By concentrating on the uniqueness of econometric methods one tends to lose sight of the fact that these methods are based on a general theory of inference. Often it is possible to gain a better understanding of econometric problems, not by emphasizing their peculiar characteristics, but rather by seeing how they fit into the broader structure of statistical theory.

This book presents an attempt at unifying certain aspects of econometric theory by embedding them in a more general statistical framework. The unifying feature is the use of a priori information and the basic tool is the traditional Cramér-Rao inequality. I believe that many confusing aspects of the simultaneous equations problem can be cleared up by viewing that problem in a broader context. Certain results on identification and efficiency turn out to be quite elementary when viewed at a general level but are not at all elementary when studied in the particular. Furthermore, the limitations of our econometric theory are made clearer when seen in the general framework of traditional statistical theory.

The present monograph can usefully be viewed as an extension of the research reported in Cowles Foundation Monographs 10 and 14. These earlier studies, many of them written by leading statisticians, represent a systematic application of the methods of mathematical statistics to econometric problems. The present book uses the same basic approach and many of the same techniques. By generalizing and extending these earlier studies it has been possible to develop a unified theory of estimation which contains many of their results as special cases.

The use of considerable mathematics is unavoidable in a book dealing with econometric theory. The reader should be familiar with the material contained in an introductory course in mathematical statistics and should be comfortable with matrix notation. Nevertheless, this is not a treatise in

mathematical statistics. My purpose has been to help bridge the gap between the textbook presentation of econometric methods and the mainstream of modern statistical theory. Many of the more difficult results are stated without proof; and most of the tedious algebra has been placed in appendixes or omitted entirely.

The theory of efficient estimation is only a part of theoretical econometrics; and theoretical econometrics is but a small part of the broad subject of empirical economics. There has been in recent years an unfortunate tendency to overemphasize formal econometric theory at the expense of the commonsense application of econometrics to real social problems. In addition to the gap between econometric theory and traditional statistical theory, there is an even greater gap between econometric theory and good econometric practice. The reader will understand that this book's concentration on the former problem in no way diminishes its author's concern over the latter.

The research for this monograph has extended over an embarrassingly long period of time. The first five chapters are a major revision of my doctoral thesis submitted to the Economics Department at M.I.T. in 1966. Chapter 6 presents the results of research that I began in 1963 while a staff member of the Econometrics Institute of the Netherlands School of Economics (Rotterdam). Various other parts of the book were written while I was associated with the Transportation Center at Northwestern University, the Cowles Foundation at Yale University, and the Center for Operations Research and Econometrics at Louvain (Belgium). I am grateful to these institutions, as well as the University of California, Berkeley, for providing me with financial support, secretarial and research facilities, and, most of all, stimulating colleagues. Additional support from the Ford Foundation, the National Science Foundation, and a U.S. Government Fulbright Fellowship is gratefully acknowledged.

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T.J.R.