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For Research in Economics at Yale University

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THEORY OF VALUE
An Axiomatic Analysis
Of Economic Equilibrium

GERARD DEBREU

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TO ALFRED COWLES

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PREFACE

The two central problems of the theory that this monograph presents are (1) the explanation of the prices of commodities resulting from the interaction of the agents of a private ownership economy through markets, (2) the explanation of the role of prices in an optimal state of an economy. The analysis is therefore organized around the concept of a price system or, more generally, of a value function defined on the commodity space.

The first solutions of the two preceding problems were achieved by L. Walras [1] and V. Pareto [1], [2], [3], [4] respectively, but neither the masters of the school of Lausanne nor their disciples for several decades gave a very rigorous account of their ideas. For example, the knot of the first problem was thought to be cut by the bold assertion that a system of equations whose number equals that of its unknowns can be solved. Only in 1935–36 did A. Wald [1], [2], [3] publish the first rigorous analysis of the problem of equilibrium. A little earlier J. von Neumann [1], [2] had begun to develop, in different contexts, a mathematical tool which was eventually to play an essential role in that area under the definitive form as a fixed point theorem it received from S. Kakutani [1]. The value of that tool for economics was demonstrated in 1950 by J. Nash's [1] proof that every finite n -person game has an equilibrium point (a concept whose origin can be traced to A. Cournot [1], Chapter 7). As for the second problem, the first rigorous study, using convex sets properties, of the equivalence be-

tween an optimum and an equilibrium relative to a price system was done by T. C. Koopmans [1] in the context of linear activity analysis of productive efficiency. The research of the last decade reported in this volume started from these contributions. But it is hardly necessary to add that many other currents of ideas have, directly or indirectly, influenced the substance or the form of that research. Outstanding among these influences has been the work of J. von Neumann and O. Morgenstern [1] which freed mathematical economics from its traditions of differential calculus and compromises with logic.

The theory of value is treated here with the standards of rigor of the contemporary formalist school of mathematics. The effort toward rigor substitutes correct reasonings and results for incorrect ones, but it offers other rewards too. It usually leads to a deeper understanding of the problems to which it is applied, and this has not failed to happen in the present case. It may also lead to a radical change of mathematical tools. In the area under discussion it has been essentially a change from the calculus to convexity and topological properties, a transformation which has resulted in notable gains in the generality and in the simplicity of the theory.

Allegiance to rigor dictates the axiomatic form of the analysis where the theory, in the strict sense, is logically entirely disconnected from its interpretations. In order to bring out fully this disconnectedness, all the definitions, all the hypotheses, and the main results of the theory, in the strict sense, are distinguished by italics; moreover, the transition from the informal discussion of interpretations to the formal construction of the theory is often marked by one of the expressions: "in the language of the theory," "for the sake of the theory," "formally." Such a dichotomy reveals all the assumptions and the logical structure of the analysis. It also makes possible immediate extensions of that analysis without modification of the theory by simple reinterpretations of concepts; this is repeatedly illustrated below, most strikingly perhaps by Chapter 7 on uncertainty.

To keep the cost associated with such an axiomatization at a minimum, the theory is consistently set forth in the simplest possible mathematical framework even when immediate generalizations, some of which will be mentioned in notes, are available. In addition, the small amount of mathematics necessary for a full understanding of the text (but not of all the notes) of Chapters 2 to 7 is given in the first chapter in a virtually self-contained fashion. In another respect the reading of this monograph will be facilitated by the excellent introductions to its problems provided by T. C. Koopmans' [2] first essay

and by R. Dorfman, P. A. Samuelson, and R. M. Solow's [1] Chapters 13, 14.

Before concluding, must one remark that the contents of this volume—which have been taught at the University of Chicago and at Yale University since the spring of 1953, and presented as a Doctor of Science thesis at the University of Paris in June 1956—do not try to exhaust the theory of value? Several important questions left unanswered are emphasized below. One may stress here the certainty assumption made, at the level of interpretations, throughout the analysis of Chapters 2 to 6, according to which every producer knows his future production possibilities and every consumer knows his future consumption possibilities (and his future resources if resources are privately owned—otherwise only the future *total* resources need be known). This strong assumption is weakened, albeit insufficiently, in the last chapter.

The Cowles Foundation has provided an uncommonly favorable environment for the research from which this monograph evolved, and I wish to express my gratitude to Alfred Cowles, its founder, and to Tjalling C. Koopmans and Jacob Marschak for the constant interest they have taken in my work. I have an exceptional debt to Kenneth J. Arrow, for several of the main ideas of this volume have been advanced either independently (K. J. Arrow [1], G. Debreu [1]), or jointly (K. J. Arrow and G. Debreu [1]) by him and by me. Tjalling C. Koopmans, Lionel W. McKenzie, Jacob Marschak, Roy Radner, and Robert M. Solow have read the whole manuscript or extensive parts of it, and I owe many searching comments to them. My concern for the theory of the school of Lausanne arose when I first met it in the treatise of Maurice Allais [1] and, a little later, in the book of François Divisia [1]. I also thank them, Georges Darmois, Wassily Leontief, Pierre Massé, René Roy, and James Tobin for having greatly helped to create the conditions which made this investigation possible. Finally I gratefully acknowledge the financial support of the Office of Naval Research and the Social Science Research Council in the writing of this text, of the Centre National de la Recherche Scientifique, the Rockefeller Foundation, and the RAND Corporation in the preliminary work that led to it.