Chapter VII

THE DEMAND RELATION

Derived Demand for Perishable Commodities

Before additional calculations pertaining to the livestock demand relation are reported, it may be useful to examine some theoretical questions raised by the way in which the demand relation has been specified in this study. One such question concerns the nature of a demand relation in which the reactions of consumers and processors are combined. As in Chapter VI, it is simpler to introduce a new notation for the discussion of fairly general theoretical points than to employ the notation previously used in developing our economic and statistical models.

Consider a situation in which a commodity is generally sold by producers to intermediaries whom we call processors, and then resold by the processors to final consumers.[1] For given values of other relevant variables,[2] we may think of quantity sold by producers, quantity sold by processors, and prices at which the two kinds of exchanges take place as being determined by four relations. These would be the producers' supply relation, the processors' demand relation, the processors' supply relation, and the consumers' demand relation. If the commodity is perishable, or if storage is sufficiently expensive for any reason, it may be expected that the same quantity will be sold by processors as is sold by producers.[3] Then there is one less variable to be determined, and the behavior of processors can be represented by a single relation. The

[1] There are, of course, usually more than two stages in the complete production and marketing process. In general, the number of stages explicitly recognized by the investigator will depend on the detail with which he chooses to study the process.

[2] The other variables need not be predetermined, but it will do no harm to regard them temporarily as predetermined in the discussion of the theoretical interpretation of our derived demand relation. The existence of additional current endogenous variables in a particular application will mean that the equations considered in this discussion do not constitute a complete model in that application.

[3] Here it is implicitly assumed that net imports are negligible. Although this is approximately true for livestock products as a whole, foreign trade would have to be allowed for in a more refined model of an open economy.

processors' behavior relation thus shows the quantities that processors are willing to handle at various combinations of producer price and consumer price.

We are interested in the latter case since, for a period as long as a year, quantities of important livestock products taken by processors are equal or nearly equal to quantities taken by consumers, if consistent units of measurement are employed. Let the relevant relations be given by

(7.1) $\varphi(x, p, r) = 0$ (producer supply relation)

(7.2) $\chi(x, p, q, w) = 0$ (processor behavior relation)

(7.3) $\theta(x, q, y, z) = 0$ (consumer demand relation)

where x = quantity exchanged
p = price received by producers
r = other factors influencing producer behavior
q = price received by processors
w = other factors affecting behavior of processors
y = consumer income
z = other factors affecting consumer behavior

Suppose that χ and θ are of such a form that it is possible to eliminate q, thus obtaining a relation among x, p, y, w, z. This may be called the derived demand relation.[4] It shows how much producers can sell at various prices when the behavior of both consumers and processors is taken into account. Let it be indicated by

(7.4) $\omega(x, p, y, w, z) = 0$ (derived demand relation)

Relations Between Corresponding Marginal Responses and Elasticities

Relations between various slopes and elasticities in ω and in θ are of some interest. Let $(\partial x/\partial p)_\omega$, $(\partial x/\partial q)_\theta$ be the limiting values of marginal

[4] To eliminate x from the two equations would yield a valid restriction among the remaining variables, but, since x appears elsewhere in the model, this would reduce the number of equations without reducing the number of endogenous variables, thus leaving the system indeterminate.

Equations obtained by simultaneously eliminating one or more equations and one or more endogenous variables from a model have been called partially reduced form equations in various discussions. In a certain fundamental sense, all equations we are likely to deal with may be regarded as partially reduced form relations. It is always possible to imagine a more fundamental explanation of the phenomena that we observe, involving more equations and more endogenous variables. If the model we use is a reasonable one, it should, in principle, be possible to derive it, either exactly or approximately, from the more fundamental model by successive elimination of variables.

responses of quantity to changes in price in (7.4) and (7.3), respectively, and let $(\partial x/\partial y)_\omega$, $(\partial x/\partial y)_\theta$ be the responses to changes in income. Let $E_{\omega p}$, $E_{\theta q}$, $E_{\omega y}$, $E_{\theta y}$ represent the price and income elasticities. Comparisons between corresponding slopes and elasticities depend on the properties of χ. Relationships showing this dependence are developed below.

The total differentials of χ and θ may be written

$$d\chi = \frac{\partial \chi}{\partial x}\,dx + \frac{\partial \chi}{\partial q}\,dq + \frac{\partial \chi}{\partial p}\,dp + \frac{\partial \chi}{\partial w}\,dw = 0 \tag{7.5}$$

$$d\theta = \frac{\partial \theta}{\partial x}\,dx + \frac{\partial \theta}{\partial q}\,dq + \frac{\partial \theta}{\partial y}\,dy + \frac{\partial \theta}{\partial z}\,dz = 0 \tag{7.6}$$

By Cramer's rule we obtain the total differential of x as a function of the total differentials of p, w, y, z.

$$dx = -\frac{\begin{vmatrix} \dfrac{\partial \chi}{\partial p}\,dp + \dfrac{\partial \chi}{\partial w}\,dw & \dfrac{\partial \chi}{\partial q} \\ \dfrac{\partial \theta}{\partial y}\,dy + \dfrac{\partial \theta}{\partial z}\,dz & \dfrac{\partial \theta}{\partial q} \end{vmatrix}}{\begin{vmatrix} \dfrac{\partial \chi}{x} & \dfrac{\partial \chi}{\partial q} \\ \dfrac{\partial \theta}{\partial x} & \dfrac{\partial \theta}{\partial q} \end{vmatrix}} \tag{7.7}$$

By principles of differentiation of implicit functions,[5] $(\partial x/\partial p)_\omega$ is equal to the coefficient of dp in (7.7); i.e.

$$\left(\frac{\partial x}{\partial p}\right)_\omega = \frac{-\dfrac{\partial \chi}{\partial p}\dfrac{\partial \theta}{\partial q}}{\dfrac{\partial \chi}{\partial x}\dfrac{\varrho\theta}{\partial q} - \dfrac{\partial \chi}{\partial q}\dfrac{\partial \theta}{\partial x}} = \frac{\left(\dfrac{\partial x}{\partial q}\right)_\theta \left(\dfrac{\partial q}{\partial p}\right)_\chi}{1 - \left(\dfrac{\partial q}{\partial x}\right)_\chi \left(\dfrac{\partial x}{\partial q}\right)_\theta} \tag{7.8}$$

The latter equality holds if $\dfrac{\partial \chi}{\partial q}\dfrac{\partial \theta}{\partial x} \neq 0$. This may generally be expected and is assumed in what follows. By finding the coefficient of dy in (7.7), we obtain

$$\left(\frac{\partial x}{\partial y}\right)_\omega = \frac{\dfrac{\partial \chi}{\partial q}\dfrac{\partial \theta}{\partial y}}{\dfrac{\partial \chi}{\partial x}\dfrac{\partial \theta}{\partial q} - \dfrac{\partial \chi}{\partial q}\dfrac{\partial \theta}{\partial x}} = \frac{\left(\dfrac{\partial x}{\partial y}\right)_\theta}{1 - \left(\dfrac{\partial q}{\partial x}\right)_\chi \left(\dfrac{\partial x}{\partial q}\right)_\theta} \tag{7.9}$$

[5] See, e.g., de La Vallee Poussin, Ch. J., *Cours d'analyse infinitesimale*, Vol. I, pp. 145–148, Dover Publications, New York, 1946.

Goursat, Edouard, *A Course in Mathematical Analysis*, translated by E. R. Hedrick, Ginn & Co., Vol. I, pp. 49–51, 1904.

Price elasticities for the derived and consumer demand relations are defined by

$$E_{\omega p} = \frac{p}{x}\left(\frac{\partial x}{\partial p}\right)_{\omega}, \qquad E_{\theta q} = \frac{q}{x}\left(\frac{\partial x}{\partial q}\right)_{\theta} \tag{7.10}$$

From (7.10) and (7.8),

$$E_{\omega p} = \frac{p}{x}\,\frac{\left(\frac{\partial x}{\partial q}\right)_{\theta}\left(\frac{\partial q}{\partial p}\right)_{\chi}}{1-\left(\frac{\partial q}{\partial x}\right)_{\chi}\left(\frac{\partial x}{\partial q}\right)_{\theta}} = \frac{E_{\theta q}\left[\frac{p}{q}\left(\frac{\partial q}{\partial p}\right)_{\chi}\right]}{1-\left(\frac{\partial q}{\partial x}\right)_{\chi}\left(\frac{\partial x}{\partial q}\right)_{\theta}} \tag{7.11}$$

The factor in brackets in the numerator of the expression on the right is (the limit of) the ratio of the relative change in retail price to the relative change in producers' price when other factors affecting processor behavior are held constant. This might be called the elasticity of price transmission.[6] Income elasticities for the two equations are defined by

$$E_{\omega y} = \frac{y}{x}\left(\frac{\partial x}{\partial y}\right)_{\omega}, \qquad E_{\theta y} = \frac{y}{x}\left(\frac{\partial x}{\partial y}\right)_{\theta} \tag{7.12}$$

From the above and (7.9),

$$E_{\omega y} = \frac{y}{x}\,\frac{\left(\frac{\partial x}{\partial y}\right)_{\theta}}{1-\left(\frac{\partial q}{\partial x}\right)_{\chi}\left(\frac{\partial x}{\partial q}\right)_{\theta}} = \frac{E_{\theta y}}{1-\left(\frac{\partial q}{\partial x}\right)_{\chi}\left(\frac{\partial x}{\partial q}\right)_{\theta}} \tag{7.13}$$

The expressions on the right of (7.8), (7.9), (7.11), and (7.13) have the same denominator. In the class of cases in which x does not appear in χ, $(\partial q/\partial x)_{\chi}$ is equal to zero, and this denominator is equal to one. We then have

$$\left(\frac{\partial x}{\partial p}\right)_{\omega} = \left(\frac{\partial x}{\partial q}\right)_{\theta}\left(\frac{\partial q}{\partial p}\right)_{\chi} \tag{7.8$'$}$$

$$\left(\frac{\partial x}{\partial y}\right)_{\omega} = \left(\frac{\partial x}{\partial y}\right)_{\theta} \tag{7.9$'$}$$

$$E_{\omega p} = E_{\theta q}\left[\frac{p}{q}\left(\frac{\partial q}{\partial p}\right)_{\chi}\right] \tag{7.11$'$}$$

$$E_{\omega y} = E_{\theta y} \tag{7.13$'$}$$

The class of cases for which these latter relations hold includes such

[6] The term was suggested by Henry Houthakker.

processor behavior as constant percentage margins, constant dollar margins, or combinations of the two.[7]

If the processor behavior relation does contain x, then it seems to us that we may typically expect $(\partial q/\partial x)_x$ to be positive, and therefore the denominator that appears in our earlier expressions may be expected to be greater than one. For a constant percentage margin, the elasticity of price transmission, $\frac{p}{q}\left(\frac{\partial q}{\partial p}\right)_x$, is equal to one; for constant dollar margin or the combined case, it is less than one. It seems reasonable to suppose that, in general, this elasticity rarely exceeds one.[8] This means that, if producers' price rises while quantity processed and such other factors as prices of inputs used by processors remain fixed, the relative change in consumer price will not exceed the relative change in producers' price. This would certainly be true if effective competition existed in processing, and might be expected to be typical of other instances as well.

If the two presumptions

$$(7.14) \qquad \left(\frac{\partial q}{\partial x}\right)_x \geqslant 0, \qquad \frac{p}{q}\left(\frac{\partial q}{\partial p}\right)_x \leqslant 1$$

are accepted, then we have

$$(7.8'') \qquad \left(\frac{\partial x}{\partial p}\right)_\omega \geqslant \left(\frac{\partial x}{\partial q}\right)_\theta \left(\frac{\partial q}{\partial p}\right)_x$$

$$(7.9'') \qquad \left(\frac{\partial x}{\partial y}\right)_\omega \leqslant \left(\frac{\partial x}{\partial y}\right)_\theta$$

$$(7.11'') \qquad E_{\omega p} \geqslant E_{\theta q}$$

$$(7.13'') \qquad E_{\omega y} \leqslant E_{\theta y}$$

The directions of the inequalities correspond to the usual case where

[7] These particular assumptions have been extensively used by the Bureau of Agricultural Economics in comparing farm and retail prices of agricultural food products. See

Price Spreads between Farmers and Consumers for Food Products 1913–44, *USDA Miscellaneous Publication 576, Bureau of Agricultural Economics*, Sep. 1945. Also Been, Richard O., Price Spreads between Farmers and Consumers, *Agricultural Information Bulletin 4*, Bureau of Agricultural Economics, U. S. Department of Agriculture, Nov. 1949.

[8] Relations between farm and retail prices for food livestock products have been discussed and analyzed statistically by Karl A. Fox. See Fox, Factors Affecting Farm Income, Farm Prices, and Food Consumption, *Agricultural Economics Research*, Vol. III, pp. 72, 73. His results tend to confirm our supposition for the important products with which we are concerned.

$(\partial x/\partial p)_w$, $(\partial x/\partial q)_\theta$ are negative and $(\partial x/\partial y)_w$, $(\partial x/\partial y)_\theta$ are positive. If the above presumptions are correct, consumer demand is at least as elastic with respect to both price and income as is the derived demand.

Long- and Short-Run Responses of Quantity Consumed

The derived livestock demand relation fitted in Chapter IV did not contain any variables corresponding to w in (7.4), though variables reflecting costs of inputs used in processing would have been included if satisfactory data had been available. Variables corresponding to the z in (7.4) were general price level (z_{7t}), population (z_{6t}), lagged population (z_{6t-1}), and lagged consumption (z_{4t} or y_{4t-1}). The presence of lagged consumption raises questions about the stability of the system and about the long-run responses of consumption to other variables. Questions of stability are not investigated here because they would be rather involved, and because the incompleteness of our model and our uncertainties about various aspects of the specification would make such an analysis almost purely illustrative.[9] Some simple observations about the long-run behavior of consumption that are helpful in interpreting our results, however, are indicated below.

Suppose the demand relation is linear in the observed variables. In the notation introduced in this chapter, it can then be written

$$x_t = \xi_0 + \xi_1 p_t + \xi_2 y_t + \xi_3 z_t + \xi_4 x_{t-1} \tag{7.15}$$

To consider long-run reactions of consumers, we assume that p_t, y_t, z_t, remain constant indefinitely at levels $\tilde{p}$, $\tilde{y}$, $\tilde{z}$. (7.15) becomes

$$x_t = \xi_0 + \xi_1 \tilde{p} + \xi_2 \tilde{y} + \xi_3 \tilde{z} + \xi_4 x_{t-1} \tag{7.16}$$

Given any initial value x_0 for consumption, we have

$$\begin{aligned} x_1 &= \xi_0 + \xi_1 \tilde{p} + \xi_2 \tilde{y} + \xi_3 \tilde{z} + \xi_4 x_0 \\ x_2 &= (1 + \xi_4)(\xi_0 + \xi_1 \tilde{p} + \xi_2 \tilde{y} + \xi_3 \tilde{z}) + \xi_4^2 x_0 \\ &\;\;\vdots \\ x_t &= (1 + \xi_4 + \xi_4^2 + \cdots + \xi_4^{t-1})(\xi_0 + \xi_1 \tilde{y} + \xi_2 \tilde{p} + \xi_3 \tilde{z}) + \xi_4^t x_0 \end{aligned} \tag{7.17}$$

If $-1 < \xi_4 < 1$, we may write

[9] Some of the stability properties of a simpler model have been considered by Richard J. Foote. See article in *Journal of Farm Economics*, Feb. 1953.

$$(7.18) \qquad \tilde{x} = \lim_{t\to\infty} x_t = \frac{1}{1-\xi_4}(\xi_0 + \xi_1\tilde{p} + \xi_2\tilde{y} + \xi_3\tilde{z})$$

Long-run responses of consumption to unit changes in price, income, and other factors are given by the coefficients $\frac{\xi_1}{1-\xi_4}, \frac{\xi_2}{1-\xi_4}, \frac{\xi_3}{1-\xi_4}$.

As before, let logarithms of observed variables be denoted by capital letters. Suppose the demand relation is linear in these logarithms and is given by

$$(7.19) \qquad X_t = \eta_0 + \eta_1 P_t + \eta_2 Y_t + \eta_3 Z_t + \eta_4 X_{t-1}$$

For constant values $\tilde{P}$, $\tilde{Y}$, $\tilde{Z}$ of P_t, Y_t, Z_t and $-1 < \eta_4 < 1$, we have

$$(7.20) \qquad \tilde{X} = \lim_{t\to\infty} X_t = \frac{1}{1-\eta_4}(\eta_0 + \eta_1\tilde{P} + \eta_2\tilde{Y} + \eta_3\tilde{Z})$$

In this case, the coefficients η_1, η_2, η_3 may be regarded as short-run elasticities of demand, and the coefficients $\eta_1/(1 - \eta_4)$, $\eta_2/(1 - \eta_4)$, $\eta_3/(1 - \eta_4)$ as corresponding long-run elasticities.

A Linear Demand Relation

Estimates of parameters in livestock-demand equations have been obtained, both under the assumption that the equation is linear in the observed variables and under the assumption that it is linear in the logs. Estimates obtained under the latter assumption were given in Table X, Chapter IV. Estimates obtained for the linear equation are given in Table XVIII. In the notation used in Chapter IV, the linear demand relation is given by

$$(7.21) \qquad \beta''_{54}y'_{4t} + y_{5t} + \gamma''_{54}z'_{4t} + \gamma''_{55}z'_{5t} + \gamma''_{57}z_{7t} + \gamma''_{50} = u_{5t}$$

where, as before,

$y'_{4t} = y_{4t}/z_{6t}$ = per-capita consumption of livestock products

y_{5t} = price of livestock products

$z'_{4t} = y_{4t-1}/z_{6t-1}$ = lagged per-capita consumption of livestock products

$z'_{5t} = z_{5t}/z_{6t}$ = disposable income per capita

z_{7t} = index of nonfarm wholesale prices

z_{1t}, z_{3t}, z_{6t}, z_{8t}, z_{9t}, z_{10t} were used as z_{**} in obtaining the limited-information estimates given in Table XVIII. y_{5t} was treated as dependent in the application of least squares. In the calculations underlying the estimates, y'_4 and z'_4 were measured in dollars worth of product (at average

TABLE XVIII

ESTIMATES FOR LINEAR VERSION OF DEMAND FOR LIVESTOCK PRODUCTS

Method	Estimates of Coefficients				
	β''_{54}	γ''_{54}	γ''_{56}	γ''_{57}	γ''_{50}
Limited information	0.0184(0.0061)	−0.0077(0.0063)	−0.0012(0.00014)	−0.0044(0.00095)	−0.283(0.546)
Least squares	0.0174(0.0057)	−0.0074(0.0062)	−0.0012(0.00014)	−0.0044(0.00094)	−0.248(0.355)

$R^2 = 0.980$

prices) per person, y_5 is a price index which takes the value 1 when products are valued at their average 1920–49 prices, z'_5 is in current dollars per person, and z_7 is an index whose average value 1935–39 equals 100.

COMPARISONS OF ALTERNATIVE ESTIMATES

Some interpretations of these results and the results previously presented in Chapter IV can be facilitated by solving each equation containing a particular set of estimated coefficients for the quantity variable or its logarithm. (7.22) below was obtained by inserting the limited-information coefficients from Table X into (4.5′) and solving. (7.23) was obtained from (4.5′), using the least-squares coefficients. (7.24), (7.25) follow from (7.21) when limited-information and least-squares estimates, respectively, from Table XVIII are inserted.

$$(7.22)\quad Y'_4 = 3.417 - 0.758Y_5 + 0.748Z'_5 + 0.373Z_7 - 0.031Z'_4$$

$$(7.23)\quad Y'_4 = 3.501 - 0.765Y_5 + 0.753Z'_5 + 0.378Z_7 - 0.035Z'_4$$

$$(7.24)\quad y'_4 = 15.38 - 54.25y_5 + 0.068z'_5 + 0.237z_7 + 0.419z'_4$$

$$(7.25)\quad y'_4 = 14.20 - 57.35y_5 + 0.072z'_5 + 0.255z_7 + 0.425z'_4$$

The constant terms in (7.22) and (7.23) have been adjusted to be consistent with the units of measurement used in the linear equations.

Coefficients in the logarithmic equations seem plausible, except for the coefficient of Z'_4. A negative coefficient for lagged consumption implies a tendency for years of relatively high consumption to be followed by years of relatively low consumption, and conversely. By relatively high and low, we mean as compared with the long-run equilibrium quantities that consumers would take if prices and incomes were stable. One way of interpreting this result is to note that it implies that, if consumers start from a position of long-run equilibrium and encounter a change in income or prices, they tend to overrespond in the first year. In the next year, they more than correct for the initial overresponse, and so forth, gradually approaching the new equilibrium consumption in an oscillatory fashion, provided the coefficient of Z'_4 is greater than -1. This would introduce a kind of cobweb effect on the demand side of the market.

Although such behavior is conceivable, it is contrary to the usual presumption of initial underadjustments of consumption to changes in prices and incomes, and would be regarded as implausible by most economists. The negative sign of this coefficient could easily have occurred by chance; it results from the positive estimates of γ'_{54} in Table X, and these are clearly not statistically significant. So far as these results go, we have no basis for saying whether or not it is reasonable to allow for the influence of past events on present consumption through a lagged consumption term. It did not turn out to be a very useful procedure in estimating these two equations.

Comparisons with Other Demand Studies

The price and income elasticities indicated by the coefficients of Y_5 and Z'_5 in (7.22) and (7.23) seem reasonable, so far as our a priori ideas as to the general orders of magnitude are concerned. They are a little higher (in absolute value) than might be expected from the results of some previous studies, but a number of economists have expressed suspicions that most statistical analyses have tended to understate price and income elasticities. The studies most directly comparable to the present one were by Karl A. Fox.[10] Using annual data, 1922–41, and first differences of logarithms of observed variables, he obtained a price elasticity of -0.41 and an income elasticity of 0.50 for a relation representing demand for all food livestock products at the farm level. This makes his relation conceptually very close to our derived demand relation. For demand for all food livestock products at retail, he obtained price elasticities of -0.52, -0.56, and -0.61 and income elasticities of 0.40, 0.47, and 0.51, using aggregate data for the same period. From family budget data he estimated income elasticity at 0.33.[11]

[10] Karl A. Fox, *op. cit.* As Mr. Fox has emphasized in correspondence, part of the difference in results between these two trials undoubtedly results from different time series used to represent the quantity variable. Fox used the BAE series of civilian consumption per capita of all food livestock products, as published in *National Food Situation.* Aside from weighting differences, this series differs from y_{4t} in excluding government purchases for military use, net exports, and net increases in commercial stocks.

[11] The figures -0.61 for price elasticity and 0.51 for income elasticity are calculated from the data in Fox, *op. cit.*, Table 3, p. 71. The cross-section elasticity is from Table 9, p. 82. It includes only food consumed at home and may be an underestimate for this reason.

There have been a number of recent studies of demand for all food. These deal with demand for food at retail. From our arguments at the beginning of this chapter, which tend to be confirmed by Fox's results, we expect retail demand to exhibit price and income elasticities (in absolute value) equal to or higher than our derived demand. On the other hand, we would expect both income and price elasticity of demand for livestock products to be higher than the corresponding elasticities of demand for food, since food includes a number of commodities (e.g. cereals) for which the elasticities are known to be very low. If quantity effects on processor behavior are negligible, then income elasticity of the derived demand is negligibly different from income elasticity in the retail demand. This is consistent with Fox's results cited above. Under these circumstances, income elasticity of demand for all food should be less than income elasticity of either retail or derived demand for livestock products. Although other comparisons depend on additional and even more hazardous assumptions,[12] some readers may be willing to make additional assumptions and may thus be interested in looking at various results on price and income elasticities of demand for all food in this connection.

Using both family budget data for 1941 and time series, Tobin[13] obtained estimates of -0.53 and 0.45 for price and income elasticity, respectively. In his discussion of Tobin's study, Professor Stone[14] reports estimates obtained by himself and jointly with Tobin that vary between -0.39 and -0.90 for price elasticity and between 0.53 and 0.83 for income elasticity. Three of the six estimates of price elasticity are within 0.06 of -0.57, and four of the six estimates of income elasticity are within 0.05 of 0.54. Girshick and Haavelmo[15] have estimated price elasticity at -0.25 and income elasticity at 0.25, using time series, 1922–41, and limited-information methods. From family budget data for 1948, Fox[16] obtained an income elasticity of 0.42 for demand for all food.

The signs of coefficients in the linear equations (7.24 and 7.25 above)

[12] If we assume that processor behavior is given by an equation of the form $(q_t/q_{t-1}) = \alpha_0(p_t/p_{t-1})^{\alpha_1}$, where q_t is retail price, p_t is farm price, and α_0, α_1 are constants, then the elasticity of price transmission is equal to α_1, and Fox's figure of 1.47 (Table 4, p. 72) for the percentage response of farm price to a 1% change in retail price is a least-squares estimate of $1/\alpha_1$. This would make α_1 approximately 0.68. Price elasticity of retail demand would be 1.47 times price elasticity of the derived demand.

[13] James Tobin, A Statistical Demand Function for Food in the U. S. A., *Journal Royal Statistical Society*, Vol. CXIII, Part II.

[14] Tobin, *op. cit.*, p. 142.

[15] M. A. Girshick and Trygve Haavelmo, *op. cit.*

[16] *Op. cit.*, pp. 79–82.

TABLE XIX

POINTS AT WHICH ELASTICITIES ARE COMPUTED

Date	Observed Values					Computed Values of y_4'	
	y_5	z_5'	z_7	z_4'	y_4'	(7.24)	(7.25)
1933	0.444	358	69.0	54.2	55.1	54.9	55.0
Avg. 1920–49	0.922	643	97.7	55.7	55.9	55.9	55.9
1949	1.685	1249	152.4	60.5	61.7	70.9	71.4

TABLE XX

ELASTICITIES OF ESTIMATED LINEAR RELATIONS AT SELECTED POINTS

Method of Estimation	Date of Selected Point	Elasticity of Quantity with Respect to:			
		Price, y_5	Income, z_5'	General, prices, z_7	Lagged consumption, z_4'
Limited information (7.24)	1933	−0.44	0.22	0.30	0.41
	Avg. 1920–49	−0.89	0.78	0.41	0.42
	1949	−1.29	1.20	0.51	0.36
Least squares (7.25)	1933	−0.46	0.47	0.32	0.42
	Avg. 1920–49	−0.95	0.83	0.45	0.41
	1949	−1.35	1.26	0.54	0.36

TABLE XXI

APPLICATION OF DURBIN-WATSON TEST TO DEMAND EQUATIONS

5 % Level of Significance

Equation No.	Method of Estimation	No. of Predetermined Variables, k'	Calculated Statistic, d	Limits of Relevant Boundary	
				Lower limit, d_L	Upper limit, d_U
4.5	Limited information	3	1.107	1.04	1.54
4.5	Least squares	4	1.102	0.96	1.65
7.21	Limited information	3	1.329	1.04	1.54
7.21	Least squares	4	1.313	.96	1.65

$n = 26$

are all plausible. In attempting to judge the plausibility of the magnitudes and form some notion of the appropriateness of the linear form for this relation, it seems convenient to convert the slopes of (7.24) and (7.25) to elasticities. The elasticities given by either linear equation vary, of course, with the point at which the elasticities are computed. To get calculations for both representative and extreme values of the variables and still stay within the general range of the observations, three sets of values for the variables on the right in (7.24), (7.25) were chosen. These were the average values for the whole period, the observed values for 1933, and the observed values for 1949. They are given in Table XIX, along with the observed values of y_4' and the values of y_4' computed from (7.24) and (7.25). The latter were used in computing the elasticities given in Table XX. The extreme increases in price and income elasticities as real income increases seem rather implausible and tend to cast some doubt on the appropriateness of the linear form for this relation.

Tests of Hypotheses for Demand Relations

For both the linear and logarithmic forms, the hypothesis of overidentification is not rejected by the Rubin-Anderson test. For the logarithmic equation, the calculated value of $T \log (1 + 1/\lambda)$ is 11.89, and for the linear version it is 11.73. The critical value of χ^2 with 6 degrees of freedom and an 0.05 level of significance is 12.6.

The Durbin-Watson test of the hypothesis of serially independent disturbances is inconclusive in all four cases: i.e., for both the linear and logarithmic forms and both limited-information and least-squares calculations. The application of this test is shown in Table XXI.