CHAPTER X

ADJUSTMENTS OF COST

1. Flexibility of Cost. As already intimated, labor costs are flexible costs of industry, and among the first to be adjusted when losses occur or appear imminent. Capital costs, bond interests, etc., and transportation are the most rigid of all costs. If raw materials are plentiful, a curtailment of some of their use may bring about drastic reductions in their costs. This is especially true for farm products for which the number of sellers (competing sellers) is very much greater than the number of buyers.

It is instructive to examine the profit function more closely to get some measure of flexibility of cost.

Substitution of the value of u_j that maximizes π_j, (7.3) of the preceding chapter, in π_j gives

$$\pi_j = \int_0^{t_1} \{[(1-H_j)^2\gamma_j^2p_0^2 - 2\,(1-H_j)B_j\gamma_jp_0 + B_j^2 - 4A_jC_j]/4A_jC_j\}\,E(0,t)\,dt.$$

Now, as already mentioned, the integrand may be negative over short periods of time, but these periods must be very short, for otherwise entrepreneurs will adjust their cost functions. Thus, in general, the integrand must be positive, that is, since A_j and C_j are positive, it follows that

$$(1-H_j)^2\gamma_j^2p_0^2 - 2(1-H_j)B_j\gamma_jp_0 + B_j^2 - 4A_jC_j > 0.$$

If, in particular, $H_j = 0$ and $\gamma_j = 1$, this simplifies into

$$p_0^2 - 2p_0B_j + B_j^2 - 4A_jC_j > 0\ ,$$

where, as already pointed out, p_0 is present market price.

Evidently for a fixed price p_0 it is possible to enlarge profits by decreasing A_j and C_j, that is, either A_j, the marginal unit cost of producing and distributing when a large number of units are already being produced and distributed,* or C_j, the overhead cost, can be adjusted.

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*If u_c represents unit cost, then
$u_c = q(u,\ p,\ t)/u = Au + B + C/u$, so that $du_c/du = A - C/u^2$,
and for u large C/u^2 is negligible; du_c/du is by definition marginal unit cost.

For a fixed minimum wage A_j can be adjusted only by reducing wages and salaries which are above the minimum, or by selecting employees for their productivity, and thus getting along with fewer employees, or by introducing labor-saving machinery. By recalling the origin of A_j, it will be noted that the wages and salaries principally affected are those of distributors.

An obvious way to reverse the inequality is to lower C_j. This can be accomplished in several ways:

1. C_j can be lowered by reducing the salaries of officers and executives and employees who do not contribute directly to production. In particular, in the United States white-collar employees can be reduced in salary or wages by firing and rehiring provided that the minimum wage section of the National Recovery Administration code is not violated. Furthermore, some white-collar workers and executives who might be said to furnish scenery, that is, those whose jobs are not indispensable, can be dismissed.

2. Capital charges can be reduced by refinancing, which lowers the interest charges on bonded and mortgage indebtedness, or by default of such indebtedness, but, of course, it is possible to do extensive refinancing on a long-time basis only if currency is not depreciating rapidly.* When bondholders foreclose, or ask for receivership, stock equities are wiped out or practically wiped out, and it merely becomes necessary to pay some kind of dividend to the bondholders.

In the case of concerns that have no bonded or mortgage indebtedness, the capital charge in C_j is one arbitrarily set by the accounting department. For such firms the capital charge can be temporarily lowered to zero, or even to a point below zero, that is, the concern may consume part of its own capital in an attempt to save the rest. During depressions many large concerns make extensive allowances for depreciation and appear to suffer losses, whereas in reality they are building up their liquid assets at the expense of dividends and replacements (see Chart XXVIII). This, of course, cannot be taken as a general criticism. It is only a natural consequence of a general situation.

It is also possible to increase profits by decreasing B_j, for

$$dQ_j/du_j = 2A_j u_j + B_j + H_j \gamma_j p_0$$

*In the U. S. the drastic Securities Act probably prevents this refinancing. It almost certainly was a hindrance to recovery in 1933.

so that B_j represents the marginal cost of *producing* the first unit, the production cost exclusive of overhead. It is, therefore, inconceivable that production would start unless $B_j < \gamma_j p_0$ or soon expected to be $< \gamma_j p_0$. Hence, in general, $B_j \gamma_j p_0 > B_j^2$, that is, if there is no expected profit, a profit may be possible by reducing the marginal cost of producing the first unit. This can be accomplished only by paying less for (1) raw materials (which may or may not be possible) or (2) by paying less for wages, especially productive wages. When a minimum wage has been set by law, it is possible only to reduce high productive wages and salaries, in other words, to make efforts to bring all wages and salaries to the minimum. In the case of B_j, however, a new factor enters, that is, if wages are too high it may be profitable to substitute machinery for men. Again it may be profitable to change from water power to electric power or vice versa. This introduces the principle of substitution of factors of production which will be considered more fully at the end of this chapter.

In some instances, decrease of H_j (removal of taxes, reductions of commissions and sales expenses, etc.) might suffice to reverse the inequality (7.3) of the preceding chapter.

2. Technological Unemployment. It is a truism to say that in any growing economy every technological improvement requires changes in the distribution of labor. Farmers become machine helpers, farmers and machine helpers become salesmen and so on. Sometimes demand greatly increases and additional machine workers, as well as salesmen or other sales or distributing help, are required. Some industries reach full maturity, i.e., reach a point of maximum demand, and then improvements in technology may produce unemployment of workers in the industry.

So long as new investments (residential building, public works, new enterprises, etc.) are being made in the economy so that savings are pressed into service and thus returned to the worker, reductions in cost by laying off "extra" labor to reduce costs have only beneficial effects on the economy as a whole. By this displacement of marginal help, labor is released for other productive enterprise. In such instances a higher standard of living is inevitable. The machine merely releases human labor for more productive enterprise for which a machine has not yet been designed. However, due to the fact that social progress, i.e., opportunities for investments in new enterprises, does not coincide with technological progress, this process of shifting labor is not always painless. Every age has had a group which has vociferously demanded limitation of the machine. Today is certainly no exception.

It appears to be universally true that industrial depressions lead to widespread attempts of industrialists and others to reduce costs. As profits shrink the entrepreneur inevitably is led to examine his costs carefully and to attempt to reduce them. Since labor costs are most flexible, they are the ones which first come in for adjustment. Thus, in an attempt to reduce costs, unnecessary and marginal labor may be laid off or other labor may be displaced by machinery. It is this latter that leads to the question of technological unemployment. Recently a great deal has been written about this subject. Much of this, like similar material written centuries ago, is sophomoric. A large amount of it has been written by so-called planners who know little or nothing about industry or, for that matter, about economic theory. As an exception, a recent paper by D. I. Vinogradoff may be mentioned.*

Suppose that a technological improvement is introduced into an industry. In order not to complicate the problem, suppose that a unit of time is chosen so that the output per unit time is equal to the consumption per unit time.

The following table summarizes notation that will be useful:

	Before Technological Improvement	After Technological Improvement
Demand = total output	U	U_1
Output per machine worker	v	v_1
Unit Cost	U_c	U_{c1}
Unit price	p	p_1
Number of employees at machines	E_m	E_{m1}
Number of employees assisting at machines	E_h	E_{h1}
Number of factory white collar workers	E_w	E_{w1}
Number of employees distributing and servicing	E_d	E_{d1}
Total number of employees	E	E_1

*D. I. Vinogradoff, "Effects of a Technological Improvement on Employment," *Econometrica*, Vol. 1, No. 4, October 1933, pp. 410-417.

See also, Arthur Dahlberg, *Machines, Jobs and Capitalism* and Philip Wernette, *Prices and Production.*

Obviously, $$E = E_m + E_h + E_w + E_d \ ,$$

and $$E_1 = E_{m1} + E_{h1} + E_{w1} + E_{d1} \ .$$

From equations (3.2), (3.3), (3.4) and (3.5) of Chapter IX it follows that

$$(2.1) \qquad E = (A_h + A_d)U^2 + (B_m + B_h + B_w + B_d)U + (C_h + C_w + C_d) + H_d pU,$$

if C_m is taken equal to 0 as was suggested likely. Now, $B_m = \frac{1}{v}$ since the number of workmen at machines making the product is the next highest integer to U/v, i.e., $E_m = B_m U = U/v$.

In simplified form (2.1) becomes

$$E = \frac{U}{v} + \bar{B}U + HpU + AU^2 + C,$$

where

$$\bar{B} = B_h + B_w + B_d, \ H = H_d, \ A = A_h + A_d,$$

and

$$C = C_h + C_w + C_d;$$

that is, the total number of employees consists of a part U/v depending upon v, the output per machine employee, and upon another part independent of v. Let the part independent of the output per worker be designated by e so that

$$E = U/v + e \ .$$

Suppose now that due to a technological improvement, the output per workman rises from v to v_1, making it possible to sell the product at a lower price p_1. The demand in this case will be $U_1 > U$. The problem is to find E_1 and other quantities.

Evidently,

$$E_1 = U_1/v_1 + \bar{B}U_1 + Hp_1U_1 + AU_1^2 + C$$
$$= U_1/v_1 + e_1 \ .$$

The condition that total unemployment be unchanged requires that

$$U/v + e = U_1/v_1 + e_1;$$

that is, that

$$U/v - U_1/v_1 + \overline{B}(U - U_1) + A(U^2 - U_1^2) + HpU - Hp_1U_1 = 0.$$

For many products the term $A(U^2 - U_1^2)$ is small and can be neglected. In such instances it follows readily that

$$U_1/U = \frac{1/v + \overline{B} + Hp}{1/v_1 + \overline{B} + Hp_1} . \tag{2.2}$$

In particular, if the demand U is given by an equation of the type

$$U = apI^{-\beta} + b ,$$

where I stands for consumer income, the condition (2.2) becomes

$$aHI^{-\beta}p_1^2 + (aI^{-\beta}[1/v_1 + \overline{B}] + Hb)p_1 + b(\overline{B} + 1/v_1) - (apI^{-\beta} + b)(1/v + \overline{B} + Hp) = 0.$$

This quadratic equation in p_1 has solutions

$$p_1 = \left\{ -(aI^{-\beta}[1/v_1 + \overline{B}] + Hb) \pm \{(aI^{-\beta}[1/v_1 + \overline{B}] + Hb)^2 - 4aHI^{-\beta}(apI^{-\beta} + b)(1/v + \overline{B} + Hp)\}^{1/2} \right\} / 2aHI^{-\beta}$$

If $1/v_1 + \overline{B} > H$ (see (2.1)), the first parenthesis is a positive quantity and hence the negative root is not permissible since it would yield a négative price p_1. Here

$$p_1 = \left\{ -(aI^{-\beta}[1/v_1 + \overline{B}] + Hb) + \{(aI^{-\beta}[1/v_1 + \overline{B}] + Hb)^2 - 4aHI^{-\beta}(apI^{-\beta} + b)(1/v + \overline{B} + Hp)\}^{1/2} \right\} / 2aHI^{-\beta}$$

The case for which $H = 0$, i.e., no commissions, is particularly interesting because of the simplicity of the solution. In fact, for this case

$$p_1 = p[(1/v + \bar{B})/(1/v_1 + \bar{B})] + \frac{b}{a} I^{\beta} [(1/v - 1/v_1)/(1/v_1 + \bar{B})] , \tag{2.3}$$

if the number of employees is the same after mechanization as before.

A similar analysis shows that the number of employees after mechanization will be greater than the number before, provided

$$p_1 < p[(1/v + \bar{B})/(1/v_1 + \bar{B})] + \frac{b}{a} I^{\beta} [(1/v - 1/v_1)/(1/v_1 + \bar{B})] , \tag{2.4}$$

and technological unemployment will occur whenever

$$p_1 > p[(1/v + \bar{B})/(1/v_1 + \bar{B})] + \frac{b}{a} I^{\beta} [(1/v - 1/v_1)/(1/v_1 + \bar{B})] . \tag{2.5}$$

The above solutions are based on the assumption that the technological change does not affect consumer income, that is, that I remains unchanged. If I is changed in the process of technological improvement, the factor $(1/v + \bar{B})/(1/v_1 + B)$ becomes

$$(I_1/I)^{-\beta} (1/v + \bar{B})/(1/v_1 + B)$$

and the I in the second term becomes I_1. Otherwise the conditions (2.3), (2.4) and (2.5) are unchanged.

Now, in many industries $\bar{B}$ is much more important than $1/v$. Suppose, however, for definiteness, that before mechanization $\bar{B} = 1/v$; i.e., that there are as many white collar workers and distributors as machine employees, and suppose further that through mechanization, v, the output per machine worker, is increased *10* per cent. Then $(1/v + \bar{B})/(1/v_1 + \bar{B}) = 1.04$. Suppose also that $\beta = 1$. If technological unemployment is not to occur, then

$$p_1 \leq 1.04p(I_1/I) + .18 \frac{b}{a} I_1 .$$

If $I_1 = I/2$; that is, if the consumer income is cut in half while the technological change is being made (not necessarily because of the change), the above expression becomes

$$p_1 \leq .52p - .18(b/|a|)I_1 ,$$

where $|a|$ denotes numerical value of a, which means that *the new price must be less than half the price before the technological change if there is to be no "technological unemployment."* But this situation seldom prevails. Hence, there is no wonder that every depression brings forth those who attribute the cause thereof to technological unemployment.

Now, if $U = apI^{-\beta} + b$, as assumed in the preceding work, then $du/dp = aI^{-\beta}$, so that the conditions (2.3) can be written in the form

$$p_1 \gtreqless p\,[(1/v + \overline{B})/(1/v_1 + \overline{B})] + b(dp/du)\,[(1/v - 1/v_1)/(1/v_1 + \overline{B})].$$

This form may be more suggestive for some purposes.

3. The Principle of Substitution. The first two kinds of coefficients of production (Section 2 Chapter IX) are ready made, but not so with the compensatory coefficients. An entrepreneur may often have a great deal of choice left him when it comes to selecting the compensatory coefficients. As already pointed out, he may substitute machine labor for hand labor, water power for electric power and so on. Thus, if a minimum wage rate is raised, the compensatory relation may be changed so that introduction of machinery gives a lower cost. An examination of the factors of production in relation to minimum cost is essential.

If the coefficients of production are represented by $f_a(u,p,t)$ where t is a parameter in the sense explained above, and if the corresponding prices are represented by $p_a(t)$, $a = 1, 2, \cdots, m$, where m is the total number of services and commodities required to produce one unit of G per unit of time, the cost of producing $u(t)$ units per unit of time will be $\sum_{a=1}^{m} f_a(u, p, t)\, p_a(t)\, u(t)$.

This quantity is the total cost function which hereafter will be called $Q(u, p, t)$. In particular, it may be the cost function Q already derived in Chapter IX.

At the time t_1 a producer may be assumed to be using amounts of each of several factors of production, say $s \leq m$, so that the initial conditions are

$$(3.1) \qquad f_i(t_1) = f_{i1}, \qquad i = 1, 2, \cdots, s \; .$$

Some of the f_{i1} may be zero, as, for example, if the producer is not producing at the time t_1 or if he is not using a particular factor of production at that time.

As a typical problem it might be supposed that for a given rate of production $u(t)$, a given rate of sales $y(t)$ and a given sales price $p(t)$ a producer starting from the situation (3.1) desires to choose his compensatory services so that his total cost of production over the period of time (t_1, t_2) is a minimum, i.e., so that

$$Q = \int_{t_1}^{t_2} \sum_{\alpha=1}^{m} f_\alpha(u, p, t)\; p_\alpha(t)\, u(t)\, dt$$

is a minimum for $f_\alpha = F_\alpha/u$ satisfying (2.3) of Chapter IX,

$$\varphi(F_1, F_2, \cdots, F_m, u, p, t) = 0.$$

Obviously since Q is a linear functional of f_α there is a set of f_α which satisfies (2.3) and minimizes Q only if (2.3) is not linear in in the f_α and satisfies certain conditions which will be discussed later. This is a problem in the calculus of variations.* To solve the problem it can be assumed that there is a solution $\overline{F}_1(t), \cdots, \overline{F}_s(t)$ and the conditions which this solution must satisfy can then be found.

By defining a quantity $\lambda(t)$ by the equation

$$\lambda(t) = (\partial \varphi / \partial \overline{F}_s) / p_s(t) \; ,$$

the conditions for solution of the problem can be written in the symmetrical form

*See, for example, G. A. Bliss, "The Problem of Lagrange in the Calculus of Variations," *American Journal of Mathematics*, Vol. 52, 1930, pp. 673-744. However since f_α does not depend upon derivatives of u and p the problem is much simplified.

$$(3.2) \qquad \partial \varphi / \partial \overline{F}_i = \lambda(t) p_i(t) \ , \qquad i = 1, 2, \cdots, s \ ,$$

where the additional equation given by $i = s$ may be taken to be the equation of definition of $\lambda(t)$.*

The equations (3.2) are also the conditions for minimum cost for a given output $u(t)$ and price $p(t)$ since in the above analysis $p(t)$ *and* $u(t)$ have been assumed constant. The partial derivative $\partial \varphi / \partial \overline{F}_i$ may be defined as the *marginal degree of productivity of the* ith factor of production. By means of this definition, which is, however, different from that usually given, the conditions (3.2) can be stated as the following theorem:

Minimum cost for a given output and a given price results when the marginal degree of productivity of each service is proportional to its price, provided that there is only one compensatory relation.

If the ith equation of (3.2) is multiplied by $\overline{F}_i$ and summed for $i = 1,2, \cdots, s$, the following equation results:

$$\sum_{i=1}^{s} \overline{F}_i \, \partial \psi / \partial \overline{F}_i = \lambda(t) \sum_{i=1}^{s} p_i(t) \overline{F}_i(t).$$

If ψ happened to be a homogeneous function of the first degree in $\overline{F}_1, \cdots, \overline{F}_s$, Euler's theorem would require that the left hand member equal $\psi = 0$, which is impossible since obviously if $\lambda(t) = 0$, there is no problem.

If $\psi(F, \cdots, F_s, u, p, t) = 0$ can be solved for u so that

$$u = \varphi(F_1, \ \cdots, F_s, p, t) \ ,$$

the same equations (3.5) with φ replacing ψ would hold since u has been kept constant. In other words it is possible to write also

$$(3.3) \qquad \partial \varphi / \partial \overline{F}_i = \lambda(t) p_i(t) \ , \qquad i = 1,2, \cdots, s \ .$$

Whenever $u = \varphi$ is a homogeneous equation of the first degree in $F_1, \cdots, F_s$, Euler's theorem can be used as above to obtain

$$\lambda(t) = u(t) / \sum_{i=1}^{s} p_i(t) \overline{F}_i(t) = u(t) / Q_f(u, p, t) = 1/p_c(t) \ ,$$

where by definition $Q_f(u, p, t)$ denotes that part of the total cost which is due to the compensatory coefficients $\overline{F}_i$, and $p_c(t)$ is that

*See Appendix VII.

part of the cost of producing one unit per unit of time due to the compensatory coefficients.

When this value of $\lambda(t)$ is substituted in (3.3) the following equations result:

$$(3.4) \qquad (\partial u/\partial \bar{F}) = [p_i(t)/p_c(t)] \ , \quad i = 1,2, \cdots , s \ .$$

This equation may be stated as the theorem:

For a minimum cost over a period of time (t_1, t_2) for a homogeneous compensatory relation $u = \varphi\ (\bar{F}_1, \cdots , F_s, p, t)$ of the first degree in $F_1, \cdots , F_s$, the marginal degree of productivity of each service should at every time t *of this interval be equal to the ratio of the price per unit of the service to the price per unit of the commodity.*

If one multiplies (3.4) by the increment of service $\Delta \bar{F}_i$ and rewrites the result in the form

$$((\partial u/\partial \bar{F}_i) \Delta \bar{F}_i) p_c(t) = \Delta \bar{F}_i p_i(t) \ , \qquad i = 1, \cdots , s \ ,$$

the following additional theorem can be stated:

The amount of each service used should be such that the increment in total compensatory cost Q_f due to an increment of this service is equal to the cost of this increment of service.

By an application of Euler's theorem it is possible to obtain from (3.4)

$$\sum_{i=1}^{s} \bar{F}_i \, \partial u/\partial \bar{F}_i = u(t) = \sum_{i=1}^{s} p_i(t)/p_c(t) \ ,$$

and hence the following theorem results:

The earning $\sum_{i=1}^{s} p_i(t)$ of the several compensatory services are equal to the total cost $up_c(t)$ due to them.

Finally, it should be remarked that all the theorems on this page are true only if $u = \varphi(\bar{F}_1, \cdots , \bar{F}_s, p, t)$ is a homogeneous equation of the first degree in $\bar{F}_1, \cdots , \bar{F}_s$.

For the case in which there are several products, say, for example, three, X, Y and Z, manufactured from the same commodities and services, Pareto has defined the coefficients of production to be the partial derivatives $\partial F_\alpha / \partial u_i$, $i = 1,2,3$; $\alpha = 1,2, \cdots, m$, where F_α is the quantity of the commodity or service required to manufacture u_1 units of X, u_2 units of Y and u_3 units of Z. For the special case of a single manufactured product one would, following Pareto, replace f_α by $dF_\alpha / du = f_\alpha$. If the f_α are compensatory, then according to Pareto

$$\psi(f_1, f_2, \cdots, f_m) = 0.$$

Various generalizations could be made (by considering more than one compensatory relation or by using Pareto's definition) but no new knowledge is to be gained by performing such mathematical exercises.