# CHAPTER II

## DEMAND FOR CONSUMER GOODS

*1. The Nature of Demand.* Almost every student of elementary economic text books is familiar with a chart showing two intersecting curves sloping in opposite directions. These curves purporting to show demand and supply as functions of price intersect in a point which is usually labeled the equilibrium point. This label means that the supply is equal to the demand at the point of intersection. Good text books mention that demand and supply shift and that instead of one downward sloping curve and one upward sloping curve there are in reality many curves of each kind; i.e., there are many demand curves and many supply curves for each commodity. A few books state that factors which are capable of causing shifts in demand are advertising, introduction of new products, etc., but usually they do not give a great deal of attention to these elements.* The position taken in this work is that there are infinitely many demand and supply curves; that is, there are factors other than price which are constantly shifting.

Price is an important factor, but it is not enough to say that demand depends upon price. There are many other determining factors, some of them often ranking in importance with price and advertising; e.g., consumer income, prices of competing or substituting goods, monetary and credit conditions, the influence of past prices, speculation and so forth.

If there were no highways, the demand for gasoline for motor cars would be very small indeed. Again, in a community where there is little consumer income other than that needed to buy food and some clothing, there is necessarily little demand for gasoline to run automobiles. As indicated in the next chapter, imposition of a sales tax might decrease consumption more than would an equal increase in price, without a tax. Thus it is seen that some factors influencing demand are of a physical nature, some are economic, and some purely psychological.

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*For diagrammatic studies of demand and some mention of shifts see E. H. Chamberlin, *The Theory of Monopolistic Competition*, Harvard Press, 1933, Chapter V.

*2. Factors Other Than Price Influencing Demand.* When an attempt is made to take all influencing factors into account, or even a few of the more important ones, the tremendous complexity of economics becomes apparent. Actually, of course, all economic quantities in the same market are related, but fortunately only a few seem to have major influences on the demand for any particular commodity.

In the older type of statistical study of demand, the chief purpose was the determination of "the demand curve," or in some cases the more limited purpose of determining the percentage decrease in demand corresponding to a one per cent increase in price (elasticity of demand). From the standpoint of such studies all factors other than price were regarded as "disturbing factors" whose effect should be eliminated. Various devices invented for performing this elimination included the method of trend ratios, of link relatives, and of first differences. On the whole this older type of study proceeded on the assumption that changes in demand due to factors other than price were of a gradual nature due to changes in habit, customs and the growth of population. Under such assumptions it appeared to be desirable to remove the effects of trends.

In the past four years it has become apparent that the demand studies of the old type which had been more or less successful prior to 1930 were faulty. In many cases the difficulty was that the only changes in the demand function of which specific account had been taken were those which could be explained by algebraic trends. Apparently these studies were empirically satisfactory prior to 1930 chiefly because shifts in demand which occurred during the period covered by the analysis were small, or were closely correlated with other factors included in the study for other purposes. An example of the latter type of factor commonly used as an omnibus was the index of wholesale prices used ostensibly to reduce "money prices" to "real prices," but also of value in explaining shifts of demand. Thorne and Beane have shown how what was formerly treated as merely an upward trend in demand for beef and pork was primarily the result of increasing business activity and payrolls.*

*3. Effects of Past Factors on Demand.* Past prices and sales

*G. B. Thorne and L. H. Bean, "The Use of 'Trends in Residuals' in Constructing Demand Curves," *Journal of the American Statistical Association,* Vol. XXVII, No. 177, March, 1932, pp. 61-67.

policies undoubtedly have important effects on current demand. As a result of continuous advertising of a product such as chewing gum at five cents per package, a demand can be built up that is almost independent of present price, consumer income, etc. Again, certain products like coffee become such integral parts of the accepted standard of living that present price must change very much before demand is appreciably affected. In such instances the effects of price are effects of past prices and policies rather than of present price. Thus, at least in many cases, the demand can be taken to be the sum of two functions, one depending upon past prices of the good and certain other pertinent past factors, the other depending upon the present price and present values of pertinent factors. A mathematical expression relating these factors can be obtained by using the limiting processes of the calculus.

Let $p(t)$ represent the average retail price of a commodity $U$ during the time $t$ to $t + \Delta t$ and suppose that the commodity has been offered continuously in a *particular* market for some time. Let $y(t)$ represent the amount purchased for consumption or speculation during the time $\Delta t$. In a general theory of demand the time required for consumption of the goods is undoubtedly an important factor which should be taken into account. It is easier and less confusing, however, to build a theory without taking this factor into account and then to extend the theory. Hence, the theory as first developed applies to demand for goods consumed immediately upon purchase, e.g., electric energy. However, it is equally valid if $\Delta t$ is sufficiently long to extend over the average time required for consumption.

Consider, first of all, goods for which there is no buying for speculative purposes. This is practically equivalent to specifying that the problem be limited to retail buying and selling of perishables, since considerable speculative activity is associated with nearly all, if not all, commodities in the wholesale markets and durable goods at retail. After the basic theory has been developed the speculative factor can be easily taken into account.

Non-speculative demand for goods certainly depends upon the price of the goods as well as upon the prices of competing or substitute goods; that is, it is a function of the present price $p(t)$, the past prices $p(t_i)$, $i = 0, 1, \cdots, n-1$, $t_0 = 0$, $t_n = t$, and the prices of $m$ competing goods, $p_1, \cdots, p_m$. Theoretically, it is also a function of the past prices of competing or substitute goods. There is, however, no serious objection to simplifying the problem by assuming that each $p_k(t_i)$, $k = 1, \cdots, m$, is equal to $p_k$. The demand, in general, also varies with the seasons and changes in social taste which may be assumed to include obsolescence, etc.; that is, demand may depend explicity upon time $t$. The above may be summarized by taking the demand as a function

$$y(t) = f[p(t_0), p(t_1), \cdots, p(t_{n-1}), p(t), p_1, \cdots, p_m, t]$$

As a first approximation it can be assumed that $f(t)$ is a linear function of each $p(t_i)$, so that

$$y(t) = \varphi(t) + [K(x_0, t)\, p(x_0) + \cdots + K(x_{n-1}, t) p(x_{n-1})] \\ + K(x_n, t)\, p(x_n)$$

where, for simplicity in writing, the arguments, $p_1, \cdots, p_m$, of $\varphi$, and $K$ have not been written. To make the theory perfectly general one need only enlarge the definition of the $p_i$ so that they include factors other than price. Thus, in particular $p_1$ might be a physical factor such as number of miles of highway, etc.

Here $K(x,t)$ is a weight function and the expression in brackets is the weighted average of the prices $p(t_0), \cdots, p(t_{n-1})$, in which, for the sake of clarity in the work that follows, $x_i$ has been used in place of $t_i$. That is, $x$ refers to past time and $t$ to present time. It is understood, of course, that time is used to express certain group effects due to numerous small factors or it is used implicitly to denote some important factor changing with time; for example,

$$f(t) = f[p_1(t), p_2(t), \cdots, p_m(t)].$$

There is no loss of generality in assuming $x_i - x_{i-1} = 1$. If this assumption is made, it follows readily that the bracketed expression is the sum of rectangles of unit width and height $K(x_i, t)p(x_i)$. This sum may be replaced by a definite integral; that is,

$$(3.1) \qquad y(t) = \varphi(t) + \int_0^t K(x,t)p(x)dx + K(t,t)p(t),$$

where $K(x,t)$ is a discontinuous step function. If price $p(x)$ is also discontinuous, the infinite sum or integral of $K(x,t)p(x)$ can be taken in the sense of Stieltjes — a finite sum. This is, however, a refinement that need not be given serious attention here since there is no important objection to replacing $K(x,t)$ by a continuous function yielding the same integral, or for that matter, by one yielding approximately the same integral.

It should hardly be necessary to point out that physicists and astronomers have been using continuous functions to represent discontinuous phenomena and have thereby achieved considerable success in discovering laws of nature. There is considerable doubt that economic phenomena are more discontinuous than physical phenomena; at least they are not more discontinuous from the mathematical point of view. Of course, the econometrist should be careful to consider only a sufficiently large number of elements (see Chapter I, Section 4). In the work that follows, $K(x,t)p(x)$ and other quantities are assumed to be continuous.

There are some examples for which $K(t,t) \equiv 0$. Thus, the demand for goods bought through a broker "at market" depends upon past prices, but not at all or only very slightly upon the price at the time $t$. In general, however, demand depends upon the present price as well as upon past prices and $K(t,t) \neq 0$. In fact, $K(t,t)$ is, in general, not zero on the interval 0 to $t$. Mathematically there is an important distinction between the two cases $K \equiv 0$ and $K \neq 0$. For the first case, equation (3.1) becomes a Volterra integral equation of the *first kind*,* whereas for the second case it becomes a Volterra integral equation of the *second kind*.†

It is quite natural to ask if, conversely, price depends upon past demand. The answer is that *whenever present demand depends upon past prices, present price depends upon past demands and the two statements are equivalent.* In fact, those familiar with the theory of integral equations will recognize that equation (3.1) yields this dual theory. As is well known, if $K(t,t) \neq 0$ in the interval $t_0$ to $t$, equation (3.1) can be written in the completely equivalent form

$$K(t,\,t)p(t) = y(t) + \varphi(t) + \int_0^t k(x,\,t)\varphi(x)dx + \int_0^t k(x,t)y(x)dx\ ,$$

where $k(x,t)$ is the resolvent kernel of $K(x,t)/K(t,t)$ and as such is completely determined as a function of $x$ and $t$. ‡

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*Vito Volterra, *Leçons sur les Équations Integrals*, Paris, 1913, p. 56.

†*Ibid.* p. 40.

‡Vito Volterra, *loc. cit.*, p. 45.

If the dependence of demand upon past prices is not linear, some other form of functional equation may be used. In fact very few demand equations are linear functions of price (other factors held constant) throughout the entire range of values of price. Most statistical studies indicate that a better approximation to the demand price relationship is given by a curve of the type $p^{-a}$ where $a$ is a positive fraction usually less than one. Thus, it is better to write

$$y(t) = \varphi(t) + K(t, t)F(p) + \int_0^t K(x,t)F_1[p(x)]dx \ ,$$

where $F$ and $F_1$ are in general non-linear functions of price. In many cases it may be expected that $F \equiv F_1$. Unfortunately for the use of such general equations by econometrists, mathematicians have not yet been able to find out a great deal about them.

For consumer goods, $p$ can generally be taken to be price. By this it is meant that, in general for consumer goods, price measures the marginal desirability of the goods with respect to other goods whose marginal desirability is also measured by prices. In the case of capital goods, $p$ is more likely to be an income factor, as is explained in a later chapter.

*4. Demand Approximations.* The sum (integral) of $K(x,t)F(p(x))$ may vary little from an average value. In some cases, then, it is possible to replace the effects of past events by a constant or at least by a quantity that differs little from a constant. This should be especially true of demand for some consumer goods. For such goods, therefore,

$$y(t) = \varphi_1(t) + K(t, t)F(p) \tag{4.1}$$

at least to a good approximation, provided $y(t)$ is taken over a period of time sufficiently long to average out some of the effects of immediate past prices, etc.

It is perhaps not amiss to point out again that both $\varphi_1(t)$ and $K(t, t)$ are denoted as functions of time to indicate that both may depend upon economic, physical and psychological factors that change with time. Over short periods of time both quantities may appear to be constant, but it need not follow that they *are* constants.

It is possible that only two or three factors, other than price $p$, say $p_1$, $p_2$, $p_3$, affect the demand $y$. In such an instance, it may be possible to write

$$y(t) = A_1F_1(p_1) + A_2F_2(p_2) + A_3F_3(p_3) + K(t,t)F(p) ,$$
$$y(t) = A_1F_0(p_1, p_2, p_3) + K(t,t) F(p)$$
$$\text{or} \quad y(t) = A_1F_0(p_1, p_2, p_3) + af_0(p_1, p_2, p_3)F(p) \text{ , etc.,}$$

in which $F_0$, $F_1$, $F_2$, $F_3$ and $f_0$ are known functions of the given arguments and $A_1$, $A_2$, $A_3$ and $a$ appear to be constants. Actually each of the quantities $A_1$, $A_2$, $A_3$ and $a$ depend upon other factors in the economy, but by hyphothesis the effect of any single factor is small and since some of the small factors increase while others decrease, it may be assumed that $A_1$, $A_2$, $A_3$ and $a$ are constants even though the elements of which they are made are constantly changing. There are many physical analogies. For example, the shape of a piece of matter may appear unchanged while the atoms of which it is composed undergo constant motion. Of course, if some new element is introduced or if some small factor becomes large, the original hypothesis is no longer valid.

Several studies of demand which appeared to be successful before 1929 failed to give satisfactory results in the subsequent years because the balance of neglected factors was upset. Correlation studies cannot be expected to give any clue to the nature of the causal relationship which exists between price and some other variable that has not shown significant variation in the period studied. Averages often cover up as much as they reveal, and must always be used with caution. Nevertheless, information gained through their use is distinctly valuable, if care is taken to allow for the introduction of new major factors. During the World War, price-fixing, propaganda, shifts in production emphasis, etc., must surely have had effects in modifying demand. It becomes desirable, therefore, to distinguish between what might be called normal economic demand and demand due to specific factors that have not normally existed or have been of no more importance than many of the other small factors.

Closest attention must be given to *a priori* reasoning concerning the causal relations that may be involved and to all inductive evidence that may bear upon the problem. Close relation between two series of data cannot of itself be considered to be of much significance as empirical evidence. Even where data are not of a cyclical nature, statistical methods alone are incapable of yielding definite evidence of causation. It is only when statistical evidence is combined with *a priori* reasoning in a closely knit argument that confidence can be felt in statistical relationships.*

Several special cases of *(4.1)* that have been used in statistical studies of demand are

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| (1) | $y = ap + b$ | $a<0, b>0$ |
| (2) | $y = b/p^a + c$ | $b > 0, c > 0$ |
| (3) | $y = b - (c-ap)^{1/2}$ | $b > 0, c > 0, a > 0$ |
| (4) | $y = ap^2 - bp + c$ | $a > 0, b > 0, c >0$ |
| (5) | $y = p^a e^{b(p-c)}$ | $a > 0, b < 0, c > 0, e = 2.718$ |

In these equations, $a$, $b$ and $c$, although written as constants, should not be strictly interpreted as such. They must be interpreted to be composites of other factors as already explained. Thus, in the case of equation (1), $b$ might be $b_1p_1 + b_2p_2 + \cdots + b_mp_m$, where $p_1, \cdots, p_m$ are $m$ additional factors influencing demand. If the quantity $\Sigma b_ip_i$ remains constant over a period of time, then $b$ can be taken to be constant. Once more for emphasis it might be pointed out that if $\Sigma b_ip_i$ is constant, it does not follow that each $p_i$ or, for that matter, that any one $p_i$ is constant. The sum $3 + 5 + 6 - 7$ is equal to the sum $4 + 9 - 8 + 2$ which is equal to 7. If $\Sigma b_ip_i$ is not constant, then it may for simplicity be regarded as a function of time unless, of course, one wishes to study the effects of particular factors. In the case of the gasoline study described in Chapter III, $b$ depends upon highway mileage, taxes and other quantities. An important point to notice, however, is that for the equation (1) as written, $a$ and $b$ are independent of price $p$.

In the discussion that follows, $a$, $b$, and $c$ will be regarded as constants. This is really no restriction on their generality since the arguments used can be carried over into $n$-dimensional space by substituting the language of the general geometry for the language used here. It seems to be undesirable, however, to complicate the description here since nothing is gained by doing it.

Formula (1) represents a straight line with negative slope. H. L. Moore† used this form to obtain a statistical law of demand for cotton. Henry Schultz‡ used it to obtain a statistical law of

*This is essentially the scientific method familiar to physicists, chemists and other scientists. The argument as presented here follows closely that given by E. J. Working in his paper "Demand Studies During Times of Rapid Economic Changes," *Econometrica*, April, 1934.

†H. L. Moore, *Forecasting the Yield and Price of Cotton*, New York, 1917, p. 143.

‡Henry Schultz, *Statistical Laws of Demand and Supply*, Chicago, 1928, p. 61.

demand for sugar in terms of link relatives and trend ratios. G. P. Scoville* used it in studies of potatoes and hay. For small price variations this form must be considered to be a fair approximation for almost any type of demand law.

Formula (2) defines an hyperbola which approaches the demand axis and the price $p = c$, asymptotically. For $c = 0$ this becomes the law $y = b/p^a$, which was popularized by Alfred Marshall† and used in statistical work by W. P. Heddon, J. Bloxom, H. H. Holland and others. It has undoubtedly led to occasional statements that the amount demanded is infinite when the price is zero and the demand is zero when the price is infinite. These statements must surely have been brought about by attempts to popularize a mathematical formula or by misconceptions of demand. G. F. Warren and F. A. Pearson‡ have used formula (2) with $c \neq 0$. Of course, it must be admitted that for values of $p$ and $y$ not close to zero a law of the type (2) with $c$ zero or not zero may give a better approximation of a law of demand than equation (1), but loose statements regarding infinite demand for zero price should not be tolerated.

Equation (3) represents the *lower branch* of a parabola open to the right, whose vertex is at $y = b$, $p = c/a$. Thus, $y$ is not defined for $p < c/a$. This curve therefore typifies the situation for which there is a finite demand $y = b$ at a minimum price $p = c/a$, which price may be taken to be the price of distribution. Furthermore, the demand is zero when the price $p = (c—b^2)/a$. Theoretically the demand should remain zero for $p > (c—b^2)/a$, so that formula (3) has no significance for $p$ greater than this value.

Formula (4) represents a parabola open above with vertex at $p = +b/2a$, $y = c—b^2/2a$. This curve crosses the $y$-axis at $y = c$ and may or may not cross the $p$-axis depending upon the values of $a$, $b$ and $c$.

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*G. P. Scoville, Per Cent of the Expected Crop Correlated with Purchasing Power of the Price of Potatoes for Each of the Last Fifty-four years, Cornell University, Dept. Farm Marketing, 1919.

†Alfred Marshall, *Principles of Economics*, London, 1920.

For a theoretical derivation of the law $y = b/p^a$, see V. Pareto, Mathématique Économie, *Encyclopedia des Sciences Mathématique*, Tome I, p. 59.

W. P. Heddon, "Measuring the Melon Market," *U. S. Dept. of Agriculture and Port of New York Authority*, August, 1924.

J. Bloxom, "Some Determining Factors in Apple Prices," New York Food Marketing Research Council, *Food Marketing Studies*, I. 1926.

H. H. Holland, "The Demand for Peaches in New York," *ibid*, III, 1926.

‡G. F. Warren and F. A. Pearson, *The Interrelationships of Supply and Price*, Cornell University, 1927.

Formula (5) defines a curve passing through the origin and approaching the $p$-axis asymptotically on the right. Thus, the curve rises from the origin, reaches a maximum (but the slope is not zero at the maximum) and then drops again to zero. H. L. Moore, Henry Schultz and H. B. Killough have used this curve in statistical studies.*

In the above discussion the quantities $a$, $b$ and $c$ have been regarded as constants. As previously indicated, for greater generality one might suppose them to be functions of time $t$, and economic factors. An important point to remember is that for typical economic situations $dy/dp$ will be negative, so that curves that satisfy this condition should be used.

5. *Demand and Consumer Income.* It has been intimated that consumer demand depends upon consumer income, but no definite formulation has been given. Most demand studies have used a "deflating" series; that is, in these studies prices have been divided by an index of wholesale prices or something of the sort. In other words, statisticians have learned that adjustments must be made for changes in the price level.

In general, purchasing power theories of economics have been condemned by orthodox economists. These condemnations may be in part warranted, but it must be admitted that consumer purchases are high when consumer income is high and they are low when it is low.

It may be assumed that when consumer income, $I$ (a particular part of it if only part of the economy is being considered), is large, the demand will not be so much affected by increase in $p$ as in the case of small $I$, hence $|\,dy/dp\,|$ is smaller for larger $I$; that is,

$$(dy/dy)^2 = f(I,p) \quad ,$$

where $f(I,p)$ decreases as $I$ increases for a fixed $p$. A simple function of $I,p$ satisfying the given conditions is $I^{-2\beta} f^2(p)$ so that

$$y = I^{-\beta} \int f(p)dp + C(t) \quad ,$$

*H. L. Moore, "The Elasticity of Demand and Flexibility of Prices," *Jour. Amer. Stat. Ass'n*, Vol. 23, 1922, pp. 8-19.

Henry Schultz, "The Statistical Measurement of the Elasticity of Beef," *Jour. Farm. Econ.*, July, 1924.

H. B. Killougn, "The Price of Oats," *U. S. Dept. Agr. Dept. Bull. 1351*, pp. 8-9, September, 1925.

where $C(t)$ is a constant or function of $t$. If $f(p)$ is equal to $a$, a negative constant or function of $t$, then

$$y = ap/I^{-\beta} + C(t) \ .$$

Other formulas can, of course, be derived. A more penetrating discussion of purchasing power and demand is given in the next chapter.