Estimated, Calibrated, and Optimal Interest Rate Rules

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Abstract

Estimated, calibrated, and optimal interest rate rules are examined for their ability to dampen economic fluctuations caused by random shocks. A tax rate rule is also considered. The results show that the estimated interest rate rule used in the paper is stable for the period beginning in 1954 except for the early Volcker period, although more observations, especially high inflation ones, are needed before much confidence can be placed on the results.

The models used for the stabilization results are large scale structural macroeconometric models, and some of the results differ from those based on small models. For example, rules with inflation coefficients less than one are not destabilizing, and rules with large inflation coefficients, such as the Taylor rule, achieve a small reduction in inflation variability at a cost of a large increase in interest rate variability.

1 Introduction

This paper considers three kinds of interest rate rules: estimated, calibrated, and optimal. The rules are examined for their ability to dampen economic fluctuations caused by random shocks. A tax rate rule is also considered. The rules are examined

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within the context of a multicountry macroeconometric (MC) model and two versions of a stand alone United States (US) model. One version of the US model has rational expectations.

An estimated interest rate rule for the United States is an attempt to approximate the behavior of the Fed. The left hand side variable is the interest rate that the Fed is assumed to control, and the right hand side variables are those that are assumed to affect Fed behavior. The first example of an estimated interest rate rule is in Dewald and Johnson (1963), where the Treasury bill rate is regressed on a constant, the bill rate lagged once, real GNP, the unemployment rate, the balance of payments deficit, and the consumer price index. The next example is in Christian (1968), and since then many rules have been estimated. I added an estimated interest rate rule to my U.S. econometric model in Fair (1978), and a version of this rule is used in the present paper. McNees (1986, 1992) when he was at the Boston Fed did some interesting work with estimated rules, where he included among the explanatory variables the Fed's internal forecasts of various variables. In the MC model, which was first presented in Fair (1984), there are estimated interest rate rules for the monetary authorities of other countries. Khoury (1990) provides an extensive list of estimated rules through 1986. Two recent studies are Judd and Rudebusch (1998), where rules are estimated for various subsets of the 1970–1997 period, and Clarida, Galí, and Gertler (2000), where rules are estimated for the different Fed chairmen.

By a ''calibrated'' rule in this paper is meant a rule that is not econometrically estimated. The most well known calibrated rule is due to Taylor (1993), where the federal funds rate is equal to 1.0 plus 1.5 times the rate of inflation over the previous four quarters plus 0.5 times the percent deviation of real GDP from a target. A number of studies have examined how calibrated rules perform. The general approach in this literature is to choose a rule and then use a model of the economy to examine how the economy would have behaved under the rule. The aim is to find a rule that gives (in some sense) a good overall performance of the economy.[1]

This paper adds to the literature in the following ways. The first concerns the estimated rule that is used. The hypothesis that the coefficients in this rule are the same in the 1954:1–1979:3 and 1982:4–1999:2 periods is tested and not rejected. The passing of a stability test like this is contrary to the general view in the recent literature that estimated interest rate rules do not have stable coefficients over time. For example, Judd and Rudebusch (1998, p. 3) state ‘‘Overall, it appears that there have not been any great successes in modeling Fed behavior with a single, stable reaction function.’’

Second, the models of the economy that are used for the stabilization experiments are large scale structural macroeconometric models. The MC and US models that are used have been extensively tested, including testing for rational expectations, and they appear to be good approximations of the economy.[2] Most recent studies have used very small models, sometimes only two to four equations and sometimes calibrated rather than estimated. For example, only one of the studies in Taylor

[1]See, for example, Feldstein and Stock (1993), Hall and Mankiw (1993), Judd and Motley (1993), Clark (1994), Croushore and Stark (1994), Thorton (1995), Fair and Howrey (1996), Rudebusch (1999), Clarida, Galí, and Gertler (2000), and the papers in Taylor (1999). Taylor (1985, fn. 1, p. 61) cites much of the literature prior to 1985.

[2]See, for example, the tests in Fair (1994) and the tests on the website listed in the introductory footnote. The rational expectations tests are briefly discussed in the appendix.

(1999)—Levin, Wieland, and Williams (1999) (LWW)—uses large scale models. LWW use linearizations of the Federal Reserve model and the Taylor multicountry model to compute unconditional second moments of the variables in the models. In the recent study of Clarida, Galí, and Gertler (2000) a four equation calibrated model is used. It will be seen that judging interest rate rules can be sensitive to the economic model used. Using small calibrated models to make policy conclusions may be risky if the models are at odds with more empirically based models.

Third, a tax rate rule is examined. Although fiscal policy rules are seldom discussed in the literature on rules, it will be seen that such rules can be of considerably help to monetary policy in its stabilization effort.

Finally, the paper shows that it is computationally feasible to use large scale nonlinear models to analyze questions that have mostly been analyzed using small linear models. The methods in this paper are computer intensive, but computer time is cheap.

The MC and US models are discussed in the appendix. The estimated rule is discussed in the next section, and the calibrated rules are presented in Section 3. The stochastic simulation and optimal control procedures are discussed in Section 4, and the stabilization results are presented in Sections 5 and 6.

2 The Estimated Rule

The rule that I added to my U.S. macroeconometric model in 1978 has been changed slightly over time. The main modification that has been made is the addition of a dummy variable term to account for the change in Fed operating procedure during period 1979:4–1982:3 (to be called the ''early Volcker'' period).[3] The stated policy of the Fed during this period was that it was focusing more on monetary aggregates than it had done in the past. The estimated interest rate rule already had the lagged growth of the money supply as an explanatory variable, and the change in policy was modeled by adding the lagged growth of the money supply multiplied by a dummy variable as another explanatory variable. The dummy variable is 1 for the 1979:4–1982:3 period and 0 otherwise.

The specification of the rule that is used in this paper is:

$$\begin{aligned} r = \alpha_1 + \alpha_2 \dot{p} + \alpha_3 u + \alpha_4 \Delta u + \alpha_5 \dot{m}_{-1} + \alpha_6 D1 \times \dot{m}_{-1} \\ + \alpha_7 r_{-1} + \alpha_8 \Delta r_{-1} + \alpha_9 \Delta r_{-2} + \epsilon \end{aligned} \tag{1}$$

where r is the three month Treasury bill rate, $\dot{p}$ is the quarterly rate of inflation at an annual rate, u is the unemployment rate, $\dot{m}$ is the quarterly rate of growth of the money supply at an annual rate, and $D1$ equals 1 for 1979:4–1982:3 and 0 otherwise. The estimates of equation (1) for three different sample periods are presented in Table 1.[4]

The endogenous variables on the right hand side of equation (1) are inflation and the unemployment rate, and two stage least squares was used to estimate the equation.

[3]Paul Volcker was chair of the Fed between 1979:3 and 1987:2, but the period in question is only 1979:4–1982:3.

[4]The data that were used for all the estimates and tests in this section, including the data on the first stage regressors, are available from the website. The results can be duplicated by downloading the data and some software from the website. The price variable that is used to construct the inflation variable is the price deflator for domestic sales. This variable was used in Fair (1978) and has been used ever since in the equation. The three month Treasury bill rate is used for the interest rate. Although in practice the Fed controls the federal funds rate, the quarterly average of the federal funds rate and the quarterly average of the three month Treasury bill rate are so highly correlated that it makes little difference which rate is used in estimated interest rate rules using quarterly data. The money supply data are taken from the flow of funds accounts.

Table 1
Estimated U.S. Interest Rate Rule
Dependent Variable is r

	1954:1–1999:2 182 obs. Coef.	t-stat.	1954:1–1979:2 103 obs. Coef.	t-stat.	1982:4–1999:2 67 obs. Coef.	t-stat.	1954:1–1999:2 182 obs. Coef.	t-stat.
constant	.895	5.60	.745	3.24	.402	1.89	.738	4.32
$\dot{p}$	.070	3.94	.070	2.87	.115	3.03	.078	4.13
u	-.143	-4.47	-.111	-2.89	-.080	-1.97	-.126	-3.92
Δu	-.856	-6.59	-.383	-2.98	-.925	-4.81	-.706	-5.28
$\dot{m}_{-1}$	.012	1.89	.027	3.48	.001	0.08	.008	1.42
$D1 \times \dot{m}_{-1}$	.218	9.59	–		–		.328	7.91
r_{-1}	.922	46.72	.894	22.02	.942	36.76	.926	38.52
Δr_{-1}	.194	3.40	.253	2.86	.301	3.16	.298	4.63
Δr_{-2}	-.352	-6.77	-.229	-2.55	-.191	-2.25	-.362	-7.26
$D1 \times \dot{p}$							-.121	-2.76
$D2 \times \dot{p}$							.044	1.37
SE	.475		.415		.313		.454	
R^2	.971		.959		.971		.974	
DW	1.83		1.86		2.05		2.09	
Wald (p-value)	10.31 (.244)							

Estimation period: 1954:1–1999:2
Estimation technique: two stage least squares
r = three month Treasury bill rate
$\dot{p}$ = inflation rate
u = unemployment rate
$\dot{m}$ = growth rate of the money supply
$D1$ = 1 for 1979:4–1982:3; 0 otherwise
$D2$ = 1 for 1982:4–1999:2; 0 otherwise

In the first stage regression inflation and the unemployment rate are regressed on a set of predetermined variables (the main variables in the US model). The predicted values from these regressions are then used in the second stage. One can look on the these regressions as those used by the Fed to predict inflation and the unemployment rate, and so it need not be assumed that the Fed has perfect foresight.

If the Fed's expectations of *future* values of inflation and the unemployment rate affect its current decision, these expectations should be added to equation (1). A way to test this is to add future values of inflation and the unemployment rate to equation (1) and then estimate the equation by Hansen's (1982) method of moments estimator, where the instruments used are the main predetermined variables in the US model. Hansen's method in this context is just two stage least squares adjusted to account for the serial correlation properties of the error term. The test is to see if the future values are statistically significant. I have performed this test on various versions of my estimated interest rate rules using different lead lengths, and the led values do not turn out to be significant.[5] There is thus no evidence that future values are needed in equation (1), and they have not been used. Clarida, Galí, and Gertler (2000) use future values in many of their specifications, but they point out (p. 164) that their conclusions are not changed if they don't use future values.

Equation (1) is a ''leaning against the wind'' equation. r is estimated to depend positively on the inflation rate and the lagged growth of the money supply and negatively on the unemployment rate and the change in the unemployment rate. Adjustment and smoothing effects are captured by the lagged values of r. The coefficient on lagged money supply growth is over ten times larger for the early Volcker period than either before or after, which is consistent with the Fed's stated policy of focusing more on monetary aggregates during this period. This way of accounting for the Fed policy shift does not, of course, capture the richness of the change in behavior, but at least it seems to capture some of the change.

The Wald value in Table 1 is for the test of the hypothesis that the coefficients in

[5]See Chapter 5 Fair (1994) for the use of this test. The latest tests are on the website.

the 1954:1–1979:3 period are the same as those in the 1982:4–1999:2 period. (The early Volcker period is excluded from this test, and so the $D1$ term is excluded.) The Wald statistic is presented in equation (3.6) in Andrews and Fair (1988). It has the advantage that it works under very general assumptions about the properties of the error terms and can be used when the estimator is two stage least squares, which it is here. The Wald statistic is distributed as χ^2 with (in the present case) 8 degrees of freedom. The estimates of the equation for the two sub periods are presented in Table 1. The value of the Wald statistic is 10.31, which has a p value of .244. The hypothesis of equality is thus not rejected at even the 10 percent level.[6]

Equation (1), estimated for the entire 1954:1–1999:2 period, was put through a number of other tests.[7] First, the lagged values of all the variables in the equation (r_{-4}, $\dot{p}_{-1}$, u_{-2}, $\dot{m}_{-2}$, $D1 \times \dot{m}_{-2}$) were added and the joint significance of these variables tested. The χ^2 value was 6.34 with 5 degrees of freedom, which has a p value of .275. Adding these variables encompasses a number of alternative hypotheses about the dynamics,[8] and these hypotheses are rejected in that the added variables are not significant. Second, the equation was estimated under the assumption of first order

[6]It is also the case that the exact specification of the interest rate rule in Fair (1978) (equation (1), p. 1170), which was specified more than 20 years ago, passes the stability test. This specification differs from the one above in the following ways: 1) the inflation rate is lagged once rather than unlagged, 2) a different measure of labor market tightness is used from the unemployment rate, 3) output growth lagged once and output growth lagged twice are included in place of the change in the unemployment rate, 4) Δr lagged once and twice are not included, and 5) the equation is estimated under the assumption of first order serial correlation of the error term. (The dummy variable term is, of course, not included since this was before 1979.) When the Wald test was performed using this specification, the value of the test statistic was even lower (8.29 versus 10.31 in Table 1), with the same number of degrees of freedom (8).

[7]See Fair (1994), Chapter 4, for a general discussion of these kinds of tests.

[8]See Hendry, Pagan, and Sargan (1984).

serial correlation of the error term. The χ^2 value was 1.32 with 1 degree of freedom, which has a p value of .251. Third, the percentage change in real GDP was added (without excluding the change in the unemployment rate). The χ^2 value was 0.20 with 1 degree of freedom, which has a p value of .653. Finally, an output gap variable[9] and the change in this variable were added (without excluding the unemployment rate and the change in the unemployment rate). The χ^2 value was 4.68 with 2 degrees of freedom, which has a p value of .096. Overall, the equation does well in these tests. The added variables, including the output gap and the change in the output gap, do not have additional explanatory power.

Returning to the stability test, the passing of this test is contrary to the general view in the literature, mentioned above. One likely reason that the stability hypothesis has generally been rejected in the literature is that most tests have included the early Volcker period, which is clearly different from the periods both before and after. The tests in Judd and Rudebusch (1998), for example, include the early Volcker period.

Clarida, Galí, and Gertler (2000) (CGG) do not perform any stability tests; they simply note that the coefficient estimates for the different periods look quite different, especially the inflation coefficient. The equations for the two sub periods in Table 1 also show a large difference in the inflation coefficient. For the first sub period the long run coefficient is 0.66 [= .070/(1.0 - .894)], and for the second sub period it is 1.98 [= .115/(1.0 - .942)]. The CGG coefficients (p. 150) are .83 for their pre-Volcker period (1960:1–1979:2) and 2.15 for their Volcker-Greenspan period (1979:3–1996:4).

[9] The output gap measure used is $(YS - Y)/YS$, where Y is actual output and YS is a measure of potential output. These variables are in the US model on the website.

Although the inflation coefficients seem quite different in Table 1, the Wald test does not reject the hypothesis of stability. It could be, however, that the test has low power, and so another test was performed. This test is represented in the last two columns in Table 1. This test is based on the assumption that all the coefficients are constant across time except the inflation coefficient, which is postulated to be different in each of the three sub periods (1954:1–1979:3, 1979:4–1982:3, and 1982:4–1999:2), which will be called ''first,'' ''early Volcker,'' and ''second.'' The coefficient estimate for $D1 \times \dot{p}$ is the estimated difference between the early Volcker period and the first period. This difference is not of much interest, since the added variable is just meant to dummy out the early Volcker period. The estimated difference is negative and significant (t-statistic of -2.76). The total coefficient for this period is -0.043 [= .078 - .121]. This negative value is not sensible, which reflects the fact that the early Volcker period is unusual and hard to model. (This is the reason the period was completely ignored for the Wald test.)

The coefficient estimate for $D2 \times \dot{p}$ is the estimated difference between the second period and the first. This estimated difference is .044 with a t-statistic of 1.37, which is not significant. Again, the long run inflation coefficient for the second period of 1.65 [= .078 + .044)/(1 - .926)] is noticeably larger than that for the first period of 1.05 [= .078/(1 - .926)].

The results thus show a large economic but not statistically significant difference for the inflation coefficient between the first and second periods. One fact that is important to keep in mind is that the variance of inflation is much smaller in the second period than in the first. The largest value of inflation in the second period is

5.33 percent in 1990:1, and no other value is above 5 percent. On the other hand, the largest value for the first period is 12.83 percent in 1974:3, and 29 other values are above 5 percent.

An interesting test of whether there has been a structural change in Fed behavior will be if inflation rises substantially in the future. The third equation in Table 1 implies a much larger Fed response than does the first equation, and the test will be which equation better predicts the actual Fed response. If the third equation predicts better, this will be strong evidence in favor of a shift in behavior from the earlier period. If the first equation predicts better, this will suggest that focusing only on the period since 1982, when inflation has been low, has given misleading estimates (in effect, a small sample problem). In short, although the statistical tests in this section suggest that there has not been a shift of behavior, more observations are needed, particularly high inflation ones, before much confidence can be placed on any conclusion. For the following stabilization results the first equation in Table 1 has been used, since this currently seems supported by the data.

3 The Calibrated Rules

Two calibrated rules are examined. The first is Taylor's (1993) rule mentioned in Section 1. In the present context this rule is:

$$r = r^* + 0.5 * 100[(Y - Y^*)/Y^*] + 1.5 * 100(\dot{P}4 - \dot{P}4^*) \qquad (2)$$

where Y is real GDP, $\dot{P}4$ is the four-quarter percentage change in the GDP deflator, and * denotes a base value. The key feature of this rule is that output deviations are weighted 0.5 and inflation deviations are weighted 1.5.

The second calibrated rule is a rule I used in Fair (1998, p. 95). This rule differs from the Taylor rule in two respects. First, the inflation variable is the one-quarter rate of inflation (at an annual rate) instead of the four-quarter rate. Second, the weight on inflation is 0.25 instead of 1.5. The rule is thus:

$$r = r^* + 0.5 * 100[(Y - Y^*)/Y^*] + 0.25 * 100(\dot{P} - \dot{P}^*) \quad (3)$$

where $\dot{P}$ is the one-quarter percentage change in the GDP deflator at an annual rate. (All percentage changes in this paper are at annual rates.) This rule will be called the ‘‘.25 rule,’’ since the weight on inflation is 0.25.

4 Stochastic Simulation and Optimal Control

The Stochastic Simulation Procedure

The focus in this paper, as in much of the literature, is on variances, not means. The aim of monetary policy is taken to smooth the effects of shocks. In order to examine the ability of monetary policy to do this, one needs an estimate of the likely shocks that monetary policy would need to smooth, and this can be done by means of stochastic simulation. Given an econometric model, shocks can be generated by drawing errors.

Of the 365 stochastic equations in the MC model, 195 are quarterly and 170 are annual. There is an estimated error term for each of these equations for each period. Although the equations do not all have the same estimation period, the period 1976–

1996 is common to almost all equations.[10] There are thus available 21 vectors of annual error terms and 84 vectors of quarterly error terms. These vectors are taken as estimates of the economic shocks, and they are drawn in the manner discussed below. Since these vectors are vectors of the historical shocks, they pick up the historical correlations of the error terms. If, for example, shocks in two consumption equations are highly positively correlated, the error terms in the two equations will tend to be high together or low together.

The period used for the stabilization experiments is 1993:1–1998:4, six years or 24 quarters. Since the concern here is with stabilization around base paths and not with positions of the base paths themselves, it does not matter much which path is chosen for the base path. The choice here is simply to take as the base path the historical path. This base path can be generated by simply adding the historical errors to the equations and taking them to be exogenous. When this is done, the solution of the model using the actual values of all the exogenous variables is the perfect tracking solution. For all the stochastic simulations in this paper the historical errors are added to the model and the draws are around these errors.

Each trial for the stochastic simulation is a dynamic deterministic simulation for 1993:1–1998:4 using a particular draw of the error terms. For each of the six years for a given trial an integer is drawn between 1 and 21 with probability 1/21 for each integer. This draw determines which of the 21 vectors of annual error terms is used for that year. The four vectors of quarterly error terms used are the four that correspond to that year. Each trial is thus based on drawing six integers. The solution of the model

[10] For the few equations whose estimation periods began later or ended earlier than the 1976–1996 period, zero errors were used for the missing observations.

for this trial is an estimate of what the world economy would have been like had the particular drawn error terms actually occurred. (Remember that the drawn error terms are on top of the historical error terms for 1993:1–1998:4, which are always used.) The number of trials taken is 20, so 20 world economic outcomes for 1993:1–1998:4 are available for analysis.[11]

For the US model alone, which is completely quarterly, historical errors are available for the 1954:1–1999:2 period (182 quarters), and these errors were used for the draws. Each vector of quarterly errors had a probability of 1/182 of being drawn. Not counting the estimated interest rate rule, there are 29 estimated equations in the US model plus the export (EX) and price of imports (PIM) equations discussed in the appendix—equations (8) and (9). Although these latter two equations are not estimated in the traditional way and have zero errors by construction, ‘‘historical’’ errors were created for them and were used for the draws. The historical errors used for equation (8) were taken to be the errors in a regression of $\log EX$ on a constant, time trend, and the first four lagged values of $\log EX$. The estimation period was 1954:1–1999:2. Similarly, the historical errors used for equation (9) were taken to be the errors in a regression of $\log PIM$ on a constant, time trend, and the first four lagged values of $\log PIM$. Otherwise, the procedure used for the US model is exactly

[11] Another way of drawing error terms would be from an estimated distribution. Let $\hat{V}$ be an estimate of the 365×365 covariance matrix V of the error terms. One could, for example, assume that the error terms are multivariate normal and draw errors from the $N(\hat{\mu}_t, \hat{V})$ distribution, where $\hat{\mu}_t$ is the vector of the historical errors for t. Because of the quarterly-annual difference, $\hat{V}$ would have to be taken to be block diagonal, one quarterly block and one annual block. Even for this matrix, however, there are not enough observations to estimate all the nonzero elements, and so many other zero restrictions would have to be imposed. The advantage of drawing the historical error vectors is that no distributional assumption has to be made and no zero restrictions have to be imposed.

the same as that used for the overall MC model.

When the estimated rule is used for the stabilization experiments, the historical errors are added to it, but no errors are drawn for it. Adding the historical errors means that when the model inclusive of the rule is solved with no errors for any equation drawn, a perfect tracking solution results. Not drawing errors for the rule means that the Fed does not behave randomly but simply follows the rule. The same procedure was followed for the Taylor rule and the .25 rule.

If in the stabilization experiments any rule called for a value of the interest rate less than 1.0 percentage point, a value of 1.0 percentage point was used. For particular shocks a rule may call for very small values of the interest rate, including negative values, and this procedure insures that these values are never used. In practice it seems unlikely that the Fed would lower interest rates much below 1.0 percentage point, and so this constraint was imposed on the model.

Let y_t^j be the predicted value of endogenous variable y for quarter t on trial j, and let y_t^* be the base (actual) value. How best to summarize the 20×24 values of y_t^j? One possibility for a variability measure is to compute the variability of y_t^j around y_t^* for each t: $(1/J)\sum_{j=1}^{J}(y_t^j - y_t^*)^2$, where J is the total number of trials.[12] The problem with this measure, however, is that there are 24 values per variable, which

[12] If y_t^* were the estimated mean of y_t, this measure would be the estimated variance of y_t. Given the J values of y_t^j, the estimated mean of y_t is $(1/J)\sum_{j=1}^{J} y_t^j$, and for a nonlinear model it is not the case that this mean equals y_t^* even as J goes to infinity. As an empirical matter, however, the difference in these two values is quite small for almost all macroeconometric models, and so it is approximately the case that the above measure of variability is the estimated variance.

makes summary difficult. A more useful measure is the following. Let L^j be:

$$L^j = \frac{1}{T}\sum_{i=1}^{T}(y_t^j - y_t^*)^2 \tag{4}$$

where T is the length of the simulation period (24). Then the measure is

$$L = \frac{1}{J}\sum_{j=1}^{J} L^j \tag{5}$$

L is a measure of the deviation of the variable from its base values over the whole period.[13]

The Optimal Control Procedure

The optimal control methodology requires that a loss function be postulated for the Fed. For the loss function used here the Fed is assumed to weight output and inflation deviations equally and to care about interest rate fluctuations. In particular, the loss for quarter t is assumed to be:

$$\begin{aligned} H_t = 0.5 * 100[(Y - Y^*)/Y^*]^2 + 0.5 * 100(\dot{P} - \dot{P}^*)^2 + \alpha(\Delta r_t - \Delta r_t^*)^2 \\ +1.0/(r_t - 0.999) + 1.0/(16.001 - r_t) \end{aligned} \tag{6}$$

where Y is real GDP, $\dot{P}$ is the percentage change in the GDP deflator, and * denotes a base value. The last two terms in (6) insure that the optimal values of r will be between 1.0 and 16.0. The value of α was chosen by experimentation. Two values were chosen, 2.0 and 9.0. A value of 2.0 results in a value of L for r roughly the same as the value obtained by the Taylor rule, and a value of 9.0 results in a value of L for

[13] L is, of course, not an estimated variance. Aside from the fact that for a nonlinear model the mean of y_t is not y_t^*, L^j is an average across a number of quarters or years, and variances are not in general constant across time. L is just a summary measure of variability.

r roughly the same as the values obtained by the estimated and .25 rules. This choice is discussed in the next section.

Assume that the control period of interest is 1 through T, where in this paper 1 is 1993:1 and T is 1998:4. Although this is the control period of interest, in order not to have to assume that life ends in T, the control problem should be thought of as one of minimizing the expected value of $\sum_{t=1}^{T+n} H_t$, where n is chosen to be large enough to avoid unusual end-of-horizon effects near T. The overall control problem should thus be thought of as choosing values of r that minimize the expected value of $\sum_{t=1}^{T+n} H_t$ subject to the model used.

If the model used is linear and the loss function quadratic, it is possible to derive analytically optimal feedback equations for the control variables.[14] In general, however, optimal feedback equations cannot be derived for nonlinear models or for loss functions with nonlinear constraints on the instruments, and a numerical procedure must be used. The following procedure was used for the results in this paper. It is based on a sequence of solutions of deterministic control problems, one sequence per trial. The US model is used.

Recall what a trial for the stochastic simulation is. A trial is a set of draws of 24 vectors of error terms, one vector per quarter. Given this set, the model is solved dynamically for the 24 quarters using a particular interest rate rule. This entire procedure is then repeated 20 times (the chosen number of trials), at which time the summary statistics are computed. As will now be discussed, each trial for the optimal control procedure requires that 24 deterministic control problems be solved, and so

[14] See, for example, Chow (1981).

with 20 trials, 480 solutions are required.

For purposes of solving the control problems, the Fed is assumed to know the model (its structure and coefficient estimates) and the exogenous variables, both past and future. The Fed is assumed *not* to know the future values of any endogenous variable or any error draw when solving the control problems.[15] The Fed is assumed to know the error draws for the first quarter for each solution. This is consistent with the use of the above rules, where the error draws for the quarter are used when solving the model with the rule.

The procedure for solving the overall control problem is as follows.

1. Draw a vector of errors for quarter 1, and add these errors to the equations. Take the errors for quarters 2 through k to be their historical values (no draws), where k is defined shortly. Choose values of r for quarters 1 through k that minimize $\sum_{t=1}^{k} H_t$ subject to the model as just described. This is just a deterministic optimal control problem, which can be solved, for example, by the method in Fair (1974). Let r_1^* denote the optimal value of r for quarter 1 that results from this solution. The value of k should be chosen to be large enough so that making it larger has a negligible effect on r_1^*. (This value can be chosen ahead of time by experimentation.) r_1^* is a value that the Fed could have computed at the beginning of quarter 1 (assuming the model and exogenous variables were known) having knowledge of the error draws for quarter 1, but not for future quarters.

2. Record the solution values from the model for quarter 1 using r_1^* and the error draws. These solution values are what the model estimates would have occurred in quarter 1 had the Fed chosen r_1^* and had the error terms been as drawn.

3. Repeat steps 1 and 2 for the control problem beginning in quarter 2, then for the control problem beginning in quarter 3, and so on through the control problem

[15] Given that EX and PIM are taken to be endogenous, the main exogenous variables in the US model are fiscal policy variables. The other exogenous variables are either unimportant or easy to forecast. Remember that since the base is the perfect tracking solution, the historical errors are always added to the model.

beginning in quarter T. For an arbitrary beginning quarter s, use the solution values of all endogenous variables for quarters $s-1$ and back, as well as the values of r^*_{s-1} and back.

4. Steps 1 through 3 constitute one trial, i.e., one set of T drawn vectors of errors. Do these steps again for another set of T drawn vectors. Keep doing this until the specified number of trials has been completed.

The solution values of the endogenous variables carried along for a given trial from quarter to quarter in the above procedure are estimates of what the economy would have been like had the Fed chosen $r^*_1,...,r^*_T$ and the error terms been as drawn.[16]

5 The Results

All the results are presented in Table 2, and the rest of this paper is essentially a discussion of this table. Values of L are presented for real GDP, the level of the GDP deflator, the percentage change in the GDP deflator, the unemployment rate, and r. The following discussion will focus on real GDP, the level of the GDP deflator, and r. The results for the unemployment rate are similar to the results for real GDP, and no further discussion is needed about the unemployment rate. The results for the percentage change in the GDP deflator are generally similar to the results for the

[16]The optimal control procedure just outlined differs somewhat for the procedure used in Fair and Howrey (1996, pp. 178-179). In Fair and Howrey (1996) the Fed is assumed not to know the exogenous variable values, but instead to use estimated autoregressive equations to predict these values for the current and future quarters. Also, the Fed is assumed not to know the error draws for the current quarter when solving its problem. In addition, stochastic simulation is not done. Instead, the error terms are set to zero (instead of to their historical values), the target values are taken to be the historical means (instead of the actual values), and the (one) trial uses for the error draws for a given quarter the actual errors for that quarter.

Table 2
Values of L

	Y	P	$\dot{P}$	U	r	$Y + \dot{P}$
Part 1						
MC Model						
1. No rule	4.28	3.57	2.02	1.42	0.00	6.30
2. Estimated rule	3.35	3.10	1.77	1.12	1.42	5.12
3. Taylor rule	3.39	2.37	1.64	1.14	3.06	5.03
4. .25 rule	3.28	3.06	1.77	1.12	1.10	5.05
Part 2						
MC Model, Interest Income Exogenous						
1. No rule	4.16	3.65	2.02	1.37	0.00	6.28
2. Estimated rule	2.76	3.06	1.74	0.92	1.10	4.50
3. Taylor rule	2.99	2.10	1.59	1.02	2.59	4.58
4. .25 rule	2.62	2.96	1.72	0.94	0.90	4.34
5. Est. & tax rules	2.37	3.10	1.93	0.83	0.94	4.30
Part 3						
US Model, Interest Income Exogenous						
1. No rule	5.38	1.42	1.99	1.19	0.00	7.37
2. Estimated rule	4.12	0.69	1.80	0.83	1.14	5.92
3. Taylor rule	4.04	0.61	1.90	0.92	2.86	5.94
4. .25 rule	3.57	0.71	1.74	0.83	1.19	5.31
5. Optimal ($\alpha = 2.0$)	2.89	0.81	1.82	0.77	2.86	4.71
6. Optimal ($\alpha = 9.0$)	3.69	0.81	1.88	0.96	1.42	5.57
Part 4						
US-RE Model, Interest Income Exogenous						
1. No rule	4.75	1.25	1.96	1.06	0.00	6.71
2. Estimated rule	3.50	0.64	1.74	0.69	1.00	5.24
3. Taylor rule	3.10	0.58	1.80	0.69	2.16	4.90
4. .25 rule	3.17	0.69	1.72	0.74	1.04	4.89

Y = real GDP
P = GDP deflator
$\dot{P}$ = percentage change in the GDP deflator
U = unemployment rate
r = three month Treasury bill rate
Simulation period = 1993:1–1998:4

level, although in some cases the differences in L across rules are fairly small for the percentage change.[17]

The sum of L for Y and $\dot{P}$ is also presented in Table 2. Note, however, that this sum is not what the optimal control procedure minimizes. The loss function in (6) includes interest rate fluctuations, and a sequence of optimal control problems is solved, not just one.

Part 1

Part 1 of the table contains results using the complete MC model. Line 1 uses no rule (r is exogenous); line 2 uses the estimated rule; line 3 uses the Taylor rule; and line 4 uses the .25 rule. All the experiments using the MC model are based on the same set of error draws, which considerably lessens stochastic simulation error across experiments.

L for real GDP falls from 4.28 for no rule, to 3.35 for the estimated rule, to 3.39 for the Taylor rule, and to 3.28 for the .25 rule. For the GDP deflator the fall is from 3.57 to 3.10, 2.37, and 3.06, respectively. For r the values for the three rules are 1.42, 3.06, and 1.10, respectively. The conclusions that emerge from these results are the following.

[17] Although not reported in Table 2, experiments were run in which the price term in the rule used the price level rather than the percentage change in the price level. In other words, the target was the price level instead of the inflation rate. The values of L were not noticeably affected by this change. This lack of sensitivity is consistent with the results in Fair (2000, Table 6), where it is shown that price equations with vastly different long run properties can have very similar short run properties. The present exercise is essentially a short run one, and it makes little practical difference whether a rule (or optimal control procedure) targets the price level or the percentage change in the price level.

1. All three rules are better than no rule.
2. The results for the estimated and .25 rules are quite close.
3. The Taylor rule has lower variability for the GDP deflator but much higher variability for r. Although not shown in the table, in some cases the Taylor rule called for values of r below 1.0, sometimes negative values, and in these cases a value of 1.0 was used.[18] The other two rules never called for a value below 1.0. This conclusion is discussed further at the end of this section.

Part 2

Part 2 of the table contains results using the MC model with interest payments for the U.S. firm and government sectors taken to be exogenous. As discussed in the appendix, this is the case where there is no interest income effect on U.S. households, which increases the effectiveness of monetary policy for a given change in r. (Ignore line 5 in Part 2 for now.) The pattern of results in Part 2 is similar to the pattern in Part 1, and so little needs to be added. Monetary policy is more effective in Part 2 than in Part 1 in the sense that for a given rule the decreases in the variability of real GDP and the GDP deflator are larger even though the variability of r is smaller.

Part 3

Part 3 of the table contains results using the stand alone US model. To save space, only the results with interest payments taken to be exogenous are presented. Part 3 is thus comparable to Part 2. All the experiments using the US model are based on the same set of error draws, although this is a different set than that used for the MC

[18] When 1.0 is used for r, this is the value used in the computation of L, not the value implied by the rule.

model. The pattern of results in Part 3 is similar to that in Part 2. One noticeable difference is that the values of L for the GDP deflator are considerably smaller in Part 3 than in Part 2. One reason for this—perhaps the main reason—is that the US model experiments (unlike the MC model experiments) draw errors from the 1950s and 1960s, and there were fewer large price shocks in the 1950s and 1960s than later. Although the level of variability for the GDP deflator is lower in Part 3 than in Part 2, the pattern of changes in variability across rules is similar.

The main interest in Part 3 is the optimal control experiments. The first experiment (line 5) uses a value of α in the loss function in (6) of 2.0. This results in a value of L for r of 2.86, which is the same as that for the Taylor rule. This experiment is thus most closely comparable to the use of the Taylor rule. Compared to the Taylor rule, the variability of output is lower (2.89 versus 4.04) and the variability of the price level is higher (0.81 versus 0.61).

The second optimal control experiment (line 6) uses a value of α in the loss function in (6) of 9.0. This results in a value of L for r of 1.42, slightly higher than the values for the estimated and .25 rules (1.14 and 1.19). Compared to the estimated rule, the variability of output for the optimal control procedure is lower (3.69 versus 4.12) and the variability of the price level is higher (0.81 versus 0.69). The .25 rule does slightly better overall than does the optimal control procedure (for $\alpha = 9.0$), although the results are close.

Part 4

Finally, Part 4 of the table contains results using the RE version of the US model, again with interest payments taken to be exogenous. In order to make Part 4 as comparable as possible to Part 3, the RE version was set up in the following way. First, the same error draws were used for the US-RE experiments as were used for the US experiments. This means that for the six equations that are different for the RE version—three consumption equations, the import equation, and two long term interest rate equations—the drawn errors are errors from the original equations, not from the RE versions. Again, the use of the same errors lessens stochastic simulation error. Second, the same coefficients were used for the RE versions of the consumption and import equations as were used for the original equations except for the income variable. The coefficient used for each of the four future income variables in each equation was taken to be one fourth of the coefficient of the current income variable in the original equation. The total income effect is thus the same between the two versions, other things being equal, but it is spread out in the future for the RE version. The two long term interest rate equations, on the other hand, were completely changed. In the RE version, each long term rate is the average of the current and seven future values of r.

The RE version of the model is solved as follows. Consider trial 1. The errors for quarter 1, but not for any future quarters, are drawn. The model is solved for quarter 1 using the extended path method in Fair and Taylor (1983, 1990). This solution requires solving the model for many quarters in the future. Since errors are drawn only for quarter 1, agents are assumed not to know the future draws when forming their expectations of the future in quarter 1. They are, however, assumed to know the future values of the exogenous variables (mostly fiscal policy variables). After quarter 1 is solved, errors are drawn for quarter 2, and the model is solved for quarter 2 using the extended path method. The solution values for quarter 1 are used as initial conditions for this solution. This process is repeated through quarter 24. This then finishes one trial. The entire process is then repeated for 19 more trials. There are thus a total of $24 \times 20 = 480$ times in which the model is solved using the extended path method.

The pattern of results in Part 4 in Table 2 is similar to that in Part 3. The overall results are thus not very sensitive to the present use of the RE assumption. Monetary policy is, however, slightly more effective under the RE assumption in that for a given rule the value of L for r is smaller in Part 4 than in Part 3 and yet the decreases in output and price variability are similar or slightly larger. The main reason for this is that long term rates respond faster to r changes in the RE version, which then makes consumption and import demand respond faster. There is also a larger income effect on demand because future income changes are larger than just the current income change (because of lags in the model). Overall, however, the differences are fairly modest, and they do not change the basic pattern of results.

General Discussion

An important result above is that the Taylor rule, which has a large coefficient on inflation, when compared to the other two rules, which have much smaller inflation coefficients, achieves a fairly small reduction in inflation variability at a cost of a large increase in interest rate variability. Some insight into this result can be gleaned from a property of the price equation in the US model, which is that the price level responds only modestly to demand (a common feature of most estimated price equations). Since the interest rate primarily affects the price level through its effects on demand, the price level responds only modestly to interest rate changes. Since the Taylor rule has a large coefficient on inflation, a large price shock leads to a large change in the interest rate, but this in turn has only a modest effect in offsetting the effects of the price shock. For the other two rules the interest rate responds much less to a price shock, and so the interest rate variance is smaller. The cost of a smaller interest rate response in terms of offsetting the effects of the price shock is modest because of the modest effect of the interest rate on the price level.

CGG show, using a four equation calibrated model of the economy, that interest rate rules that have inflation coefficients less than one can be destabilizing. Why aren't the estimated and .25 rules destabilizing, as they would be in the CGG model? The answer is that the response of output to a price shock is much different in the CGG model than it is in the US model. Consider a positive price shock with no change in the nominal interest rate. In the CGG model this is expansionary because the real interest rate, which has a negative effect on output, is lower. In the US model, on the other hand, a positive price shock with no change in the nominal interest rate is contractionary. In the short run the aggregate price level rises more than do wage rates, and so there is a fall in real income. Real wealth also falls. These effects are contractionary on demand. In addition, the empirical results suggest that households respond to nominal interest rates and not real interest rates, and so there is no positive household response to lower real interest rates. The net effect of a positive price shock with no change in the nominal interest rate is contractionary in the US model. If this is true, then in response to a positive price shock the Fed does not have to increase the nominal interest rate more than the increase in inflation to achieve a contraction. There will be a contraction even if there is no increase in the nominal interest rate at all!

Judging interest rate rules can thus be sensitive to the economic model used. Using an economic model in which positive price shocks are expansionary, as CGG do, leads to a quite different conclusion than using a macroeconometric model like the US model, where positive price shocks are contractionary. Using small calibrated models to make policy conclusions may be risky if the models are at odds with more empirically based models. It may be that the specification and calibration have not captured reality well.

Another interesting result is that the optimal control procedure with $\alpha = 9.0$ does not do much better than the estimated rule and is about tied with the .25 rule. In other words, if the optimal control procedure is restricted to have an interest rate variability about equal to the historical variability, little improvement over the estimated or .25 rules seems possible. If the estimated rule is a good approximation of Fed behavior, then the Fed seems to be doing well.

6 Adding a Tax Rate Rule

A tax rate rule is proposed in this section that might help monetary policy in its stabilization effort. The idea is that a particular tax rate or set of rates would be automatically adjusted each quarter as a function of the state of the economy. Congress would vote on the parameters of the tax rate rule as it was voting on the general budget plan, and the tax rate or set of rates would then become an added automatic stabilizer.

Consider, for example, the federal gasoline tax rate. If the short run demand for gasoline is fairly price inelastic, a change in the after-tax price at the pump will have only a small effect on the number of gallons purchased. In this case a change in the gasoline tax rate is like a change in after-tax income. Another possibility would be a national sales tax if such a tax existed. If the sales tax were broad enough, a change in the sales tax rate would also be like a change in after-tax income.

For the results in this paper a constructed federal indirect business tax (IBT) rate based on data from the national income and product accounts is used for the tax rate rule. In practice a specific tax rate or rates, such as the gasoline tax rate, would have to be used, and this would be decided by the political process. The constructed tax rate for quarter t, denoted τ_t, is the ratio of overall federal indirect business taxes to total consumption expenditures. In the regular version of the model τ_t is taken to be exogenous.

The following equation is used for the tax rate rule:

$$\begin{aligned}\tau_t = \tau_t^* &+ 0.125[.5((Y_{t-1} - Y_{t-1}^*)/Y_{t-1}^*) + .5((Y_{t-2} - Y_{t-2}^*)/Y_{t-2}^*)] \\ &+0.125 * [.5(\dot{PF}_{t-1} - \dot{PF}_{t-1}^*) + .5(\dot{PF}_{t-2} - \dot{PF}_{t-2}^*)]\end{aligned} \quad (7)$$

where Y denotes real GDP and $\dot{PF}$ denotes the percentage change in a private nonfarm price deflator. It is not realistic to have tax rates respond contemporaneously to the economy, and so lags have been used in (7). Lags of both one and two quarters have been used to smooth tax rate changes somewhat. The rule says that the tax rate exceeds its base value as output and the inflation rate exceed their base values. PF is used instead of the GDP deflator because by construction the GDP deflator is affected by indirect business taxes (and thus by τ_t).

Results using this rule along with the estimated interest rate rule are reported in the fifth line in Part 2 in Table 2. The use of the rule lowers L for real GDP from 2.76 when only the estimated interest rate rule is used to 2.37 when both rules are used. The rule is thus of considerable help in lowering output variability. Although not reported in Table 2, the rule does lower the variability of PF (from 3.61 to 3.42). On the other hand, as shown in Table 2, it does not lower the variability of the GDP deflator. This is because, as just mentioned, the GDP deflator is directly affected by indirect business taxes. When the tax rate rule is used, the variability of indirect taxes is greater, which, other things equal, increases the variability of the GDP deflator.

7 Conclusion

The main conclusions have been discussed at the end of Sections 2 and 5. They are in brief: 1) The estimated interest rate rule passes a number of fairly stringent tests, including the stability test. 2) When using the MC and US models, the Taylor rule when compared to rules with smaller inflation coefficients achieves a small reduction in inflation variability at a cost of a large increase in interest rate variability. Also, interest rate rules with small inflation coefficients are not destabilizing. 3) The optimal control procedure is not much of an improvement over the estimated rule.

Finally, it is interesting to note that even when both the estimated interest rate rule and the tax rate rule are used, the values of L in Table 2 are nowhere close to zero. Monetary policy even with the help of a fiscal policy rule cannot come close to eliminating the effects of typical historical shocks. In this sense Fed power is quite limited.

Appendix

The MC and US Models

The MC Model

The MC model in Fair (1994) is used for the results in this paper. An updated version of this model has been used for the present work, and this version is presented on the website mentioned in the introductory footnote. There are 38 countries in the MC model for which stochastic equations are estimated.[19] There are 31 stochastic equations for the United States and up to 15 each for the other countries. The total number of stochastic equations is 363, and the total number of estimated coefficients is 1650. In addition, there are 1050 estimated trade share equations. The total number of endogenous and exogenous variables, not counting the trade shares, is about 4500. Trade share data were collected for 59 countries, and so the trade share matrix is 59×59.[20]

The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. They end between 1996 and 1999. The estimation technique is two stage least squares except when there are too few observations to make the technique practical, where ordinary least squares is used. The estimation

[19]The 38 countries are the United States, Canada, Japan, Austria, France, Germany, Italy, the Netherlands, Switzerland, the United Kingdom, Finland, Australia, South Africa, Korea, Belgium, Denmark, Norway, Sweden, Greece, Ireland, Portugal, Spain, New Zealand, Saudi Arabia, Venezuela, Colombia, Jordan, Syria, India, Malaysia, Pakistan, the Philippines, Thailand, China, Argentina, Chile, Mexico, and Peru.

[20]The 21 other countries that fill out the trade share matrix are Brazil, Turkey, Poland, Russia, Ukraine, Egypt, Israel, Kenya, Bangladesh, Hong Kong, Singapore, Vietnam, Nigeria, Algeria, Indonesia, Iran, Iraq, Kuwait, Libya, the United Arab Emirates, and an all other category.

accounts for possible serial correlation of the error terms. The variables used for the first stage regressors for a country are the main predetermined variables in the model for the country. A list of these variables is available from the website.[21]

There is a mixture of quarterly and annual data in the MC model. Quarterly equations are estimated for 14 countries (the first 14 in footnote 19), and annual equations are estimated for the remaining 24. However, all the trade share equations are quarterly. There are quarterly data on all the variables that feed into the trade share equations, namely the exchange rate, the local currency price of exports, and the total value of imports per country. When the model is solved, the predicted annual values of these variables for the annual countries are converted to predicted quarterly values using a simple distribution assumption. The quarterly predicted values from the trade share equations are converted to annual values by summation or averaging when this is needed.

Since the MC model is discussed in detail in Fair (1994) and on the website, it will not be discussed in detail here. The key properties of the model that are relevant for present purposes are the effects of interest rates on the economy, and these properties will now be outlined.

The main U.S. short term interest rate in the model is the three month Treasury bill rate, which the Fed is assumed to control. This rate is denoted r in the text, but RS in the model, and the RS notation will be used in this appendix. A change in RS affects the U.S. economy in the following ways:

[21]Some of the equations in the model are changed beginning in 1999 to incorporate the EMU. Beginning in 1999, the exchange rate equations of the individual EMU countries are replaced with one exchange rate equation, and the individual interest rate rules are replaced with one rule.

1. Long term interest rates depend on current and lagged values of RS.

2. Interest rates appear as explanatory variables in the consumption, import, and housing investment equations, all with negative coefficient estimates.

3. Interest rates have a negative effect on stock prices in the stock price equation, and stock prices appear in the consumption and housing investment equations through a wealth variable, which has a positive effect on consumption and housing investment.

4. Interest payments of firms and the government—and thus interest income of households—change when interest rates change, and household interest income appears in the consumption, import, and housing investment equations through a disposable income variable, which has a positive effect in these equations.

5. A change in RS leads to a change in the value of the dollar vis-a-vis the other major currencies through exchange rate equations—an increase in RS leads to an appreciation of the dollar and a decrease leads to a depreciation. A change in the value of the dollar leads to a change in U.S. import prices, which then results in a change U.S. domestic prices through an import price variable in the domestic price equation. The change in the value of the dollar also leads to a change in the demand for U.S. exports through the trade share equations, and it leads to a change in U.S. import demand through an import price variable in the U.S. import equation.

6. RS appears as an explanatory variable in some of the other countries' interest rate rules, and so foreign interest rates in part follow U.S. rates.

The net effects of, say, a decrease in RS on U.S. output and the price level are positive. Output increases because there is an increase in the demand for U.S. domestically produced goods, and the price level increases because of the increase in demand and the depreciation of the dollar.[22]

[22]The dynamics of the estimated U.S. price equation in the model are discussed in Fair (2000). An initial increase in the price level caused by changes in the explanatory variables or by positive shocks leads to further increases in the price level in the future and thus to inflation. The inflation generated from these changes eventually dies out.

The interest income effect listed in point 4 above is now fairly large because of the large stock of federal government debt that was accumulated in the 1980s and early 1990s. U.S. households own much of this debt, much of which is short term, and so there is a large change in their interest income when interest rates change. This positive income effect offsets some of the negative intertemporal substitution effect, and so the net effect of an interest rate change on output is considerably smaller than would be the case without the income effect. It is not easy empirically to link interest rate changes to interest income changes in the national income accounts, and it may be that the model has overestimated the income effect. Consequently, an alternative version of the model has been used for some of the experiments, which treats interest income as exogenous. In this version there is no interest income effect on households.

The US Model

The optimal control procedure and the use of rational expectations are too costly in terms of computer time to be able to be used in the MC model. For this work a stand alone model of the United States has been used. This US model is exactly the same as the model for the United States in the MC model except for the treatment of U.S. exports (EX) and the U.S. price of imports (PIM). These two variables change when RS changes—primarily because the value of the dollar changes—and the effects of RS on EX and PIM were approximated in the following way.

An experiment was run using the MC model in which the RS rule was dropped and RS was decreased by one percentage point from its base value in 1999:1. The values of RS from 1999:2 on were kept unchanged from the base values. (The base values are values from a forecast that was made using the MC model.) No other changes were made to the MC model, which means, for example, that all the other countries' interest rate rules were retained. The MC model was solved for the RS change for the 1999:1-2001:4 period, and the percentage deviations in EX and PIM from their base values were recorded for each quarter. Let β_i denote the percentage deviation in EX in quarter i, and let γ_i denote the percentage deviation in PIM in quarter i. i is 1 for 1999:1, 2 for 1999:2, and so on.

The approximating equation used for EX is:

$$\begin{aligned} EX/EX^* = 1.0 + \beta_1(RS - RS^*) + \beta_2(RS_{-1} - RS^*_{-1}) + ... \\ +\beta_{12}(RS_{-11} - RS^*_{-11}) \end{aligned} \tag{8}$$

where EX^* is the base value of EX and RS^* is the base value of RS. The approximating equation for PIM is:

$$\begin{aligned} PIM/PIM^* = 1.0 + \gamma_1(RS - RS^*) + \gamma_2(RS_{-1} - RS^*_{-1}) + ... \\ +\gamma_{12}(RS_{-11} - RS^*_{-11}) \end{aligned} \tag{9}$$

where PIM^* is the base value of PIM. If these two equations are added to the US model, then any change in RS relative to its base values will change EX and PIM relative to their base values, and the changes in EX and PIM will be approximately what would be the case in the MC model.

The US Model–RE Version

If agents use the model to form expectations of future values, then expectations are said to be ''rational'' or ''model consistent.'' A method is proposed in Fair (1993) for testing whether expectations are rational. Consider explaining a long term interest rate as a function of expected future short term rates. If expected future short term rates are assumed to depend on current and past short term rates, then the long term rate can be regressed on current and past short term rates. This is done in the regular version of the US model. If instead expectations of future short term rates are model consistent, the explanatory variables in the long term rate equation are the model's predictions of the future short term rates. A test in this context is to add future values of the short term rate to the long term rate equation, estimate the equation by a consistent method, and test the significance of the future values. A limited information method that can be used is Hansen's (1982) method of moments, where the instruments used are the main predetermined variables in the model.[23]

Tests of the kind just described have been performed on most of the estimated equations in the MC model.[24] The overall results are not generally supportive of the rational expectations hypothesis in that in most cases the future values are not significant. (This includes future short term interest rate values in long term interest rate equations.) An important exception, however, concerns the three consumption equations in the US model—explaining respectively service, nondurable, and durable consumption—where future values of income are significant.

To see how sensitive the results are to the treatment of expectations, a ‘‘rational expectations’’ (RE) version of the US model was specified. In each of the three consumption equations and in the import equation, the current value of income was

[23]An alternative, full information, method is FIML— see Fair (1993, p. 183).

[24]See the results in Chapters 5 and 6 in Fair (1994) and updated results on the website.

replaced with the value of income led one, two, three, and four quarters. In addition (although the tests do not support this), each of the two long term interest rate equations (explaining the AAA bond rate and a mortgage rate) was replaced with an equation in which the long term rate is equal to the average of the current short term rate and the one- through seven-quarter-ahead short term rates. The coefficients that were used for these equations are discussed in Section 5. The RE version of the US model was solved using the extended path method in Fair and Taylor (1983, 1990).

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