# Vertical Integration, Networks, and Markets

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*Abstract*: The organization of supply relations varies across industries. This paper builds a theoretical framework to compare three alternative supply structures: vertical integration, networks, and markets. The analysis considers the relationship between uncertainty in demand for specific inputs, investment costs, and industrial structure. It shows that network structures are more likely when productive assets are expensive and firms experience large idiosyncratic shocks in demand. The analysis is supported by existing evidence and provides empirical predictions as to the shape of different industries.

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## 1. Introduction

The organization of supply relations varies across industries. This paper studies three alternative structures: markets, networks, and vertical integration. Case studies show an abundance of industries organized as networks, but most economic theory considers only vertical integration and markets.[1] This paper develops a theoretical framework to study all three structures, and the relationship between structure, demand uncertainty for specific inputs, and investment costs. The analysis is supported by existing evidence and provides empirical predictions as to the shape of different industries.

Networks (we provide examples below) are distinct from vertically integrated firms and decentralized markets. Manufacturers maintain on-going contact with their suppliers. They train them, provide specialized equipment and know-how, and otherwise invest in the relationship. Suppliers also invest in assets that allow them to produce inputs to buyers' specifications. A manufacturer-supplier relationship is typically non-exclusive; buyers have several suppliers for each input, and suppliers have several clients. In contrast, a vertically integrated firm has its own supply facilities for specialized inputs. General Motors in the 1950's is the canonical example. In input "markets," manufacturers do not maintain relationships with sellers and only standardized inputs can be obtained.

The existence of buyer-seller networks is puzzling from a theoretical perspective. Well-known arguments imply vertical integration is likely when technology involves costly specific investments that improve the value of inputs: Vertical integration eliminates ex post non-cooperative bargaining between a buyer and seller and thus improves investment incentives [Williamson (1975, 1985), Klein, Crawford and Alchian (1978), Grossman and Hart (1986), Hart and Moore (1990)]. This paper does not counter this argument. Rather, we posit a counterveiling incentive for vertical

[1] We review the case study literature below. See Holmstrom and Roberts (1998) for a review of the economic literature on vertical integration and markets. Helper and Levine (1992) compare competitive markets to supplier networks in which each buyer has a single supplier for each input. The networks we consider are more general: there could be multiple suppliers for any given input and multiple buyers for any supplier. Networks of different forms been used to study particular industries such as airlines [Hendricks, Piccione, and Tan (1995)], communications [Economides and Himmelberg (1995), Henriet and Moulin (1996)], and energy [Smith, Backerman, and Rassenti (1996)]. Kranton and Minehart (1998a) introduces a general theory of buyer-seller (i.e., bipartite) networks, and we use that theory here to build a model of industrial structure.

disintegration rooted in uncertainty in demand for specialized inputs.[2]

A simple example illustrates. Consider an industry of designer clothing consisting of $N > 2$ manufacturers each with its own "style." In each season, exactly two of the $N$ are fashion "winners" and secure half of the consumer demand each.[3] Normalize this demand to two, and suppose that each potential supplier can invest in one unit of costly capacity. Then an efficient industrial structure involves exactly two suppliers that sell to whichever firms are the fashion "winners." The firms may be thought of as a network (assuming that each manufacturer invests in the suppliers, by training them, loaning equipment, explaining designs, etc.) in which the $N$ manufacturers share the capacity of two suppliers.[4] The suppliers are flexible because, thanks to their own and the manufacturers' investments, they can produce clothing for any of them.[5]

This connection between uncertainty and economies of scale has its origin in the "repairman problem" (Feller (1950), Rothschild and Werden (1979)). This paper adds the requirement of links between trading partners and demonstrates the connection between the "repairman problem" and network industrial structures. We show how the position of a link determines its contribution to economic welfare and relate that contribution to the distribution of buyers' idiosyncratic shocks. Moreover, we examine the strategic incentives of firms to build these links.

We discuss below two cases, the garment industry in New York City and the Japanese electronics industry, where buyers have uncertain demand for specialized inputs and the industries are organized as networks. In both settings, links between a buyer and seller allow the seller to be able to make specialized inputs to a buyer's specifications. We also discuss industries whose structures have changed over time and different sources of demand uncertainty.

This paper analyzes how demand uncertainty as well as contractual incompleteness may affect industrial structure. We first characterize the efficient industrial structure in terms of demand uncertainty and investment costs. We then ask whether strategic firms will build their own supply

[2]Other advantages of networks put forward in the case study literature include technological diffusion, information sharing, and economies of scope.

[3]In our model, the uncertainty in buyers' valuations is i.i.d., not perfectly correlated as in this example. However, the economic intuition and results are qualitatively the same.

[4]Uzzi (1996) discusses the many facets of the information transfer from garment manufacturers to their suppliers. This transfer assures the manufacturer that the supplier will produce inputs according the buyer's "style."

[5]The network structure allows the $N$ manufacturers to avoid the inefficiency which would result if they each built their own dedicated capacity.

facilities (i.e., vertically integrate), join an anonymous market, or invest in a network. In networks, ex ante investments will affect ex post bargaining positions and thus individual investment incentives may diverge from social incentives. A buyer that invests in its own exclusive supply facility does not face this problem. We show that despite suboptimal equilibrium investment in networks, network industrial structures are often "second-best." As buyers face greater demand uncertainty, networks are equilibrium outcomes and yield greater welfare than vertical integration and markets.

These results provide some new intuitions about multiple sourcing arrangements. First, according to the traditional view, multiple sourcing by a buyer reduces the bargaining power of sellers (Demski et. al. (1987), Scheffman and Spiller (1992), Riordan (1996)). When, as happens in networks, both buyers and sellers have multiple trading partners, this "multilateral" sourcing may balance the bargaining power of buyers and sellers to mitigate (if not entirely eliminate) the hold-up problems that would otherwise distort investment incentives. Second, according to the traditional view, firms tend not to make specific investments when buyers and sellers are not integrated. In networks, buyers make many specific investments, possibly even more than under vertical integration. Buyers in fact make multiple specific investments, by building links to several sellers. In addition to increasing the gains from trade, this multiplicity ensures that sellers have the incentive to invest in flexible assets and allows a savings on overall investment costs because buyers share the productive capacity of fewer sellers. Third, our multilateral setting reveals a new consequence of vertical merger. When an upstream firm has at most one relationship to a downstream firm, Bolton and Whinston (1993) find that vertical merger can lead to inefficiently high levels of investment in a specific asset. In our network setting, which allows for many vertical relationships, we find that downstream firms that own a network productive facility might not invest in relationships with other upstream firms, even when such links would be efficient. Thus, the overall investment effect of vertical merger is ambiguous.

A series of earlier papers has considered the impact of demand uncertainty on firm and market behavior. In this work, firms must set prices or quantities before demand uncertainty is resolved [Baron (1971), Leland (1972), Holthausen (1976), Carlton (1978)]. Carlton (1979) shows that when competitive sellers cannot adjust their prices of a homogeneous input to the numbers of randomly arriving buyers, vertical integration is always inefficient (because when buyers withdraw from the market, demand variation increases). Buyers, however, may vertically integrate to avoid input rationing. In the present paper, as in Bolton and Whinston (1993), prices adjust after uncertainty is realized, so there is no input rationing. We consider different reasons for vertical integration. Under certain demand and cost conditions, vertical integration may be the efficient industrial structure. When it is not, firms may still have the incentive to vertical integrate because of the incomplete contracting environment. Overall, in our setting there is a tension, both in terms of social welfare and individual profits, between vertical integration and vertical disintegration.

Our theory further distinguishes between firm-specific and aggregate demand uncertainty. This distinction helps clarify discussions in the literature on the benefits of networks. Piore and Sabel's (1984) influential work on networks of "flexible specialists" argues that networks emerge in times of greater economic uncertainty, and in case studies of networks, demand fluctuations figure prominently. Our results indicate that uncertainty, per se, does not does not lead to networks. Idiosyncratic shocks, not aggregate shocks, are the source of network benefits. If, however, firms face greater idiosyncratic shocks during recessions,[6] then industrial structure could become more "network-like" during business slowdowns.[7]

The next section discusses industry examples. Section 3 presents the basic model of demand uncertainty, investments, and vertical integration, networks, and markets. Section 4 determine when each structure yields the greatest social welfare. Section 5 examines the strategic incentives of firms to vertically integrate, to join an anonymous market, or invest in a network. Section 6 considers vertical merger in networks. Section 7concludes.

## 2. Examples

Our first network example is the Women's Better Dress sector of the garment industry in New York City [Uzzi (1996, 1997)]. Manufacturers (a.k.a. jobbers) design and market garments, hiring "contractors" to fabricate them. The manufacturers and contractors are linked by long-term, on-

[6]A number of economic series are known to be more uncertain during recessions (Schwert (1989)), suggesting that firm-specific uncertainty might also be greater.

[7]Lilien (1982) argues that the movement of labor out of declining industries causes unemployment in recessions. By analogy, flexible suppliers should fare better during recessions than suppliers dedicated to a particular firm or industry.

going relationships. These links embody "fine-grained information" acquired over time about a manufacturer's particular "style." A contractor needs this information to make a garment correctly. For example, there are many different properties of fabrics, how they "fall," "run," "stretch," "forgive stitching," to which production procedures must be subtly adjusted. The necessary adjustments are impossible to specify in advance.[8] Suppliers with experience making such adjustments can allow the manufacturer to take advantage of rapidly changing market conditions.

The market for Better Dresses is highly fashion-sensitive. Firms face significant idiosyncratic demand uncertainty. Some designs succeed, others fail. When a manufacturer's design is "hot," it has a surge in orders. The manufacturer must then be able to locate an experienced contractor on short notice (that is, links must be established ex ante). To help insure production, manufacturers often have long-term relationships with multiple sellers. Conversely, to protect themselves against the difficulties of any one manufacturer, sellers have long-term relationships with multiple manufacturers.[9] Manufacturers often spread their work among their contractors to cushion them against demand uncertainty.[10] Uzzi (1996) finds that contractors with long term on-going relationships with several manufacturers have a lower failure rate than those that primarily engage in "arms-length" transactions with many manufacturers. The value of their output is higher, and they have a more reliable stream of orders.

Our second example is the electronics industry in Japan. Here, Nishiguchi (1994) describes vertical supply networks for finished products (as opposed to components), where long-term specific investments, i.e., links, are important. Assemblers need "customer-specific knowledge," training, tools, and machines that have little use in assembly for other manufacturers. Relationships between manufacturers and contractors develop over many years, with contractors only gradually taking the complex assignment of finished product assembly.[11] This slow qualification process

[8]A manufacturer relates: "If we have a factory that is used to making our stuff, they know how it's supposed to look. [...] They will know how to work the fabric to make it look the way we intended. A factory that is new will just go ahead and make it. They won't know any better." (Uzzi 1996, p.678)

[9]Over a sixteen month period in 1990-91, 25% of manufacturers hired 5 or fewer contractors, 30% hired 5-12 contractors, and 40% hired 20 or more contractors. As for contractors, 35% sold to 3 or fewer manufacturers, 45% sold to 4-8 manufacturers, and 20% sold to 9 or more manufacturers [Uzzi (1996, p. 690)].

[10]A manufacturer relates, "[w]here we put work all depends on the factory. If it's very busy [with another manufacturer's orders] I'll go to another factory that needs the work to get by in the short-run." [Uzzi (1997, p.54)]

[11]At Fuji Electric, in 1983, 25% of Fuji's subcontractors had done business with Fuji for 21 years or more. For

is sometimes formalized as a grading sytem in which manufacturers score the subcontractors' performance. Subcontractors are only given high level work after they have performed well in lower level tasks (pp. 133-134).

Assemblers work for several manufacturers to protect themselves from demand uncertainty and indeed may be encouraged to do this by their clients.[12] As in the garment industry, firms have multiple links.[13] Links to a few manufacturers in different lines of business help to protect a contractor against drops in demand in any one of them.

More generally, demand uncertainty characterizes many industries with network supply structures. We can divide industry case studies into two broad categories. The first is fashion, culture, and craft industries such as garments, textiles, shoes, leather goods, and toys.[14] In these industries, volatile consumer preferences underlies uncertainty in a manufacturer's demand for inputs. The second category is "high-tech" industries such as electronics, engineering, computers, and semiconductors, custom machinery, and automobile parts.[15] In these industries, uncertainty over firms' success in innovation [16] and demand for new products both translate into idiosyncratic uncertainty in input demands.

We now turn to a formal model that explores the connection between input demand uncer-

63%, the business relationship had lasted at least 6 years (p. 117).

[12]Nishiguchi relates, "During recession, it became general practice for the large customers not only to give advance warning ...about the forthcoming reduction in subcontracting orders but also to help those subcontractors most likely to be severely affected to change their products and look elsewhere for business. The large customers also frequently helped the subcontractors find stopgaps (e.g. by finding other, less affected business entitites to work with or even by sharing parts of the customers' own in-house operations not as affected by the recession), in order to keep the subcontractors' factories running." (p.118)

[13]"First tier" electronics assembly contractors had on average 3.36 regular customers who each placed orders several times over the period of a year (p. 151).

[14]See Lazerson (1993) and Brusco (1982) for the garment industry in Emilia-Romagna, Italy. Cawthorne (1995) and Banerjee and Munshi (1998) for cotton knitwear in Tirupur, India. Schmitz (1995) analyzes a shoe manufacturing network in the Sinos Valley, Brazil. Rabelotti (1995) compares Italian and Mexican show manufacturing networks. "The Puppet-master of Toytown," *Economist* 1997, Sept. 6, p. 88, discusses the toy industry.

[15]Saxenian (1994) studies Silicon Valley and Route 128. Nohria (1992) also studies Route 128. Scott (1987) analyzes defense subcontracting in Orange County, CA. Lorenz (1989) studies engineering and electronics industries in France. Nishiguchi (1994) discussed in the text studies Britain as well as Japan. We discuss the automobile industry below. Scott (1993), Nishiguchi (1994), and Lorenz (1989) all consider NC tools. Piore and Sabel (1984, p. 217) also discuss NC tools in Japan.

[16]See for instance the literature on "quality ladders"(Grossman and Helpman (1991)).

tainty, investment in links between firms, and industrial structure.

## 3. The Basic Model: Technology and Industrial Structure

There are $B \geq 2$ buyers each of whom demands one (indivisible) unit of a *specialized input,* that is, an input made to its specifications. Each buyer $i$ has a random valuation for such an input $v_i = v + \varepsilon_i$, where $v \geq 0$ is an aggregate shock with mean $\overline{v}$ and $\varepsilon_i$ is an idiosyncratic shock. We assume $\varepsilon_i$ is an i.i.d. random variable with continuous distribution $F$ with mean 0, and variance $\sigma^2$. $\mu^{n:B}$ denotes the expectation of the $n^{th}$ order statistic of $B$ draws from the distribution. We will sometimes write $\mu^{n:B}(\sigma^2)$ to denote the dependence of the order statistic on the variance. We assume that $v_i \geq 0$ for all possible realizations of $v$ and $\varepsilon_i$. We do not specify further the distribution of the aggregate shock as only the mean will affect outcomes in the model.[17]

Specialized inputs can be produced in two ways.

*Buyer Production of Specialized Inputs.* A buyer can produce a specialized input for itself by making an investment in its own productive capability. A unit of productive capacity which can produce one (indivisible) unit of specialized input exclusively for one buyer costs $\alpha_e$. Marginal production costs are zero. We call a buyer that builds an exclusive unit of productive capacity a *vertically integrated firm.*

*Network Production of Specialized Inputs.* $S$ specialized sellers, $S \leq B$, can each potentially produce produce one (indivisible) unit of input. For a seller to be able to produce a specialized input for a buyer, the buyer must invest in a "link" to the seller, incurring a cost $c$. The seller must also invest in productive capacity which allows it to produce a specialized input for any linked buyer. This "flexible" capacity costs $\alpha_f$, where $\alpha_f + c \geq \alpha_e$. The combination of productive capacity and links to specific buyers makes a seller a "flexible specialist;" i.e., it can produce specialized inputs for a number of different buyers. Sellers that invest in productive capacity and their linked buyers are called a *network of firms.*

Notice that networks involve both *specific* investments and *quasi-specific* investments. The link between a buyer and seller is a specific investment since it has no value to any other firm.

[17] We are assuming that the distribution of the idiosyncratic shock is independent of the distribution of the aggregate shock and of the realization of $v$. Further specifications of the model could incorporate correlation between the two shocks or other relationship between their distributions.

We call a seller's productive asset *quasi-specific* since it can have value to more than one buyer but its value is limited. The asset has no value to any buyer to whom the seller is not linked.

For most of the analysis, we assume that sellers in networks own all productive assets. At the end of the paper we explore the possibility that some buyers own flexible units of productive capacity and can, therefore, produce specialized inputs for themselves and other buyers. (This would be a different type of vertical integration than that defined above.)

*Market Production of Standardized Inputs.*[18] Buyers can also forgo purchase of specialized inputs. We assume that there is a competitive fringe of sellers (different from the sellers enumerated above) that produce standardized inputs. We normalize the value buyers have for these inputs to zero and normalize all production costs to zero. We refer to this option as a *market*. We emphasize here that a market actually involves different technology and sales of a different type of good: standardized, not specialized, inputs.[19]

*Industrial Structure*. The investments of the $B$ buyers and $S$ specialized sellers form an *industrial structure.* Firms are divided into markets, networks, and vertical integration. We represent an industrial structure as a *graph,* $\mathcal{G}$.[20] Figure 1 shows an industrial structure for 4 buyers and 1 specialized seller. Buyers 1 and 2 are in a network with seller 1 which has invested in a flexible productive asset, as indicated by the box. Buyers 3 and 4 have invested in exclusive productive assets, also as indicated by boxes, and are vertically integrated firms. No buyers procure standardized inputs in a market.

[18] Alternatively, a market could be defined as the limit of a network as the cost of links $c$ goes to 0. When $c =$ 0, buyers may costlessly link to all sellers so that trading is unrestricted. We might think of these structures as markets for high quality, non-specific inputs. Our results on networks when $c = 0$ would characterize such markets, which may still be contrasted with markets for low quality goods.

[19] Standardized inputs products procured in a "market" might include solvents, industrial cleansers, and paper towels. Manufacturers do not typically invest in relationships with producers of these products, nor do producers make these inputs to match a specific buyer's needs.

[20] We present formal notation for industrial structures in the appendix. This notation builds on the framework developed in Kranton and Minehart (1998a). We refer the reader to that paper for technical exposition of the network model.

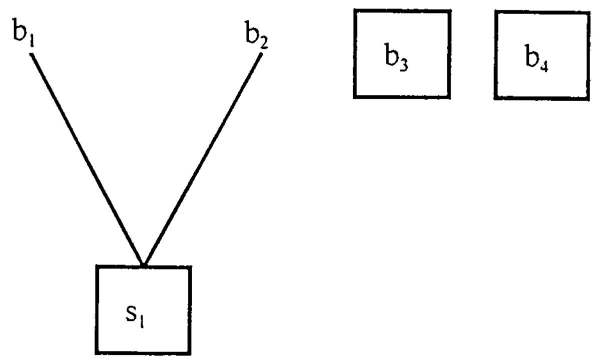


Figure 1

*Timing.* We assume that all investments by buyers and sellers must be made before demand uncertainty is resolved. That is, firms invest in anticipation of future, short-term, demand for inputs. This assumption captures aforementioned observations of "real-world" industrial settings where firms must respond rapidly to changing demand.

## 4. Economic Welfare and Efficient Industrial Structures

In this section we compare the welfare generated by different industrial structures and characterize *efficient industrial structures.* Since investments are made before uncertainty is resolved, we evaluate welfare from an ex ante perspective - the difference between the ex ante investment costs and the expectation of ex post gains from trade. In our welfare analysis we assume that, given an industrial structure, ex post trade is efficient, i.e., the highest possible gains from trade are realized. We make this assumption both as a benchmark and because any bargaining process with sufficiently small renegotiation costs should yield an efficient allocation.[21]

For an industrial structure $\mathcal{G}$, we first describe the maximal expected ex post gains from trade. Let $\mathbf{v} = (v_1, ..., v_B)$ be a vector of buyers' realized valuations, and let $A$ be an *allocation*

[21]This assumption contrasts with Carlton (1978, 1979) and other papers cited above where prices do not adjust after demand uncertainty is realized. In an industrial setting where a few buyers and sellers may negotiate face-to-face, we would expect that prices would adjust to demand conditions. Indeed, as we assume below, with sufficiently small renegotiation costs, prices should be *pairwise stable.* Payoffs should be such that no linked buyer and seller could strike a different deal that would make them both better off.

*of goods.*[22] The economic surplus associated with an allocation $A$ is the sum of the valuations of the buyers that secure specialized inputs in $A$.[23] We denote this surplus $w(\mathbf{v},A)$. For a given $\mathbf{v}$ and industrial structure $\mathcal{G}$, an allocation $A$ is *efficient* if and only if there does not exist another feasible allocation that yields greater surplus. The word "feasible" is important. Every vertically integrated buyer can always obtain a good. But in a network, the pattern of links will constrain which buyers can obtain goods from which sellers. Let $A^*(\mathbf{v},\mathcal{G})$ denote an efficient allocation.[24] With $A^*(\mathbf{v},\mathcal{G})$ for each ordering of buyers' valuations, we can determine the maximal expected ex post gains from trade for a given industrial structure: $E_{\mathbf{v}}\left[w(\mathbf{v},A^*(\mathbf{v},\mathcal{G}))\right]$, where the expectation is taken over all the possible realizations of buyers' valuations.

The welfare generated by an industrial structure, $W(\mathcal{G})$, is the maximal expected ex post gains from trade minus total investment costs:

$$W(\mathcal{G}) \equiv E_{\mathbf{v}}\left[w(\mathbf{v},A^*(\mathbf{v},\mathcal{G}))\right] - \alpha_e \cdot \sum_{i=1}^{B} \upsilon_i(\mathcal{G}) - c \cdot \sum_{i=1}^{B} l_i(\mathcal{G}) - \alpha_f \cdot \sum_{j=1}^{S} \kappa_j(\mathcal{G}),$$

where $\upsilon_i(\mathcal{G}) = 1$ when buyer $i$ is a vertically integrated firm and equals 0 otherwise, $l_i(\mathcal{G})$ is the number of buyer $i$'s links, and $\kappa_j(\mathcal{G}) = 1$ when seller $j$ has invested in productive capacity and equals 0 otherwise. An industrial structure $\mathcal{G}$ is *efficient* if and only if there does not exist another structure $\mathcal{G}'$ such that $W(\mathcal{G}') > W(\mathcal{G})$. That is, efficient industrial structures balance ex post expected gains from trade and ex ante investment costs. In our analysis of efficient structures below, we will assume that $\alpha_e = \alpha_f = \alpha$. This assumption simplifies the presentation, and the implications of a divergence in these costs ($\alpha_e < \alpha_f$) are easy to see.

We next characterize the efficient industrial structure. Our propositions apply to an industry with an arbitrary number of buyers $B$. We will illustrate our general results with a four-buyer-industry example. We begin by calculating the welfare for three basic types of structures: (1) all four buyers procure standardized inputs in a competitive market, (2) all buyers are vertically integrated, and (3) all buyers are in a network with two specialized sellers.

Let $\mathcal{M}$ denote the industrial structure in which all four buyers obtain standardized inputs in a market. Since we have normalized the production costs and value of standardized inputs to

[22] Formal notation for an allocation is given in the appendix. In Figure 1, an example of an allocation is: buyer 1 procures an input from seller 1, buyer 2 does not procure an input, buyers 3 and 4 each procure an input internally. Call this allocation $\widetilde{A}$.

[23] For the allocation $\widetilde{A}$, $w(\mathbf{v},\widetilde{A}) = v_1 + v_3 + v_4$.

[24] The allocation $\widetilde{A}$ is efficient if and only if $v_1 \geq v_2$.

zero, we simply have $W(\mathcal{M}) = 0$.

Let $\mathcal{V}$ denote the industrial structure where all four buyers are vertically integrated firms. Since each buyer builds its own exclusive unit of productive capacity, we have $W(\mathcal{V}) = 4[\overline{v} - \alpha]$.

In a network industrial structure, welfare depends on how the buyers are linked to sellers. Figure 2 illustrates the two networks with four buyers and two sellers that yield the highest welfare.[25]

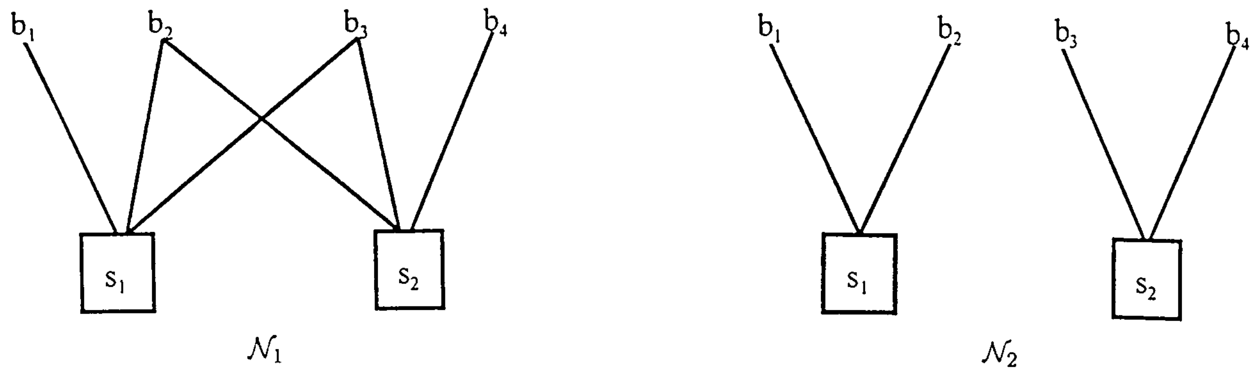


Figure 2

To calculate network welfare, we determine the efficient allocation for each ordering of buyers' valuations. In $\mathcal{N}_1$ suppose $v_1 > v_2 > v_3 > v_4$. In the efficient allocation for this ordering, $b_1$ obtains a good from $s_1$ and $b_2$ obtains a good from $s_2$. Indeed, in $\mathcal{N}_1$ for any ordering of buyers' valuations, the buyers with the two highest valuations obtain inputs.[26] Thus $W(\mathcal{N}_1) = 2\overline{v} + \mu^{1:4} + \mu^{2:4} - 6c - 2\alpha$. To calculate $W(\mathcal{N}_2)$, suppose again $v_1 > v_2 > v_3 > v_4$. In $\mathcal{N}_2$ it is not possible for both buyers 1 and 2 to obtain inputs. In the efficient allocation $b_1$ obtains a good from $s_1$, and $b_3$ obtains a good from $s_2$. For every ordering of buyers' valuations, the efficient allocation involves the buyer with the highest valuation of each pair obtaining a good. We therefore have $W(\mathcal{N}_2) = 2(\overline{v} + \mu^{1:2}) - 4c - 2\alpha$. In what follows, we will compare the welfare of these two networks, and other industrial structures, using the "triangle rule" which provides the relationship between order statistics from different size draws from a given distribution: $\mu^{n:m-1} = \frac{m-n}{m}\mu^{n:m} + \frac{n}{m}\mu^{n+1:m}$.[27]

[25]The two links that differentiate $\mathcal{N}_1$ and $\mathcal{N}_2$ carry the same value. That is, if it increases welfare to remove one, it is increases welfare to remove both. Networks with fewer links than $\mathcal{N}_2$ are dominated by either vertical integration or markets.

[26]$\mathcal{N}_1$ is an *allocatively complete network* (Kranton and Minehart (1998a)).

[27]See David (1981) for a derivation of this formula. We can use it to obtain, for example, $W(\mathcal{N}_2) = 2\overline{v} + \mu^{1:4} +$

**Advantages of Networks: Capacity Sharing and Flexiblity** A network may generate greater welfare than vertical integration or markets because buyers face shocks to their valuations and gain by sharing sellers' capacity. A vertically integrated buyer who suffers a large negative shock may "regret" having built the productive capacity. In a network, however, there are fewer units of productive capacity and buyers suffering the largest negative shocks do not procure inputs. Instead, inputs are allocated flexibly to the buyers with higher valuations.

We see these benefits of networks by comparing $\mathcal{N}_1$ and $\mathcal{V}$. $W(\mathcal{N}_1) \geq W(\mathcal{V})$ when

$$\left[\mu^{1:4} + \mu^{2:4}\right] - 6c \geq 2(\overline{v} - \alpha)$$

The left hand side captures the relative benefits of the network. Because the efficient allocation selects the two highest valuations, there is a gain of $\left[\mu^{1:4} + \mu^{2:4}\right]$. However, the multiple links that create the "flexibility" in the network generate an investment cost of $6c$. As for vertical integration, the two additional units of capacity each generate a surplus of $\overline{v}$ but add the investment cost $\alpha$.

We provide next two preliminary results which allow us to evaluate the expected gains from trade in any network and thereby allow us to compare networks, in general, to vertical integration and markets. First, we show that the maximal expected gains from trade in a network can always be written as a constant plus a sum of order statistics. The summation of order statistics reflects that buyers with higher valuations obtain goods in networks whenever possible.

**Lemma 1.** *The maximal expected ex post gains from exchange in a network with $\widetilde{B}$ buyers and $\widetilde{S}$ sellers can be expressed as the sum $\widetilde{S}\overline{v} + \sum_{i=1}^{\widetilde{B}} \beta_i \mu^{i:\widetilde{B}}(\sigma^2)$, where $\beta_i \in \mathbb{R}$ and $\sum_{i=1}^{\widetilde{B}} \beta_i \mu^{i:\widetilde{B}}(\sigma^2) \geq 0$.*

**Proof.** The Appendix provides all proofs not provided in the text.

Second, we show that the expectation of an order statistic has a particularly simple relationship, homogeneous of degree one, to the variance of buyers' valuations.

**Lemma 2.** *For $B$ draws from distribution $F$ with mean $0$ and variance $\sigma^2$, the expectation of the $k^{th}$ order statistic, $\mu^{k:B}(\sigma^2)$, for all $k \leq B$, is homogeneous of degree one in $\sigma^2$.*

With these two results we can readily see that expected gains from trade in a network are always increasing in the variance of buyers' valuations. An implication is that networks will generally yield greater welfare in industries where firms face larger idiosyncratic demand shocks.

$\frac{2}{3}\mu^{2:4} + \frac{1}{3}\mu^{3:4} - 4c - 2\alpha.$

**Proposition 1.** *In any network, the expected ex post gains from trade are increasing in the variance of buyers' idiosyncratic shocks.*

Our next proposition shows when networks are efficient industrial structures. In any industry of $B$ buyers, if $\sigma^2 > 0$, there are sufficiently link costs, $c$, and intermediate levels of investment costs, $\alpha$, such that a network yields greater welfare than either markets or vertical integration (even when vertical integration yields positive welfare). Moreover, these ranges of costs where networks are efficient expand as $\sigma^2$ increases.

**Proposition 2.** *In any industry where buyers face firm-specific demand shocks, i.e., $\sigma^2 > 0$, there are investment costs $\overline{\alpha} > \overline{v} > \underline{\alpha}$ and $\overline{c} > 0$ such that a network is the efficient industrial structure for $c \leq \overline{c}$ and $\overline{\alpha} \geq \alpha \geq \underline{\alpha}$. Furthermore, $\overline{\alpha}$ and $\overline{c}$ are increasing in $\sigma^2$, and $\underline{\alpha}$ is decreasing in $\sigma^2$.*

To illustrate, notice that $W(\mathcal{N}_1) \geq W(\mathcal{V})$ when $\alpha$ is above the critical investment cost

$$\underline{\alpha}(\sigma^2) \equiv \overline{v} - \frac{1}{2}\left[\mu^{1:4}(\sigma^2) + \mu^{2:4}(\sigma^2)\right] + 3c.$$

For $\sigma^2 > 0$ and $c$ sufficiently small, $\underline{\alpha}(\sigma^2) < \overline{v}$, implying that even when vertical integration is profitable, $\mathcal{N}_1$ yields greater welfare. For higher $\alpha$, by sharing the capacity of suppliers in networks, buyers can do better than by forgoing specific inputs altogether (unless, of course, investment costs are very high). $W(\mathcal{N}_1) \geq W(\mathcal{M})$ when $\alpha$ is smaller than the critical investment cost

$$\overline{\alpha}(\sigma^2) \equiv \overline{v} + \frac{1}{2}\left[\mu^{1:4}(\sigma^2) + \mu^{2:4}(\sigma^2)\right] - 3c.$$

It is clear that $\overline{\alpha}(\sigma^2) > \underline{\alpha}(\sigma^2)$, for $c$ sufficiently small, and this range is increasing in $\sigma^2$.

**Network Density: Link Costs vs. Flexibility** The welfare comparison of vertical integration, networks and markets also depends on network *density*. Networks with fewer links may yield greater welfare when $c$ is high. There is less flexibility in input allocation but there is a savings of link costs. Formally, we say a network $\mathcal{N}'$ is less *dense* than a network $\mathcal{N}$ when $\mathcal{N}'$ is a subgraph of $\mathcal{N}$ (i.e., removing links from $\mathcal{N}$ yields $\mathcal{N}'$).

To illustrate, consider $W(\mathcal{N}_2)$ and $W(\mathcal{N}_1)$. $\mathcal{N}_2$ is less dense than $\mathcal{N}_1$. In $\mathcal{N}_2$ all the buyers are in fact single sourcing. $W(\mathcal{N}_2) \geq W(\mathcal{N}_1)$ when $c$ exceeds the critical link cost

$$\underline{c}(\sigma^2) \equiv \frac{1}{6}(\mu^{2:4} - \mu^{3:4});$$

i.e., when the savings in link cost exceeds the losses from allocating an input to the buyer with the third rather than second highest valuation in some events.[28] Here we see directly how demand uncertainty creates an economies of scale. As in the "repairman problem" (Feller (1950), Rothschild and Werden (1979)), one four-buyer-two-seller network yields greater gains from trade than two two-buyer-one-seller networks. The inputs in the combined network may be more efficiently allocated to the four buyers.[29] Since $\mu^{2:4}(\sigma^2)$ and $\mu^{3:4}(\sigma^2)$ are homogeneous of degree one in $\sigma^2$, as $\sigma^2$ increases, $\mu^{2:4} - \mu^{3:4}$ increases, and the ability to allocate inputs to the buyer with the second highest valuation becomes more important.[30]

In general, we show that the difference between the welfare of any network and a less dense network is increasing in the variance of buyers' idiosyncratic shocks. The result implies that networks should tend to be more densely linked when firm-specific shocks in an industry are high.

**Proposition 3.** *The difference between the welfare of any network and a less dense network is increasing in the variance of buyers' idiosyncratic shocks.*

Propositions 1, 2, and 3 together describe a strong connection between idiosyncratic demand shocks and the efficiency of networks. The example concretely shows this connection, and Figure 3 summarizes the welfare comparison of the four industrial structures as a function of the cost of productive assets, $\alpha$, and the cost of links, $c$, for a given variance $\sigma^2$ of buyers' valuations.[31]

[28] See footnote 28 for calculation of $W(\mathcal{N}_2)$.

[29] The differences between these networks are also reflected in the comparison between $\mathcal{N}_2$ and vertical integration. We see that $W(\mathcal{N}_2) \geq W(\mathcal{V})$ is $\alpha > \overline{v} - \left[\frac{1}{2}\mu^{1:4} + \frac{1}{3}\mu^{2:4} + \frac{1}{6}\mu^{3:4}\right] - 2c$. Compare this to the inequality $W(\mathcal{N}_1) \geq W(\mathcal{V})$.

[30] Comparing $W(\mathcal{N}_1)$ and $W(\mathcal{N}_2)$ also shows that, while adding links increases the surplus from exchange in a network, there are diminishing returns to adding links. Adding a link to a network with few links increases surplus more than adding a link to a network with a greater number of links (given the existing links are arranged efficiently). Removing a link from $\mathcal{N}_2$ reduces the surplus of exchange by $\frac{1}{2}(\mu^{1:4} - \mu^{4:4}) + \frac{1}{6}(\mu^{2:4} - \mu^{3:4})$ whereas removing a link from $\mathcal{N}_1$ reduces the surplus of exchange by only $\frac{1}{6}(\mu^{2:4} - \mu^{3:4})$.

[31] $\underline{\alpha}(\sigma^2)$, $\overline{\alpha}(\sigma^2)$, and $\underline{\alpha}(\sigma^2)$ at $c = 0$ can be derived from the formulas in the text above. At $c = \overline{c}(\sigma^2) = \frac{1}{2}\left[\mu^{1:2}\right]$ and $\alpha = \overline{v}, W(\mathcal{M}) = W(\mathcal{V}) = W(\mathcal{N}_2)$. The equations for all boundaries of the regions are provided in the Appendix.

networks are more often the socially preferred industrial structure.[32]

## 5. Strategic Firms and Industrial Structure

In this section we consider whether strategic firms, acting in their own self-interests, will form efficient industrial structures. We analyze a two-stage non-cooperative game. In the first stage, firms invest in productive capacity and links. In the second stage, production and exchange takes place. This stage represents the possibly many period returns to first-stage investments.

This formulation implicitly assumes that firms cannot use long-term contingent contracts to assign investments, future prices, or allocations of goods. It thus embodies the now standard Grossman and Hart (1986) and Hart and Moore (1990) incomplete contracts framework: agents must make investments before uncertainty is resolved and contingent contracts are not possible. Rather, firms make their first-stage investment keeping in mind how their decisions will affect their ability to obtain inputs and their competitive or bargaining positions in the second-stage.

In this situation, individual investment incentives are not necessarily aligned with economic welfare. Vertically integrated buyers, of course, need not worry about bargaining and hold-up. But in networks, the nature of the second-stage competition for inputs and the division of surplus from exchange will influence firms' investment decisions. A buyer's investment in a link to a seller is a specific investment, and the buyer must concern itself with the possiblity of hold-up. A seller's investment in a productive asset is quasi-specific, and it also must be concerned with obtaining a return sufficient to justify its investment.

We consider how two formulations of second stage revenues in networks affect investment incentives. The first is firms' Shapley values. The Shapley value captures the notion of equal bargaining power; a buyer and seller share equally the gains from their relationship. It is a weighted average of a firm's contribution to all possible groups (coalitions) of firms and is thus also a standard way to associate "bargaining power" with an agent's position in a network (Aumann and Myerson (1988)).

[32] Comparing all possible industrial structures for four buyers and $S = \{1, 2, 3\}$ sellers, we find the same general tradeoffs between vertical integration, networks, and markets. The Appendix provides the complete characterization of the efficient industrial structures for four buyers for given ranges of parameter values. Compared to Figure 3, the area in which networks are efficient is larger (because more networks are considered), and the area is more finely subdivided into networks with greater or fewer sellers.

The second formulation of revenues derives from a common representation of competition in pairwise settings. As in the "assignment games" (e.g. marriage problems) studied by Shapley and Shubik [1979] and Roth and Sotomayer [1988], we consider revenues that are pairwise stable: no linked buyer and seller can strike a deal that would make both better off.[33] We consider, in particular, the stable payoffs that give buyers the highest possible level of surplus. These revenues are equivalent to those that would arise in an ascending-bid auction model of competition [Demange and Gale (1988), Kranton and Minehart (1998a,b)]. As in a competitive market with a "Walrasian auctioneer," this formulation of revenues emphasizes the interaction of supply and demand in a network.

These formulations provide theoretical benchmarks of how competition for inputs and bargaining can affect firms' investment decisions. In the competitive framework, with its emphasis on supply and demand, a buyer earns the marginal value of its participation in a network. The Shapley value, in contrast, gives each firm a share of the inframarginal gains from trade. We will see that these very different divisions of surplus lead to different predictions as to whether an efficient industrial structure will emerge.

### 5.1. The Game

There are $B$ buyers, $S$ specialized sellers, and a competitive fringe of standardized sellers who supply the standardized input at a price of 0.

Stage One: Buyers simultaneously choose to build exclusive productive capacity, to form links with specialized sellers, or to procure standardized inputs in the market. A buyer incurs a cost $\alpha$ if it vertically integrates, and incurs a cost $c$ for each link to an independent seller. At the same time each of the $S$ sellers chooses whether or not to invest in a productive asset, incurring a cost $\alpha$ if it does. These actions yield an industrial structure $\mathcal{G}$ which is observable to all players.

Stage Two: Buyers' valuations of goods are realized and in the simplest case observed by all players.[34] Production and exchange takes place. Firms earn revenues which we express by a reduced form *revenue rule*. For a given realization of buyers' valuations, let $r_i^b(\mathbf{v}, \mathcal{G})$ be buyer $i$'s revenues in industrial structure $\mathcal{G}$, and let $r_j^s(\mathbf{v}, \mathcal{G})$ be the revenue of the independent seller

[33] Kranton and Minehart (1998b) provides a full analysis of competition in a network as an assignment game.

[34] In the analysis below, we note where the results extend to the case that buyer's valuations are private information.

$j$.[35] We make several assumptions on the revenue rule. A vertically integrated buyer earns $r_i^b(\mathbf{v}, \mathcal{G}) = v_i$. A buyer who procures a market input earns $r_i^b(\mathbf{v}, \mathcal{G}) = 0$. The rest of the firms are in networks, and we assume that the surplus a network generates is fully distributed to its constituent firms.[36] Both revenue rules we consider for networks satisfy this property.

A firm's expected profits in the game are its second-stage expected revenues minus its first stage investment costs. Let $\Pi_i^b(\mathcal{G}) \equiv E_{\mathbf{v}}\left[r_i^b(\mathbf{v}, \mathcal{G})\right] - \alpha \cdot v_i(\mathcal{G}) - c \cdot l_i(\mathcal{G})$ be the expected profits of buyer $i$, and let $\Pi_j^s(\mathcal{G}) \equiv E_{\mathbf{v}}\left[r_j^s(\mathbf{v}, \mathcal{G})\right] - \alpha \cdot \kappa_j(\mathcal{G})$ be seller $j$'s profits.

This game is effectively a (one-stage) simultaneous-move game. The graph $\mathcal{G}$ summarizes the firms' strategies, and the profits $\Pi_i^b(\mathcal{G})$ and $\Pi_j^s(\mathcal{G})$ give firms' payoffs for each profile. We solve for pure-strategy Nash equilibria. In equilibrium, each firm's investments maximize its profits, given the investments of other firms.[37]

### 5.2. Equilibrium Industrial Structures

It is easy to support equilibria in which either all buyers are vertically integrated ($\mathcal{V}$) or all buyers are in the market ($\mathcal{M}$). Markets and vertical integration do not require any coordination; a buyers' payoffs from either are independent of the actions of other firms. One of these two structures, therefore, is always an equilibrium.

**Proposition 4.** *When $\overline{v} \geq \alpha$ vertical integration ($\mathcal{V}$) is an equilibrium outcome and when $\overline{v} \leq \alpha$ a market ($\mathcal{M}$) is an equilibrium outcome.*

**Proof.** Given all buyers are vertically integrated and no seller has invested in a productive asset: (1) no buyer has an incentive to deviate to a market if and only if $\overline{v} \geq \alpha$, (2) no buyer has an incentive to deviate and establish a link to a seller. The same argument holds for a market for $\alpha \leq \overline{v}$. ■

[35] The sellers in the competitive fringe always get 0. We do not specify a revenue rule for them.

[36] Formally, we require that the revenue rule to be *component balanced* (Jackson and Wolinsky (1996)). That is, the revenue rule distributes all the surplus from each maximally connected subgraph to nodes in that subgraph.

[37] Formally, for an industrial structure $\mathcal{G}$, let $\mathcal{G}'_k$ be an industrial structure that differs from $\mathcal{G}$ only by firm $k$'s investments. We say an industrial structure $\mathcal{G}$ is an *equilibrium structure* if and only if for each buyer $i$ and there does not exist a structure $\mathcal{G}'_i$ such that $\Pi_i^b(\mathcal{G}'_i) > \Pi_i^b(\mathcal{G})$ and for each seller $j$ there does not exist a structure $\mathcal{G}'_j$ such that $\Pi_j^s(\mathcal{G}'_j) > \Pi_j^s(\mathcal{G})$.

For both vertical integration and market procurement, a buyer's expected profits are also exactly its contribution to economic welfare ($\overline{v} - \alpha$ for vertical integration, 0 for a market). Therefore, vertical integration is the unique equilibrium outcome when it is efficient. The same is true for a market.

**Proposition 5.** *When the industrial structure $\mathcal{V}$ or $\mathcal{M}$ is efficient, it is also the unique equilibrium outcome (up to welfare equivalence).*[38]

**Proof.** In the industrial structure $\mathcal{V}$, each buyer earns $\overline{v} - \alpha$ and welfare is $B \cdot (\overline{v} - \alpha)$. When $\mathcal{V}$ is efficient, any alternative non-welfare equivalent industrial structure $\mathcal{I}$ generates a strictly smaller welfare. It follows that at least one of the non-integrated buyers profits are strictly less than $\overline{v} - \alpha$ in $\mathcal{I}$. This buyer would do better to vertically integrate because then it earns $\overline{v} - \alpha$. Therefore $\mathcal{I}$ is not an equilbrium. A similar argument holds for markets. ■

Firms may not form efficient networks for exactly the opposite reasons. First, in networks firms' payoffs depend on the investments of the other firms, so coordination failure is possible. If too few buyers, for example, invest in links to sellers, then a particular network will not be an equilibrium even when it is efficient. Second, a buyer's or seller's individual payoffs may not match its contribution to economic welfare.

We see this latter problems in the equilibrium conditions, or *stability conditions,* for a network. We examine when the following conditions are met under the two formulations of firms' revenues and how these conditions relate to the welfare benefits of networks. Given other firms' investments, a network is an equilibrium outcome if and only if (1) no buyer can earn greater profits from either vertical integration or markets (2) no seller can earn greater profits by not investing in an asset, and (3) no buyer has an incentive to change its links within the network. For a network industrial structure $\mathcal{G}$, these conditions are, in turn,

$$\Pi_i^b(\mathcal{G}) \geq \max\{\overline{v} - \alpha, 0\} \tag{1}$$

for all buyers $i$;

$$\Pi_j^s(\mathcal{G}) \geq 0 \tag{2}$$

[38] Two industrial structures are *welfare equivalent* if they generate the same economic welfare. For instance, in the degenerate case that $\overline{v} = \alpha$, $\mathcal{V}$ and $\mathcal{M}$ are welfare equivalent because both yield 0 economic welfare. If, in addition, prohibitively expensive links rule out a network alternative, then $\mathcal{V}$ and $\mathcal{M}$ are both equilibrium oucomes.

for all sellers $j$; and

$$\Pi_i^b(\mathcal{G}) \geq \Pi_i^b(\mathcal{G}_i') \tag{3}$$

for each buyer $i$ and all graphs $\mathcal{G}_i'$ where $\mathcal{G}_i'$ differs from $\mathcal{G}$ only in the links of buyer $i$.

In the first revenue rule we consider, firms earn their Shapley values. As mentioned above, this rule gives buyers and sellers *equal bargaining power*. Two agents sharing equally from their relationship reflects notions of "fairness" (Myerson (1977)).[39] Formally, for a graph $\mathcal{G}$, let $\mathcal{G} - ij$ represent the graph $\mathcal{G}$ except for any link between buyer $i$ and seller $j$. We say a revenue rule satisfies the *equal bargaining power property* if and only if

$$r_i^b(\mathbf{v}, \mathcal{G}) - r_i^b(\mathbf{v}, \mathcal{G} - ij) = r_j^s(\mathbf{v}, \mathcal{G}) - r_j^s(\mathbf{v}, \mathcal{G} - ij)$$

for all $\mathcal{G}$, buyers $i$, sellers $j$, and all realizations of buyers' valuations $\mathbf{v}$. The Shapley value is the only revenue rule satisfying this property.[40] Because it is a weighted average of a firm's contribution to all possible groups (coalitions) of firms,[41] it is also considered a reasonable way to define "bargaining power" in a network.

While the equal bargaining power property may seem quite natural, Shapley values significantly distort firms' investment incentives. Intuitively, a firm's Shapley value is based on both the inframarginal and marginal value that it carries in a network. We will see that because of the role of the inframarginal value, all three stability conditions diverge from efficiency criterion. We illustrate with network industrial structure $\mathcal{N}_1$ from Figure 2 above.

The first stability condition requires the buyer to choose the network over market procurement and vertical integration. In vertical integration and market procurement, a buyer earns exactly its marginal contribution to economic welfare. The same must therefore also hold in a network in order that the buyer's overall incentives be exactly aligned with economic welfare. When the

[39]Many social norms governing splits of surplus from a relationship involve an equal split rule [Young (1998)].

[40]See Jackson and Wolinsky (1996). They extend a result of Myerson (1977) showing that the Shapley value is the only revenue rule which is both component balanced and satisfies the equal bargaining power property.

[41]The revenue rule which gives the Shapley value for buyer $i$ is

$$r_i^b(\mathbf{v}, \mathcal{G}) = \sum_C [w(A^*(\mathbf{v}|_{C+b_i}, \mathcal{G}|_{C+b_i})) - w(A^*(\mathbf{v}|_C, \mathcal{G}|_C))] \frac{|C|!((S+B) - |C| - 1)!}{(S+B)!},$$

where $C$ is a set of firms, $\mathbf{v}|_C$ are the valuations of the buyers restricted to $C$ and $\mathcal{G}|_C$ is the industrial structure restricted to investments of the firms in $C$. Buyer $i$'s Shapley value is then the expected revenues according to this rule $E_{\mathbf{v}}[r_i^b(\mathbf{v}, \mathcal{G})]$. We have a similar formular for a seller $j$.

revenue rule is given by the Shapley value, however, this is not the case. In $\mathcal{N}_1$, the Shapley value for buyer 1 (and 4) yields the following expected revenues:

$$E_{\mathbf{v}}[r_1^b(\mathbf{v},\mathcal{N}_1)] = \frac{7}{60}\overline{v} + \frac{1}{6}\mu^{1:4} + \frac{47}{360}\mu^{2:4} - \frac{11}{360}\mu^{3:4}.$$

In contrast, the marginal contribution of buyer 1 to surplus from exchange is $\frac{1}{2}\left[\mu^{1:4} - \mu^{3:4}\right]$.[42]

In general, the Shapley value may be greater or less than a buyer's marginal contribution to a network. The Shapley value is based on inframarginal contributions which bear no simple relationship to the marginal contribution.[43] In this above example, buyer 1's Shapley value is less than the marginal contribution.[44] This reflects the fact that buyer 1 has only one link. Links to sellers are important for bargaining power under the Shapley value, and firms with relatively few links tend to be undercompensated relative to what they add to social welfare. Because buyer 1 earns less than its marginal contribution to $\mathcal{N}_1$, it will sometimes prefer vertical integration or market procurement even when $\mathcal{N}_1$ is socially preferred.

The second stability condition requires that sellers in the network invest in productive assets. In the network $\mathcal{N}_1$, the seller's marginal contribution to surplus is $\overline{v} + \frac{1}{4}\mu^{1:4} + \frac{3}{4}\mu^{2:4}$.[45] Taking other firms' investments as given, the seller should invest if this marginal contribution exceeds $\alpha$. Calculations show that the Shapley value gives expected revenues of:

$$E_{\mathbf{v}}[r_i^s(\mathbf{v},\mathcal{N}_1)] = \frac{43}{60}\overline{v} + \frac{2}{15}\mu^{1:4} + \frac{5}{24}\mu^{2:4} + \frac{7}{120}\mu^{3:4}.$$

Again, there is no general relationship between the Shapley value and a seller's marginal contribution. In this example, the Shapley value is strictly less than the seller's marginal contribution.[46] So the seller has insufficient incentive to invest in the productive asset.

[42] The difference in surplus from exchange when buyer 1 is in the network and when buyer 1 is not is $\mu^{1:4} + \mu^{2:4} - \left[\mu^{1:3} + \mu^{2:3}\right]$. Using the triangle rule, this simplies to $\frac{1}{2}\left[\mu^{1:4} - \mu^{3:4}\right]$

[43] The Shapley value involves inframarginal value as measured by a firm's contributions to coalitions of firms other than the grand coalition. A firm's marginal contribution is its contribution to the grand coalition.

[44] Formally, we need two mild restrictions on a buyer's idiosyncratic shock: (1) the distribution of $\varepsilon_i$ is symmetric around 0, and (2) $\mu^{1:4} > \frac{1}{2}\overline{v}$.

[45] Each seller's marginal contribution to surplus from exchange in $\mathcal{N}_1$ is $\overline{v} + \mu^{1:4} + \mu^{2:4} - \mu^{1:3}$. The seller allows an additional unit to be sold. With only one seller, one input would be sold to a buyer with the highest valuation out of three. With both sellers in the network, two inputs are sold to the buyers with the two highest valuations. Using the triangle rule, the marginal contribution simplifies to $\overline{v} + \frac{1}{4}\mu^{1:4} + \frac{3}{4}\mu^{2:4}$.

[46] Formally, we need $\varepsilon_i$ to be symmetric about 0 for this.

The third stability condition requires that buyers maintain their links inside a network. Again we find that a buyer's return to a link does not match the welfare value of the link. One problem is that a link can increase the bargaining power of a buyer, even if it does not add welfare to the network. In the network $\mathcal{N}_1$, for example, buyer 1 has a higher Shapley value if it adds a link to seller 2.[47] On the flip side, a buyer must share the value of a link with the seller (equal bargaining power property). This sometimes means that a buyer would underlink.

In general, with the Shapley revenue rule the region where a network is both stable and efficient is restricted by all three stability conditions.

We now consider stability and efficiency for our second formulation of revenues, which we call the *competitive revenue rule*. As mentioned above, this rule is of interest because revenues are pairwise stable and a focal outcome of an assignment game for the network setting. Also, competition between buyers and sellers in a network can be represented as an ascending-bid, or English, auction. Ascending-bid and Vickrey auctions are known to have many efficiency properties, particularly when buyers' valuations are private information.[48] Both the auction and assignment games capture a competitive environment where the interaction between supply and demand determines final revenues.[49]

The supply and demand character of these revenues can be seen most easily in the auction formulation. In a network, suppose sellers simultaneously hold ascending-bid auctions; that is, the price rises at the same time in each auction. Buyers can bid only in the auctions of their linked sellers. The price rises until demand no longer exceeds supply for some subset of sellers. These sellers then sell their goods at that price, and the price continues to rise until all sellers have sold their output. In this auction it is an equilibrium following elimination of weakly dominated

[47]Since $\mathcal{N}_1$ is allocatively complete, there is no welfare gain from adding links. However, buyers 1 (and 4) have an incentive to add links. Let $\mathcal{N}_1'$ denote the structure that arises when a link between buyer 1 and seller 2 is added to $\mathcal{N}_1$. The difference in buyer 1's Shapley value is:

$$\frac{12}{360}\overline{v} + \frac{9}{360}\mu^{1:4} + \frac{35}{360}\mu^{2:4} - \frac{2}{360}\mu^{3:4} > 0$$

The link changes the Shapley value, because it increases buyer 1's contribution to small coalitions of firms such as for instance the coalition consisting of seller 2 alone.

[48]For instance, in many cases where bidders' valuations are private information, a Vickrey auction ensures that buyers' valuations are revealed and goods are allocated efficiently.

[49]See Kranton and Minehart (1998a) for auction details and proofs and Kranton and Minehart (1998b) for analysis of the assignment game.

strategies for each buyer to remain in the bidding of its linked sellers' auctions until the price reaches its valuation of an input.[50]

To see this outcome, consider the network $\mathcal{N}_1$ and suppose buyers' idiosyncratic shocks are realized in the following order: $\varepsilon_1 > \varepsilon_2 > \varepsilon_3 > \varepsilon_4$. The price rises until $p = v + \varepsilon_4$, when buyer 4 drops out of the bidding. Buyers 1, 2, and 3 remain in the bidding for the two sellers' goods and so demand for these goods exceeds their supply. At $p = v + \varepsilon_3$, buyer 3 drops out of the bidding. The two sellers are now collectively linked to only two buyers, and there is an allocation in which each buyer procures a good. So both auctions clear at the common price $p = v + \varepsilon_3$. The price $p$ is the lowest price such that supply for a subset of sellers' goods equals the demand.

We derive the competitive revenue rule from from the prices and allocations that emerge from the auction or equivalently from the assignment game. For every graph and every realization of buyers' valuations, there is an efficient allocation of goods, and prices determine the split of surplus between buyers and sellers.

With these competitive revenues, a buyer's expected payoff exactly equals its marginal contribution to the network [Kranton and Minehart (1998a)]. Buyers do not earn any of the inframarginal surplus. Rather, a buyer who obtains an input earns the difference between its valuation and the valuation of the "next-best" buyer. In other words, the buyer earns the opportunity cost of obtaining for the good. In the example above, buyer 1 paid a price $v + \varepsilon_3$ which is the surplus that would have accrued had it not purchased an input.[51]

An immediate implication is that the network stability conditions for buyers are aligned with economic welfare. That is, buyers make the efficient choice between vertical integration, networks, and markets, and if a buyer participates in a network, it chooses its links efficiently given the investments of the other firms. We have:

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[50]See Kranton and Minehart (1998a), Proposition 2.

[51]The revenue rule (which is given by expected payoffs from this equilibrium of the auction) is straightforward to calculate. In $\mathcal{N}_1$, the revenue rule for all $\mathbf{v}'s$ with the above ordering is:

$$r_1^b(\mathbf{v},\mathcal{N}_1) = \varepsilon_1 - \varepsilon_3 \quad r_2^b(\mathbf{v},\mathcal{N}_1) = \varepsilon_2 - \varepsilon_3 \quad r_3^b(\mathbf{v},\mathcal{N}_1) = 0 \quad r_4^b(\mathbf{v},\mathcal{N}_1) = 0$$
$$r_1^s(\mathbf{v},\mathcal{N}_1) = v + \varepsilon_3 \quad r_2^s(\mathbf{v},\mathcal{N}_1) = v + \varepsilon_3$$

The expectation of the revenue rule is :

$$E_{\mathbf{v}}[r_i^b(\mathbf{v},\mathcal{N}_1)] = \tfrac{1}{4}(\mu^{1:4} - \mu^{3:4}) + \tfrac{1}{4}(\mu^{2:4} - \mu^{3:4}) \quad E_{\mathbf{v}}[r_j^s(\mathbf{v},\mathcal{N}_1)] = \overline{v} + \mu^{3:4}$$

**Proposition 6.** *When firms' revenues are given by the competitive revenue rule, buyers make first-stage choices optimally given the choices of the other firms.*

Unfortunately, sellers' investment incentives are not perfectly aligned with economic welfare. They earn less than their marginal contribution to the network. A seller's marginal contribution equals the valuation of the additional buyer who obtains a good. A seller's revenues, however, are the valuation of the "next-best" buyer of the good. For example, in network $\mathcal{N}_1$ each seller earns expected revenues of $\overline{v} + \mu^{3:4}$. This is less than the seller's marginal contribution which we have shown to be $\overline{v} + \frac{1}{4}\mu^{1:4} + \frac{3}{4}\mu^{2:4}$.

Under the competitive revenue rule, the region where a network is both stable and efficient is therefore restricted only by the seller's stability condition. Sellers do not always have sufficient bargaining power vis buyers to cover their investment in the productive asset. This means that sellers tend to underinvest.

This outcome is a form of team-agency problem and a consequence of the incomplete contracting framework. Any balanced revenue rule which gives buyers the correct incentives to invest in a seller-specific assets will not provide adequate incentives for the sellers. On the other hand, giving sellers greater surplus, as in the pairwise stable payoffs that are optimal for the sellers, will improve sellers' incentives to invest in quasi-buyer-specific assets but distort buyers' incentives. Of course, if firms could write complete contingent contracts such a conflict would not arise.[52]

With competitive revenues there is a particularly simple relationship between demand uncertainty, stability and efficiency. For a network $\mathcal{N}$, let $E$ be the ranges of costs $(\alpha, c)$ for which $\mathcal{N}$ is efficient,and let $E_S$ be the ranges of investment costs $(\alpha, c)$ for which $\mathcal{N}$ is efficient and stable. Let $\overline{E}$ and $\overline{E_S}$ denote the areas corresponding to these ranges. As the variance in the idiosyncratic shocks to demand increase, networks become more efficient relative to vertical integration and markets. Intuitively, then both $\overline{E}$ and $\overline{E_S}$ should be increasing in $\sigma^2$. We find that these areas do increase in $\sigma^2$, and in fact increase proportionately:

**Proposition 7.** *$\overline{E_S}(\sigma^2)$ and $\overline{E}(\sigma^2)$ are increasing in $\sigma^2$, and the ratio $\overline{E_S}(\sigma^2)/\overline{E}(\sigma^2)$ is constant with respect to $\sigma^2$.*

**Proof.** We prove in the Appendix that both areas $\overline{E}$ and $\overline{E_S}$ are homogeneous functions of degree 2 in the variance. Therefore, the ratio does not depend on $\sigma^2$. ■

[52]In Section 6, we will ask how a buyer's ownership of a flexible productive asset might mitigate this problem.

### 5.3. Second-Best Equilibrium Networks

Despite the problems in achieving an efficient network in equilibrium, network equilibria exist and these networks are always welfare enhancing. Because of limitations on long-term contracting, the above results show that when the efficient industrial structure involves a network, the structure need not be an equilibrium outcome. The next propositions show, however, that there always exist network equilibria in industries where buyers face idiosyncratic shocks. Furthermore, these network equilibria always yield greater welfare than vertical integration and markets. That is, an equilibrium network structure, while not first-best, is second-best.

With the competitive revenue rule, network equilibria exists for any $B \geq 3$ and $\sigma^2 > 0$. The result mirrors Proposition 2 illustrated in Figure 3. Networks are equilibria for small link costs and capacity investment costs in an intermediate range. Moreover, the greater $\sigma^2$, the greater the range of $\alpha$ and $c$ where networks are equilibria. As $\sigma^2$ increases, the area where network equilibria exists unambiguously increases.

**Proposition 8.** *With the competitive revenue rule, for any $\sigma^2 > 0$ and $B \geq 3$, there exist critical costs: $c^* > 0, \underline{\alpha}^* < \overline{v}$, and $\overline{\alpha}^* > \overline{v}$ such that a network equilibrium exists for all $\{c, \alpha\}$ satisfying $c \leq c^*$ and $\overline{\alpha}^* \geq \alpha \geq \underline{\alpha}^*$. Furthermore, the equilibrium set of $\{c, \alpha\}$ pairs is expanding in $\sigma^2$; $c^*$ and $\overline{\alpha}^*$ are increasing in $\sigma^2$, while $\underline{\alpha}^*$ is constant.*

This result is robust to our other formulation of revenues. With the Shapley revenue rule, network equilibria exist for $\sigma^2$ and $B$ sufficiently high and link costs $c$ sufficiently low. A sufficiently high variance and number of buyers ensures that there is enough surplus from in the network so that all equilibrium conditions are simultaneously satisfied.

**Proposition 9.** *With the Shapley revenue rule, there exists network equilibria for a sufficiently high $B$ and $\sigma^2$. In this case, there exist critical costs: $c^* > 0, \underline{\alpha}^* < \overline{v}$, and $\overline{\alpha}^* > \overline{v}$ such that a network equilibrium exists for all $\{c, \alpha\}$ satisfying $c \leq c^*$ and $\overline{\alpha}^* \geq \alpha \geq \underline{\alpha}^*$.*

Not only do network equilibria exist, but these equilibrium structures always yield greater welfare than vertical integration or markets. Proposition 5 tells us that when vertical integration (or a market) is the efficient industrial structure, it is the unique equilibrium outcome. The proof also implies that when a network industrial structure is an equilibrium outcome, it must yield higher welfare than vertical integration or markets. Otherwise, a buyer would deviate and build its own exclusive productive capacity. Thus, despite the inefficiencies that arise from incomplete contracting, firms may form welfare-enhancing, disintegrated industrial structures.

Network equilibria counter the standard intuition that buyers have stronger incentives to make specific investments when they are vertically integrated with sellers (Grossman and Hart (1986), Hart and Moore (1990), Joskow (1985), Monteverde and Teece (1982) ). In networks, ex post bargaining may balance payoffs in such a way that firms do wish to undertake these investments. Under the competitive revenue rule, for instance, we found that buyers always build links efficiently. Moreover, in networks buyers make many specific investments, possibly even more than under vertical integration. In building links to multiple sellers, a buyer in a sense "duplicates" its specific investments. This multiplicity allows buyers to pool the uncertainty in their valuations. It also allows a savings on overall investment costs; buyers share the productive capacity of fewer sellers.

## 6. Vertical Merger in Networks

Because network equilibria may be second-best, the question arises of whether alternative ownership structures might improve on network welfare. In this section we consider an ownership structure where a buyer in a network may be merged with a flexible seller. We ask whether such a mixed ownership structure improves on investment incentives.[53]

We build on our basic model as follows. Consider a network with $S$ upstream units and $B$ downstream units. The value of an input produced by a downstream unit $i$ linked to an upstream unit that has invested in flexible productive capacity is $v_i$. In the first stage of the game, owners of the units decide whether or not to invest in links and/or flexible productive capacity. In the second stage, valuations are realized and production and exchange take place. We assume that second-stage revenues accrue to each unit according to the competitive revenue rule. These revenues then accrue to the units' owners. With this rule, asset use is efficient.[54] By fixing the

[53]One possibility is for all the units to be under common ownership. Indeed, complete integration must lead to efficient investments. However, we take the position that complete integration is either illegal because of antitrust considerations or suboptimal for unmodeled reasons such diminishing returns to managerial effort.

[54]Regardless of the ownership structure, asset use will be efficient if the owner of an upstream asset produces an input for a linked downstream asset whenever it is efficient to do so. We can see this easily in the auction

revenue rule, we are able to identify the changes in investment incentives that come from a change in ownership structure.[55]

Consider the network $\mathcal{N}_1$ with four buyers and two sellers shown in Figure 2. We compare the equilibrium conditions to the efficiency conditions for this industrial structure under two different ownership structures. The first is the ownership structure previously analyzed. The second is the same except buyer 2 owns seller 1.

First, consider the decentralized structure. By Proposition 7, the buyers make link investments optimally given the choices of other firms. However sellers earn expected revenues that are less than the seller's marginal contribution.[56] Because of this shortfall, network $\mathcal{N}_1$ may fail to be an equilibrium when it is efficient.

Consider next the ownership structure in which buyer 2 owns seller 1. We will refer to this merged entity as M. We ask whether M has a greater incentive than seller 1 to invest in the flexible asset, taking the other investments as given.[57] We find that M's incentive to invest in $\alpha$ is indeed higher, because both the upstream and downstream unit earn returns from use of the productive capacity.[58] Thus, merger mitigates the team agency problem for the seller. We next consider M's incentive to invest in the "internal" link between buyer 2 and seller 1. As in Bolton and Whinston (1993, Proposition 4.1, part (3), p. 134), the incentive to invest in this link is inefficiently strong.[59] In events where M sells the input of seller 1 to buyer 3, M receives a higher

formulation of the competitive revenue rule. The owner of downstream unit $i$ with a link to its own upstream asset will produce an input for a linked downstream asset $j$ when $j$ is willing to pay a price higher than $v_i$.

[55]A different approach would have the revenue rule depend on ownership structure. These revenue rules could derive from an extensive form game of competition where the game itself changes when the ownership structure changes. For example, one might imagine a richer strategy space in an ascending-bid auction where a buyer and seller under the same ownership might collude. We have avoided this approach for two reasons. First, it is difficult to associate different revenue processes with different ownership structures in a consistent and meaningful way. Second, the results about investment incentives would be harder to interpret, since they would depend on both the change in ownership and the change in the revenue process.

[56]See the discussion following Proposition 7.

[57]We take all of buyer 2's link investments as given, including the link to seller 1. This is because we want to change one investment at a time. Even if we assume that the owner does not build a link to a noninvesting seller, we still find that the incentive to invest in $\alpha$ is stronger under joint ownership.

[58]These calculations are available from the authors on request. They are not difficult.

[59]Bolton and Whinston develop a model where there are 2 downstream firms and one upstream firm. The downstream firms make welfare-enhancing investments in the upstream firm that are somewhat analagous to our

price when it has the link than when it does not. This strategic effect raises the value of the link to M above its productive value. Finally, we consider M's incentive to invest in an "external" link to seller 2. This link increases the payoff of buyer 2 but decreases the payoff of seller 1 who now sometimes loses buyer 2's business to seller 2. We find that this gain and loss exactly cancel: M has no incentive to build the link even though the link is welfare enhancing.[60]

This example reveals a new consequence of vertical merger in a multilateral setting. Buyers that own a network productive facility might not invest in relationships with other sellers, even when such links would be efficient. This result has no analogy in Bolton and Whinston's (1993) setting, because they do not consider the possibility of multiple investments by upstream firms. The example, then, demonstrates a general point that in a multilateral environment partial merger need not unambiguously improve investment incentives even for the merged firms. A possible direction for future research would be to use the network model developed here to systematically examine the interaction between ownership structure, industrial structure, and investments. Because the model allows for many different configurations of links, a wide range of implications of ownership changes may be easily captured.

## 7. Conclusion

This paper develops a theory of alternative industrial structures. We contrast markets, where buyers obtain standardized inputs, to vertical integration and networks, where buyers obtain specialized inputs made to order. Specialized input production requires investment in specific assets. Vertically integrated firms invest in their own supply facilities. In networks, buyers invest in links to specific sellers, who in turn invest in a quasi-buyer-specific assets. These assets make sellers "flexible specialists" who can produce specialized inputs to linked buyers' specifications.

We examine both the social and individual incentives to form these industrial structures. The welfare comparison depends on investment costs and demand uncertainty. We find that when buyers face sufficiently large idiosyncratic shocks and productive capacity is costly, dense networks are the efficient industrial structure. Buyers should have links to multiple sellers and share their

link investments. The upstream firm has a random capacity to produce 1 or 2 units. When the upstream firm can produce only one unit of input, their bargaining process gives the same division of payoffs as our competitive renvenue rule.

[60]The surplus added by the link is earned by other agents.

capacity. The links allow inputs to be allocated to the buyers with the highest valuations.

When contracts are incomplete, the efficient industrial structure might not be an equilibrium outcome. Vertically integrated firms' investment incentives are always aligned with economic welfare. Networks investment incentives need not be because ex ante investments can affect the ex post division of surplus. Despite these distorted investment incentives, however, we find that network equilibria are always second-best industrial structures.

These network equilibria might resolve part of the puzzle [see Holmstrom and Roberts (1998)] of why non-integrated firms are observed to make large specific investments. In the networks we analyze, buyers invest in assets specific to more than one, but not an unlimited number, of sellers. Both by making these investments and by limiting their number, buyers increase sellers' incentives to invest in assets that enhance the value of specialized inputs. Overall, there is a savings in investment costs and a welfare improvement.

The analysis has further implications for evaluating "real world" supply relations. If an industry is organized as a network, our results indicate that a network must be the efficient industrial structure. (If vertical integration were the efficient structure, networks would not be an equilibrium.) The network we observe may not be the first-best network, but it does yield greater welfare than vertical integration or markets. On the other hand, if an industry is vertically integrated (or a market), vertical integration (or market) is not necessarily the efficient structure. A network structure may be efficient but does not emerge because of distorted investment incentives or simple coordination failure.

This study also yields several predictions about differences in supply structures across industries when buyers face idiosyncratic shocks. First, when investments in quasi-specific productive assets are relatively inexpensive, an industry is more likely to have a vertically integrated structure. Second, for intermediate costs of quasi-specific assets an industry is more likely to have a network structure. This will be especially true when the links to sellers - communication, training, etc. - are not prohibitively expensive.[61] Finally, markets for standardized inputs may be an indication of prohibitively high costs of quasi-specific productive capacity for specialized inputs.

The analysis indicates that observed differences in supply structures may be due to different magnitudes of firm-specific shocks. Case studies show that, over time, changes in demand uncer-

[61]A related point is that reductions in the costs of links, such as enhancements in communication technologies, should increase the comparative advantage of networks.

tainty may affect supply arrangements. Helper, MacDuffie, and Sabel (1997) argue that starting in the late 1920's, there was an increase in uncertainty in the U.S. automobile industry because of competition from the emerging used car market and new independent manufacturers. The big automakers GM and Ford moved away from vertical integration to flexible, collaborative arrangements with independent suppliers. This trend to disintegration lasted through World War II, then reversed.[62] Similarly, Storper (1989) holds that volatility in the demand for Hollywood movies increased in the late 1940's with the advent of television, leading to a replacement of the vertically integrated studio system with outsourcing for many aspects of a film's production.[63] These studies suggest that disintegrated supply structures are a response to underlying environmental uncertainty.

Our analysis finds that high firm-specific demand shocks should be associated with a more network-like industrial structure. Greater variances of these shocks should be associated with a denser network structure. Finally, aggregate or industry-wide demand shocks may not be related to these differences in industrial structures, unless they affect the distributions of firm-specific shocks.

[62]The move towards vertical integration post WWII and towards networks in the 1980's is explained similarly. In the post WWII expansion, the automakers were more concerned with market expansion than with innovation. This concern reversed in the 1980's with the increase of foreign competition.

[63]See also Faulkner and Anderson (1987) and Aksoy and Robins (1992).

## 8. Appendix

### 8.1. Formal Notation for the Model

**Industrial Structures:** We use matrices to model an industrial structure. The graph $\mathcal{G}$ can be represented with two matrices, $\mathcal{G} \equiv (G, Z)$. The first matrix, $G$, is a $B \times (S+1)$ matrix $[g_{i,j}] \in \{0,1\}$, that represents the links and status of the buyers. When buyer $i$ has a link to seller $j$ $[g_{i,j}] = 1$, otherwise $[g_{i,j}] = 0$. When a buyer $i$ is a vertically integrated firm, $[g_{i,S+1}] = 1$, otherwise $[g_{i,S+1}] = 0$. If all elements of row $i$ are equal to zero, buyer $i$ is in the market for standardized inputs. $Z$ is a $(S+1) \times (S+1)$ matrix that represents the status of sellers. For seller $j$ that has invested in productive capacity, $[z_{j,j}] = 1$, otherwise $[z_{j,j}] = 0$. The last diagonal element $[z_{(S+1),(S+1)}]$ is equal to one. The off-diagonal elements of $Z$ are all equal to zero. The product $G \cdot Z$ is a $B \times (S+1)$ matrix which shows the number of vertically integrated firms, the links between buyers and sellers, and sellers' productive assets that can actually be used for exchange. By multiplying $G$ and $Z$, we "activate" buyers' links to sellers as well as buyers' investments in productive capacity. A link to a seller that has not invested in productive capacity, for example, will not be shown in $G \cdot Z$ but will be shown in $G$.

**Allocations:** An allocation is a $B \times (S+1)$ matrix, $A$, $[a_{ij}] \in \{0,1\}$, where $[a_{i,j}] = 1$ indicates that buyer $i$ obtains a good from seller $j$, and $[a_{i,S+1}] = 1$ indicates that a vertically integrated firm obtains a good from itself. We restrict attention to *feasible allocations of goods* - a buyer that is allocated a good from a particular seller must be linked to that seller, and no buyer (seller) is paired to more than one seller (buyer). That is, $A$ is a subgraph of $\mathcal{G} \cdot Z$, and for $[a_{i,j+1}] = 1$, we must have $[a_{ik}] = 0$ for all sellers $k \neq j$, and $[a_{lj}] = 0$ for all buyers $l \neq i$.

Given a vector $\mathbf{v}$ of buyers' valuations, the economic surplus of an allocation $A$ is $w(\mathbf{v},A)$. We can write $w(\mathbf{v},A) = \mathbf{v} \cdot A \cdot \mathbf{1}$, where $\mathbf{1}$ is an $(S+1)$ $\times 1$ matrix where each element is 1.

### 8.2. Proofs and Other Material

**Proof of Lemma 1.**

For notational convenience, consider an industrial structure $\mathcal{G}$ where all $B$ buyers and $S$ sellers are in a network. Consider any allocation $A$ in which every seller's input is allocated to a distinct buyer. If this allocation arose in every state of the world, the ex ante expected surplus would be $S\overline{v}$. In each state of the world, the efficient allocation $A^*(\mathbf{v},\mathcal{G})$ yields weakly higher surplus than $A$. Therefore, $E_{\mathbf{v}}\left[w(A^*(\mathbf{v},\mathcal{G})\right] \geq S\overline{v}$. For each realization of buyers' valuations $\mathbf{v}$, the efficient allocation depends only on the ordering of buyers valuations, not the level. Because of this we can write the surplus $E_{\mathbf{v}}\left[w(A^*(\mathbf{v},\mathcal{G})\right]$ as $S\overline{v}+\sum_{i=1}^{B}\beta_i\mu^{i:B}$ for some constants $\beta_i$. Finally $S\overline{v}+\sum_{i=1}^{B}\beta_i\mu^{i:B} \geq S\overline{v}$ implies that $\sum_{i=1}^{B}\beta_i\mu^{i:B} \geq 0$. ■

**Proof of Lemma 2.**

For any distribution $X \sim F(x)$ with density $f(x)$, with mean 0 and variance normalized to 1, we consider the family of distributions obtained by changing the variance by a factor $\sigma^2$.

Let $X_\sigma \sim F_\sigma(x)$ denote the $\sigma^{th}$distribution and let $f_\sigma(x)$ denote its density.

Formally:

$$f_\sigma(x) = \frac{1}{\sigma}f(\frac{x}{\sigma})$$

and

$$F_\sigma(x) = F(\frac{x}{\sigma})$$

The variance of the distribution $F_\sigma(x)$, $Var_\sigma(x)$, is then

$$\begin{aligned}
Var_\sigma(x) &= \int_{-\infty}^{\infty}\frac{x^2}{\sigma}f(\frac{x}{\sigma})dx \\
&= \sigma^2\int_{-\infty}^{\infty}\frac{x^2}{\sigma^2}f(\frac{x}{\sigma})\frac{dx}{\sigma} \\
&= \sigma^2\int_{-\infty}^{\infty}y^2 f(y)dy = \sigma^2
\end{aligned}$$

where the last equation follows from the change of variables $y = \frac{x}{\sigma}$.

The density for the $k:B$ order statistic $X_\sigma^{k:B}$ is:

$$\binom{B}{B-k}\left[F_\sigma(x)\right]^{B-k+1}\left[1-F_\sigma(x)\right]^{k-1}f_\sigma(x).$$

The expectation of $X_\sigma^{k:B}$ is:

$$\begin{aligned}
E[X_\sigma^{k:B}] &= \int_{-\infty}^{\infty}\binom{B}{B-k}\left[F_\sigma(x)\right]^{B-k+1}\left[1-F_\sigma(x)\right]^{k-1}f_\sigma(x)xdx \\
&= \int_{-\infty}^{\infty}\binom{B}{B-k}\left[F(\frac{x}{\sigma})\right]^{B-k+1}\left[1-F(\frac{x}{\sigma})\right]^{k-1}\frac{x}{\sigma}f(\frac{x}{\sigma})dx
\end{aligned}$$

$$
\begin{aligned}
&= \sigma \int_{-\infty}^{\infty} \binom{B}{B-k} [F(y)]^{B-k+1} [1-F(y)]^{k-1} f(y) y dy \\
&= \sigma E[X^{k:B}]
\end{aligned}
$$

where the third equation follows from the change of variables $y = \frac{x}{\sigma}$. ■

**Proof of Proposition 1.** Consider a network with $B$ buyers and $S \leq B$ sellers.[64] By Lemma 1, the gross surplus of a network is $S\overline{v} + \sum_{i=1}^{B} \beta_i \mu^{i:B}$. Since, by Lemma A2, $\mu^{i:B}$, for all $i$ is homogeneous of degree one in $\sigma^2$, $\sum_{i=1}^{B} \beta_i \mu^{i:B}$ is weakly increasing in $\sigma^2$. The gross surplus of the network is thus also weakly increasing in $\sigma^2$. If $\sum_{i=1}^{B} \beta_i \mu^{i:B} > 0$, then the gains from trade are strictly increasing in $\sigma^2$. ■

**Proof of Proposition 2.** Consider an industry with $B$ buyers in a network with $S < B$ sellers, and variance $\sigma^2 > 0$. For $c$ sufficiently small, the network that yields the greatest welfare is an allocatively complete network with the least number of links. Such a network yields welfare $S(\overline{v} - \alpha) + \sum_{i=1}^{S} \mu^{i:B}(\sigma^2) - S(B-S+1)c$, where $\sum_{i=1}^{S} \mu^{i:B}(\sigma^2) > 0$ by Lemma 1 and $B - S + 1$ links per seller is necessary and sufficient for an allocatively complete network (Kranton and Minehart (1998)). This network yields greater welfare than a fully vertically integrated structure for $\alpha > \underline{\alpha}(\sigma^2) = \overline{v} - \frac{1}{(B-S)} \sum_{i=1}^{S} \mu^{i:B}(\sigma^2) + \frac{S(B-S+1)}{(B-S)} c$. Since $\sum_{i=1}^{S} \mu^{i:B}(\sigma^2) > 0$, $\underline{\alpha}(\sigma^2) < \overline{v}$, and for $\alpha > \underline{\alpha}(\sigma^2)$ and $c$ sufficiently small, the network yields greater welfare. The network yields greater welfare than a market industrial structure when $\alpha < \overline{\alpha}(\sigma^2) = \overline{v} + \frac{1}{S} \sum_{i=1}^{S} \mu^{i:B}(\sigma^2) - (B - S + 1)c$. For $c$ sufficiently small, $\overline{\alpha}(\sigma^2) > \underline{\alpha}(\sigma^2)$. Since $\sum_{i=1}^{S} \mu^{i:B}(\sigma^2)$ is increasing in $\sigma^2$ by Lemma 2, this range is increasing in $\sigma^2$. ■

**Proof of Proposition 3**. The difference between the welfare of any network $\mathcal{N}$ and a strict subgraph $\mathcal{N}'$ is a linear function of $\overline{v}$ and of differences of the form $\mu^{k:n} - \mu^{l:n}$ where $k > l$. This is because in the more dense network there can be more inputs allocated in the efficient allocation, and inputs can be allocated to buyers with valuations that are higher in the ordering. For any $k > l$, $\mu^{k:n} - \mu^{l:n} \geq 0$. Since the expectation of each order statistic is homogeneous of degree one in $\sigma^2$, the difference is increasing in $\sigma^2$. Therefore, the sum of these terms is increasing in $\sigma^2$. ■

**Boundaries of Regions in Figure 3**

[64]Formally, we need to assume that there is a feasible allocation in $\mathcal{N}$ where each sellers' input is allocated to a distinct buyer. This means that there are no idle suppliers, as will always be true for an efficient network.

For $W(\mathcal{N}_1) = W(\mathcal{M}) = 0$ we have: $\overline{\alpha}(\sigma^2) \equiv \overline{v} + \frac{1}{2}\left[\mu^{1:4} + \mu^{2:4}\right] - 3c$.

For $W(\mathcal{N}_1) = W(\mathcal{N}_2)$ we have from the text $\underline{c}(\sigma^2) \equiv \frac{1}{6}(\mu^{2:4} - \mu^{3:4})$.

For $W(\mathcal{N}_1) = W(\mathcal{V})$ we have from the text: $\underline{\alpha}(\sigma^2) \equiv \overline{v} - \frac{1}{2}\left[\mu^{1:4} + \mu^{2:4}\right] + 3c$.

For $W(\mathcal{N}_2) = W(\mathcal{V})$ we have $\overline{\alpha}' = \overline{v} - \left[\frac{1}{2}\mu^{1:4} + \frac{1}{3}\mu^{2:4} + \frac{1}{6}\mu^{3:4}\right] + 2c$.

For $W(\mathcal{N}_2) = W(\mathcal{M}) = 0$ we have $\overline{\alpha}' = \overline{v} + \left[\frac{1}{2}\mu^{1:4} + \frac{1}{3}\mu^{2:4} + \frac{1}{6}\mu^{3:4}\right] - 2c$.

## The Efficient Industrial Structures for a Four-Buyer Industry

Below we show in Figure A1 the four other networks ($\mathcal{N}_3, \mathcal{N}_4, \mathcal{N}_5, \mathcal{N}_6$)which can be the efficient industrial structures for a four-buyer industry. Figure A2 then gives a schematic representation of the efficient industrial structures for a four-buyer industry, as a function of $c$ and $\alpha$ for a given $\sigma^2$. The calculations deriving this picture are available from the authors upon request.

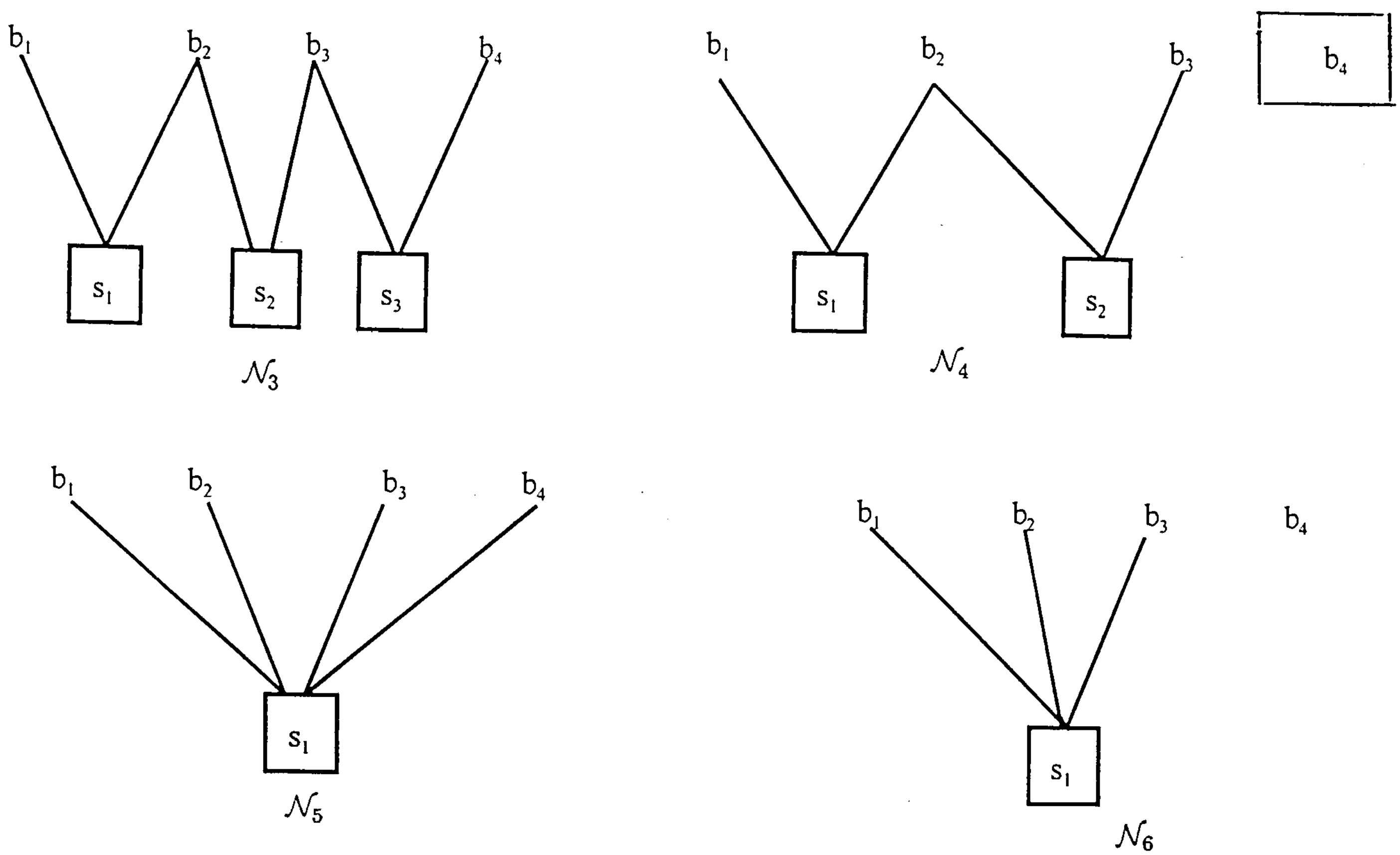


Figure A1

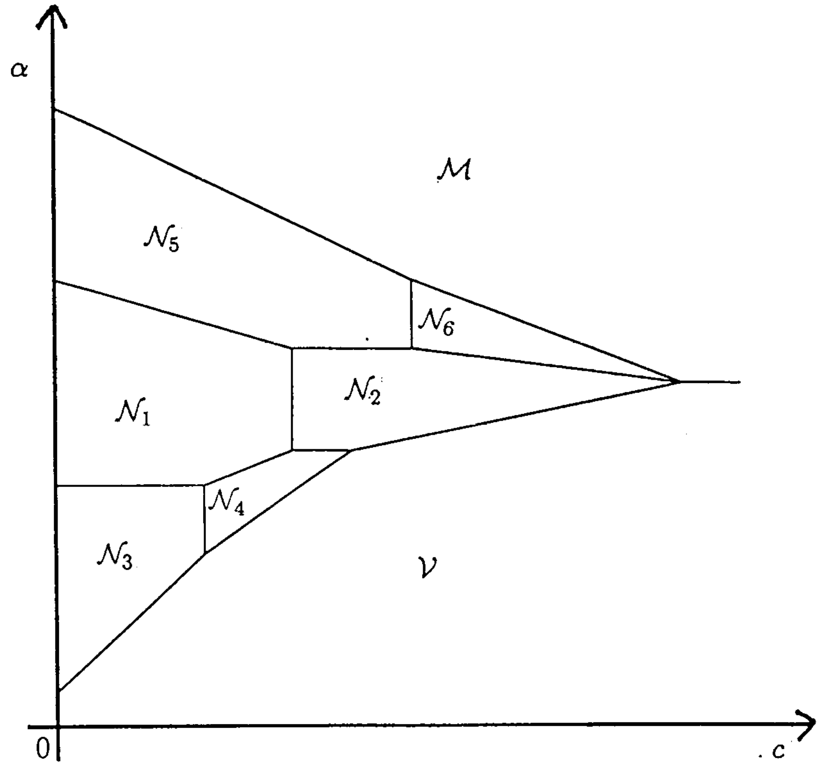


Figure A2

Proof of Proposition 7.

Consider first the area in $c, \alpha$ space where a network $\mathcal{N}(c, \alpha)$ is efficient. The boundaries of the area are of the form $\alpha(c) = \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B} + qc$ where $\beta_i \in \mathbb{R}$ and $q$ is an integer or of the form $c = \sum_{i=1}^{B} \beta_i \mu^{i:B}$. The first boundary comes from the fact that, for a given $c$, as $\alpha$ increases, the efficient network involves fewer sellers, where the number of sellers decreases one at a time. To see this, consider the efficient network for the same $c$ but for slightly higher $\alpha$. The only way to change welfare from the network is to remove a seller. A change in the links (number of links, buyers connected, and pattern) that increases welfare would imply that $\mathcal{N}$ was not the efficient network at $(\alpha, c)$. Call the network with one fewer seller $\mathcal{N}'$. $W(\mathcal{N}') - W(\mathcal{N})$ is of the form $\alpha - \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B}$ where $\beta_i \in \mathbb{R}$, since there is one less input produced. As $\alpha$ increases, eventually we have $W(\mathcal{N}') > W(\mathcal{N})$ and $\mathcal{N}$ is not the efficient network. We have entered a new region. A comparison of the welfare of all the networks with one fewer seller will identify the efficient network. It is possible that a change in link pattern of $\mathcal{N}'$ (number and position of links) will yield a network $\mathcal{N}''$ with even greater welfare. $W(\mathcal{N}'') - W(\mathcal{N}')$ is of the form $\sum_{i=1}^{B} \beta_i \mu^{i:B} + qc$ where $\beta_i \in \mathbb{R}$ and $q$ is an integer. The new link pattern changes the total link costs and the feasible allocations of goods among the existing sellers and buyers. Therefore, the difference between $W(\mathcal{N}'')$ (or $W(\mathcal{N}')$) is of the form $\alpha - \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B} + qc$, giving us the form of the boundary equation above. The second boundary form $c = \sum_{i=1}^{B} \gamma_i \mu^{i:B}$ derives from the fact that changes in link patterns yield changes in welfare of the form $\sum_{i=1}^{B} \gamma_i \mu^{i:B} + qc$ For a given number of sellers and buyers, as $c$ increases this welfare change may be positive, and the network $\mathcal{N}(\alpha, c)$ is no longer efficient. With boundaries of these two forms, any coordinates on the boundaries of these regions are of the form $\alpha = \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B}, c = \sum_{i=1}^{B} \gamma_i \mu^{i:B}$.

The region where $\mathcal{N}$ is efficient can be divided into triangles and rectangles. For triangles, the base of the triangle would be measured along the $c$ axis and the height along the $\alpha$ axis. The base is then of the form $\sum_{i=1}^{B} \gamma_i \mu^{i:B} - \sum_{i=1}^{B} \theta_i \mu^{i:B}$. The height is of the form $\overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B} - \left( \overline{v} - \sum_{i=1}^{B} \phi_i \mu^{i:B} \right) = \sum_{i=1}^{B} \beta_i \mu^{i:B} + \sum_{i=1}^{B} \phi_i \mu^{i:B}$. The same holds for the sides of rectangles. Therefore, the formula for the total area is of the form $\sum_{i=1}^{B} \delta_i \mu^{i:B} \cdot \sum_{i=1}^{B} \lambda_i \mu^{i:B} = \sigma^4 \sum_{i=1}^{B} \delta_i x^{i:B} \cdot \sum_{i=1}^{B} \lambda_i x^{i:B}$ where $x^{i:B}$ is the $ith$ order statistic for the parent distribution $F$.

Now consider the area where a network is both efficient and stable. The boundaries of the region are also of the form $\alpha(c) = \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B} + qc$ where $\beta_i \in \mathbb{R}$ and $q$ is an integer or of the form $c = \sum_{i=1}^{B} \gamma_i \mu^{i:B}$. Because each buyer earns the marginal value of its contribution to a network, its profits from a network are of the form $\sum_{i=1}^{B} \beta_i \mu^{i:B} - qc$. A buyer earns $\overline{v} - \alpha$ from vertical integration. Therefore the boundaries of the equilibrium region that derive from the buyers incentive to deviate to vertical integraion are of the form $\alpha(c) = \overline{v} - \sum_{i=1}^{B} \beta_i \mu^{i:B} + qc$. Sellers' expected profits from a network are of the form $\overline{v} + \sum_{i=1}^{B} \beta_i \mu^{i:B}$ because they expect to earn the valuation of the "next-best" buyer. Therefore the boundaries of the equilibrium region that derive from the sellers are of the form $\alpha(c) = \overline{v} + \sum_{i=1}^{B} \psi_i \mu^{i:B}$. Finally, we consider the boundaries that derive from buyers' incentives to change links. Any change in a buyer's links would yield a change in expected profits of $\sum_{i=1}^{B} \xi_i \mu^{i:B} + qc$. This leads to a boundary of $c = \frac{1}{q} \sum_{i=1}^{B} \xi_i \mu^{i:B}$. With boundaries of these forms, the formula for the area of the region where the network is an equilibrium is homogenous of degree 2 in $\sigma^2$, as argued above for the efficiency region. Finally, note that the region where the network is both efficient and stable must have boundaries of the same forms. ■

**Proof of Proposition 8**.

Suppose $\sigma^2 > 0$ and $B \geq 3$. Consider a network where all $B$ buyers invest in one link to one specialized seller which invests in a flexible asset. Since by Lemma 2 $\mu^{k:B}(\sigma^2)$ is homogeneous of degree one, we will write the expectation of the $k^{th}$ order statistic as $\sigma^2\mu^{k:B}$.

Under the competitive revenue rule, each buyer's equilibrium condition is $\frac{\sigma^2}{B}\left[\mu^{1:B} - \mu^{2:B}\right] - c \geq \overline{v} - \alpha$. Consider a link cost $c^*(\sigma^2) > 0$ such that a buyer earns $\lambda > 0$ profits in the network $c^*(\sigma^2) \equiv \frac{\sigma^2}{B}\left[\mu^{1:B} - \mu^{2:B}\right] - \lambda$. (Since $\sigma^2 > 0$ and $\mu^{1:B} > \mu^{2:B}$, such a strictly positive cost exists.) Define the investment cost $\underline{\alpha}^*(c^*) < \overline{v}$ to be the cost where a buyer earns exactly the same profits from vertical integration as in the network $\underline{\alpha}^*(c^*) \equiv \overline{v} - \left[\frac{\sigma^2}{B}\left[\mu^{1:B} - \mu^{2:B}\right] - c^*(\sigma^2)\right] = \overline{v} - \lambda$. For all $\{c, \alpha\}$ such that $c \leq c^*(\sigma^2)$ and $\alpha \geq \underline{\alpha}^*(c^*)$, the buyer's equilibrium condition is satisfied. Clearly, $c^*(\sigma^2)$ is increasing in $\sigma^2$ and $\underline{\alpha}^*(c^*)$ is constant.

Under the competitive revenue rule, the seller's equilibrium condition is $\overline{v} + \sigma^2\mu^{2:B} \geq \alpha$. This condition is satisfied for all $\alpha$ below the critical investment cost $\overline{\alpha}^*(\sigma^2) \equiv \overline{v} + \sigma^2\mu^{2:B}$ which is weakly increasing in $\sigma^2$.

It remains to prove that $\overline{\alpha}^*(\sigma^2) > \underline{\alpha}^*(c^*)$ for some $c^*(\sigma^2) > 0$; i.e., there exists a non-empty range of investment costs $\alpha$ where the buyers' and the seller's equilibrium conditions are all satisified. Consider the lowest possible $\underline{\alpha}^*(c^*)$. This occurs when $c^*(\sigma^2)$ is the lowest possible, so that buyers earn the highest possible profits from a network. As a benchmark, set $c^*(\sigma^2) = 0$. We then have $\underline{\alpha}^*(c^*) \equiv \overline{v} - \frac{\sigma^2}{B}\left[\mu^{1:B} - \mu^{2:B}\right]$ and $\overline{\alpha}^*(\sigma^2) > \underline{\alpha}^*(c^*)$ if and only if $\overline{v} + \sigma^2\mu^{2:B} > \overline{v} - \frac{\sigma^2}{B}\left[\mu^{1:B} - \mu^{2:B}\right]$. This condition simplifies to $\frac{1}{B}\mu^{1:B} + \frac{B-1}{B}\mu^{2:B} > 0$. The claim below shows that this condition is always satisfied for $B \geq 3$ draws from a distribution $F$ with mean 0. Since it is a strict inequality, we can relax our assumption that $c^*(\sigma^2) = 0$ and find an arbitrarily small $c^*(\sigma^2) > 0$ such that all conditions are satisfied. (For $B = 2$, $\frac{1}{2}\mu^{1:2} + \frac{1}{2}\mu^{2:B} = 0$, and no such $c^*(\sigma^2) > 0$ exists.)

**Claim:** $\frac{1}{B}\mu^{1:B} + \frac{B-1}{B}\mu^{2:B} > 0$ for $B \geq 3$.

**Proof of Claim.** From the triangle rule, for a sample of $n$ draws from any distribution $F$ with mean $\mu$, $\sum_{i=1}^{B}\frac{1}{B}\mu^{i:B} = \mu$. Therefore, for a distribution with mean 0, $\frac{1}{B}\mu^{1:B} = -\frac{1}{B}\sum_{i=2}^{B}\mu^{i:B}$. Substituting for $\frac{1}{B}\mu^{1:B}$ makes the left-hand-side of our hypothesized inequality $\frac{B-1}{B}\mu^{2:B} - \sum_{i=2}^{B}\frac{1}{B}\mu^{i:B}$.

Since $\mu^{2:B} > \mu^{i:B}$ for all $i \geq 3$, we have $\frac{B-1}{B}\mu^{2:B} - \sum_{i=2}^{B} \frac{1}{B}\mu^{i:B} > 0$. ■

**Proof of Proposition 9.**

Suppose $\sigma^2 > 0$ and $B \geq 3$. Consider a network where all $B$ buyers invest in one link to one specialized seller which invests in a flexible asset. Since by Lemma 2 $\mu^{k:B}(\sigma^2)$ is homogeneous of degree one, we will write the expectation of the $k^{th}$ order statistic as $\sigma^2\mu^{k:B}$.

The Shapley value is a weighted sum of an agent's contribution to different coalitions. The weight placed on a coalition of size $n$ in this network with $B+1$ agents is $\frac{n!(B-n)!}{(B+1)!}$.

With the Shapley revenue rule, each buyer's revenues derive from its contribution to coalitions of size $n = 1 \cdots B$. The only coalitions to which a buyer would add surplus are coalitions that include the seller. From all the size $n$ coalitions, there are only $\binom{B-1}{n-1}$ such coalitions. These coalitions consist of $n-1$ buyers and one seller. For $n = 1$, the contribution is $\overline{v}$. For $n \geq 2$ a buyer's contribution is $\sigma^2\left[\mu^{1:n} - \mu^{1:n-1}\right]$. By the triangle rule, this contribution is $\frac{\sigma^2}{n}\left[\mu^{1:n} - \mu^{2:n}\right]$. The equilibrium condition for each buyer is then

$$\frac{1!(B-1)!}{(B+1)!}\overline{v} + \sum_{n=2}^{B} \frac{n!(B-n)!}{(B+1)!}\binom{B-1}{n-1}\frac{\sigma^2}{n}\left[\mu^{1:n} - \mu^{2:n}\right] - c \geq \overline{v} - \alpha$$

which simplifies to

$$\frac{1}{(B+1)B}\overline{v} + \sum_{n=2}^{B} \frac{\sigma^2}{(B+1)B}\left[\mu^{1:n} - \mu^{2:n}\right] - c \geq \overline{v} - \alpha$$

Consider a link cost $c^*(\sigma^2) > 0$ such that a buyer earns $\lambda > 0$ profits in the network. (Since $\sigma^2 > 0$ and $\mu^{1:n} > \mu^{2:n}$, such a strictly positive cost exists.) $c^*(\sigma^2) = \frac{1}{(B+1)B}\overline{v} + \sum_{n=2}^{B} \frac{\sigma^2}{(B+1)B}\left[\mu^{1:n} - \mu^{2:n}\right] - \lambda$. Define the investment cost $\underline{\alpha}^*(c^*) < \overline{v}$ to be the cost where a buyer earns exactly the same profits from vertical integration as in the network: $\underline{\alpha}^*(c^*) = \overline{v} - \left[\frac{1}{(B+1)B}\overline{v} + \sum_{n=2}^{B} \frac{\sigma^2}{(B+1)B}\left[\mu^{1:n} - \mu^{2:n}\right] - c^*\right] = \overline{v} - \lambda$. For all $\{c, \alpha\}$ such that $c \leq c^*(\sigma^2)$ and $\alpha \geq \underline{\alpha}^*(c^*)$, the buyer's equilibrium condition is satisfied. Clearly, $c^*(\sigma^2)$ is increasing in $\sigma^2$ and $\underline{\alpha}^*(c^*)$ is constant.

Now consider the seller's equilibrium condition. The seller's revenues derives from its contribution to coalitions of buyers. For a coalition of size $n$, a seller contributes $\overline{v} + \sigma^2\mu^{1:n}$ and there are $\binom{B}{n}$ such $n$-size coalitions. The equilibrium condition for the seller is then

$$\sum_{n=1}^{B} \frac{n!(B-n)!}{(B+1)!}\binom{B}{n}\left[\overline{v} + \sigma^2\mu^{1:n}\right] \geq \alpha$$

which simplifies to

$$\frac{B}{B+1}\overline{v}+\sum_{n=2}^{B}\frac{\sigma^2}{(B+1)}\mu^{1:n}\geq\alpha$$

since $\mu^{1:1}=0$. This condition is satisfied for all $\alpha\leq\overline{\alpha}^*(\sigma^2)=\frac{B}{B+1}\overline{v}+\sum_{n=1}^{B}\frac{\sigma^2}{(B+1)}\mu^{1:n}$ and clearly $\overline{\alpha}^*(\sigma^2)$ is increasing in $\sigma^2$.

It remains to show that for $\sigma^2$ sufficiently high, $\overline{\alpha}^*(\sigma^2)>\underline{\alpha}^*(c^*)$ so that the there exists a non-empty set of investment costs $\alpha$ such that buyers' and seller's equilibrium conditions are all satisfied. As a benchmark, set $c^*=0$. $\overline{\alpha}^*(\sigma^2)>\underline{\alpha}^*(c^*)$ then becomes

$$\sigma^2\left[\sum_{n=2}^{B}\mu^{1:n}+\frac{1}{B}\sum_{n=2}^{B}\left[\mu^{1:n}-\mu^{2:n}\right]\right]>\frac{B-1}{B}\overline{v}.$$

Since $\mu^{1:n}>0$, $\left[\mu^{1:n}-\mu^{2:n}\right]>0$ and $\left[\frac{B-1}{B}\right]\to1$ as $B$ increases, this inequality is satisfied for $\sigma^2$ and $B$ sufficiently high. Since it is a strict inequality, we can relax our assumption that $c^*(\sigma^2)=0$ and find an arbitrarily small $c^*(\sigma^2)>0$ such that all equilibrium conditions are satisfied. ■

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