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THE PRICE FOR THE WIDOW'S CRUSE:
OR THE VALUE OF AN INFINITELY PRODUCTIVE ASSET

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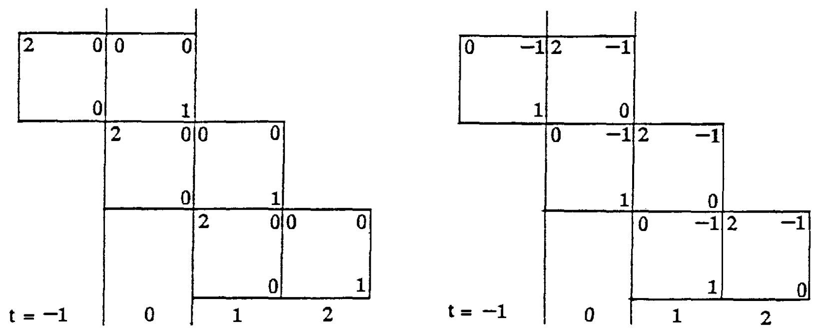
1. INTRODUCTION

Two basic problems are considered here, The first is the necessity for introducing government money (as contrasted with individual credit) and an infinitely lived government in an overlapping generations economy. The second concerns the evaluation of the price of an infinitely productive asset in an economy without a natural discount factor.

The seminal work of Allais (1947) and Samuelson (1956) made clear the need for the introduction of government as an infinitely lived player in an economy with overlapping generations. In our search for minimal mechanisms or institutions to help bring about optimal production and exchange (note no comment has been made about government's role in supplying public goods) a natural question to ask concerning government and outside money is what are the necessary conditions on the debt relationship between individuals and the government to achieve optimality?

In essence the problem is as follows. In either a finite horizon general equilibrium economy or an infinite horizon economy with one or more agents who are infinitely lived and have bounded utility functions (for example a discounted sum) the presence of borrowing and lending among the traders is all that is needed. At any point in time debts and credits among all agents net out to zero and at the end of time all books balance. When the economy, however consists exclusively of overlapping generations of finitely lived agents it is easy to construct examples where personal credit is not enough. The key example in Samuelson's paper shows this. In particular if all individuals produce a perishable when young, but live for two periods and produce nothing when old they would like to lend when young and be repaid when old. But the individuals who wish to borrow will never be able to repay thus the internal credit markets never go active and economic efficiency cannot be obtained. By introducing an infinitely lived government who has the power to issue its paper money and enforce repayments through say bankruptcy laws it can set up conditions to enable the private sector to achieve optimal trade.

The argument here is illustrated by means of a series of elementary examples. We begin with Sam's[1] example already noted above. Figure 1 shows the government that is implicit in this model. We have not specified a utility or objective function for government as a player. There are many alternatives. Muller and Woodford (1988) introduce infinitely lived players with a "natural" time discount, but I suggest that there does not appear to be any overwhelming reason to do so except for dubious mathematical convenience.[2] We could, for example, consider that the goal of the government player is to provide a policy which enables all "natural persons" to achieve an efficient outcome. Otherwise at a more sophisticated level we could consider grafting onto government a political control mechanism where the politicians strive to select economic policies which optimize political goals.



a　　　　b

Figure 1

Suppose that we were to try to play the overlapping generations model illustrated in Figure 1a as an experimental game. We can do this in the following way. As referee we need to announce the initial conditions at time $t = 0$ and if we wish to stop play at some time $t = T$ we can do this by informing the live players of the strategies of the (virtual) players to be born after time T.

[1]In keeping with Professor Samuelson's economical abbreviation of Boehm-Bawerk's name to Boehm it seems appropriate to follow the lead and abbreviate Samuelson to Sam.

[2]There are several axiom systems arguing for a "natural time discount", but this still does not avoid the wiping out of the time discount in overlapping generations. The possible exception is if a dynasty or inheritance discount is introduced in each cohort, thus one's utility for future generations has a discount in it.

In Figure 1a the number in the upper left of a box is the initial holding of the good; upper right is the level of debt and lower right is the amount of money held.

For simplicity assume that each generation lives for two periods and possesses two units of nonstorable manna when young, but has nothing when old. Let the utility function of the typical individual born at time t be:

$$\sqrt{x_t} + \sqrt{x_{t+1}} + \alpha \min[\text{debt}, 0] .$$

Where α is a parameter measuring the disutility of default. Without the government as banker, Sam observed there will be no lending and in this simple example each generation will achieve a "score" of $\sqrt{2}$ and starve in its old age. It is obvious that there is a nice simple stationary state lurking in the wings where each individual consumes one unit in each period and everyone achieves a score of 2. How do we arrange for this? At this point government steps in as the central bank controller. We can imagine the game as starting at $t = 0$. There will be two types of individuals alive, those born at $t = -1$ and those born at $t = 0$. In order to fully specify the game we must assign a consumption level to those born earlier for their consumption during $t = -1$.[3] We must also specify credit relations between the bank and the players at $t = 0$.

Suppose we select as the initial condition that the government issues 1 unit of fiat money to the old of the first generation (see Figure 1a). The young have no money, but expect that the next generation will accept what they earn in payment. Trade takes place using fiat money and the young sell the old 1 unit of manna and as the new old they have one unit of the money and face the new young with two units of manna and no fiat money. The process repeats itself and as long as there is no end this Ponzi game works and yields each a Pareto optimal outcome of two. It is trivially easy to see that in this formulation the bankruptcy penalty α is irrelevant. There is no credit granted and no individual is ever in debt. The fiat money is supported only by its acceptability and the expectation of its future acceptability.[4] There is only outside money and it exists in positive quantity as a store of value supported by the convention and expectation of its acceptability.

When we turn to the case illustrated in Figure 1b where it is the old who have the two units of manna and the young have nothing. The same trick does not work. Giving the young or the old one unit of fiat will not

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[3] Or if we are considering a playable game we can introduce a special "first generation" who enters the game only as old.

[4] Formally expectations are modeled by giving ending prices exogenously in a truncated finite representation of the economy.

promote trade under any expectation. The monetary engineering that will work is to give the old a debt of one unit of fiat and to permit the young to borrow one unit of fiat from the central bank in exchange for their IOU notes. Thus initially the old have two units of manna and owe the government one unit of outside money; the young have one unit of fiat and owe the government one unit of outside money. The constant net holdings of all real agents is plus one unit of fiat and minus two units of personal IOUs (denominated in fiat). The fiat money here is the symbolic store of value which enables value for value transactions to take place. But unlike in the previous case the default or bankruptcy conditions are critical. There will be no incentive to trade unless α is large enough. For values of one or more then it pays the old to fully discharge the debt to the government. For values less than one some level of default is more profitable. In this instance *both* expectations and a default penalty must be introduced exogenously. There is a nonsymmetry between trading forward and backwards in time. The examination of these two extreme cases appears to be fully generalizable and suggests that the various paradoxes of the OLG models[5] can be regarded as arising from the need to introduce $m+n$ parameters m representing a vector of final prices at which the now young can trade after the finite economy is over (expectations) and n representing a vector of marginal disutility of loss to be suffered by the old if they fail to pay back any debt they have incurred (default).

The financial gluing together with the young owning the manna requires one special financial instrument, to wit fiat money which enters in positive supply. The financial gluing together with the old owning all of the manna requires two financial instruments, fiat money and private debt to the government. In the first instance only expectations are sufficient to achieve optimal trade, in the second instance a default penalty is needed to provide the (negative incentive) for the discharge of debt.

2. THE PRICE OF THE WIDOW'S CRUSE

We turn now to a simple paradox of the OLG economy. Consider an extremely simple economy with one type of trader who lives for three periods. There are equal numbers of each age group. An individual born at time t has a utility function of form:

[5] There are more difficulties and more results when we introduce nonseparable utility functions, production, durable assets and population growth. These require more boundary conditions and parameters when trying to construct a finite playable game. However in return the outside rate of interest can be studied.

$$2\sqrt{x_t} + 2\sqrt{x_{t+1}} + 2\sqrt{x_{t+2}} + \alpha \min[\text{debt}, 0] \ .$$

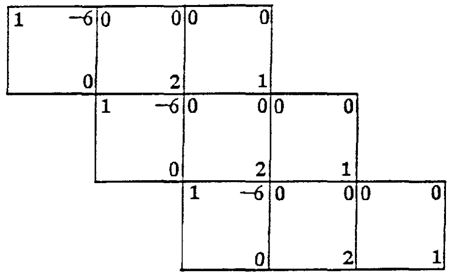

Figure 2

There is one infinitely durable commodity, a "Widow's Cruse" which produces three meals every period forever. The initial conditions are given with the youngest person alive owning the cruse, but owing an outside agency (the government bank) \$6 the middle aged individual has \$2 and the old has \$1. We can check the following stationary state noncooperative equilibrium efficient solution. The price of a meal is \$1 and the price of the cruse is \$6. At the start of time t the youngest individual alive buys the cruse from the individual born at time $t-1$. He finances his purchase by borrowing \$6 from the central bank at a zero money rate of interest. Figure 2 shows the asset positions of all individuals. The number in the upper right corner indicates the debt position of each individual after the asset has been traded but before meals have been sold. The number in the left hand corner indicates the ownership of the cruse. The number in the lower right is money holdings.

It is easy to check that after the youngest individual buys the cruse he sells a meal to each of the others and keeps one for himself. He enters the next period with a net debt of \$4 and the cruse. He sells the cruse for \$6 and hence now has \$2 after settling his debt. This he can use to buy a meal this period and next. Thus there is a stationary state where every individual obtains a meal each period and obtains a payoff of 6.

The autarky solution gives the first individual a payoff of $6\sqrt{3} - 6\alpha$ and the payoffs to the others depend explicitly on assumptions concerning what happens to the unsold cruse.[6]

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[6] For $\alpha < .732$ it pays the first individual to opt for autarky. For $\alpha > .732$ trade is better.

We have shown that there is a price system and competitive markets which yield a Pareto optimal solution. We have not shown that the price is unique. A little further consideration indicates that any price $p \geq \$6$ will also give rise to an equilibrium where the price of a meal is \$1.

The presence of the bankruptcy penalty $\alpha = .732$ sets a lower bound on prices and the availability of borrowing consistent with any level of expectations on the price obtainable for the cruse can sustain that price.

This example is suggestive of an asset market such as an art market for some paintings or jade carvings. Price may be regarded as being factored into two: the benefit derived from the services of the object and the expected price of resale. The result would be influenced if there were a rate of depreciation present but the possibility for a greater fool price is still there. In order to prevent this a formal upper bound to the amount of credit that the government is permitted to issue is called for.

Viewing Figure 2, the upper left hand side indicates the ownership of the cruse. The upper right indicates the IOU owed to the government and the lower right the fiat in circulation. In this model there are three units of fiat always in circulation. But there is an implicit amount that does not appear because (unlike in Figure 1a) the government is financing the full purchase of the capital stock directly by lending the young \$6. If the government was willing to lend the young \$100 instead of \$6 there would be another solution still with \$3 of fiat as circulating capital being used to pay for meals and with the price of the cruse at \$100.

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