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THE BALANCE OF PAYMENTS ADJUSTMENT MECHANISM  
IN A RATIONAL EXPECTATIONS EQUILIBRIUM

by

R.H. Clarida

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Richard H. Clarida

Yale University, The Cowles Foundation for Research in Economics,  
and The National Bureau of Economic Research

This paper provides a choice theoretic, general equilibrium account of the balance of payments adjustment process and the determination of national price levels in a world comprised of countries populated by rational households. Balance of payments adjustment dynamics arise in the equilibrium of this model from the precautionary saving behavior of risk-averse households who self-insure against random productivity fluctuations by accumulating, via balance of payments surpluses in productive periods, buffer stocks of domestic money which can be drawn down to finance payments deficits, and thus a less variable profile of consumption relative to output, when productivity is unexpectedly low. Precautionary saving is shown to exhibit the partial-adjustment-to-target behavior typically postulated in the monetary approach literature. The existence of a rational expectations equilibrium in which the distribution of international reserves among central banks is stationary is established.

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1. Introduction

This paper provides a choice theoretic, general equilibrium account of the balance of payments adjustment process and the determination of national price levels in a world comprised of countries populated by rational households. The setting is similar in some respects to that proposed by Mundell (1968) and Dornbusch (1973) in that households consume and produce a single good, save by accumulating balances of domestic money, and exchange domestic for foreign currencies at rates which are fixed by central bank intervention. However, in contrast to the approaches prevalent in the modern monetary approach literature (as surveyed by Frenkel and Johnson (1976) and Frenkel and Mussa (1984)), we study the balance of payments adjustment process in an explicit general equilibrium framework in which expenditure, saving, and money demand relationships are based upon the intertemporal, utility maximization problems of risk-averse, forward looking households who confront random fluctuations in their labor productivities.

Non-trivial balance of payments adjustment dynamics arise in the equilibrium of this model from the precautionary saving behavior of risk-averse households who attempt to self-insure against unanticipated, country specific productivity fluctuations by accumulating, via balance of payments surpluses in relatively productive periods, buffer stocks of domestic money

balances which can be drawn down to finance payments deficits, and thus a less variable profile of spending relative to income, when productivity is unexpectedly low.<sup>1</sup> An appeal of our inventory theoretic approach is that we are able to derive the typically postulated properties of savings behavior in a rational expectations general equilibrium framework which does not require that money holdings yield positive utility. In particular, the balance of payments adjustment process exhibited in the equilibrium of this model is driven by the precise sort of partial-adjustment-to-target savings behavior typically postulated in the monetary approach literature.

The model developed here is adapted from the work of Foley and Hellwig (1975) and Lucas (1980) and employs the device of a continuum of countries which experience independently and identically distributed productivity fluctuations in each period. With no fluctuations in the per capita world supply of output, there exists a rational expectations equilibrium which is characterized by a stationary distribution of international reserves among countries and constant national price levels which depend in an explicit way on household's attitudes toward risk and the probability distribution of productivity shocks. Thus, our analysis makes rigorous the key role played by the distribution of international reserves among countries, and not just their aggregate supply, in the determination of national price levels--a role often emphasized in the monetary approach literature (see especially Dornbusch (1973)).

The plan of the paper is as follows. In Section 2, we set up the general equilibrium model of international trade and reserve flows, establish the homogeneity and monotonicity properties of optimal expenditure and savings functions, and establish the existence of a stationary rational

expectations equilibrium. In Section 3, we show that equilibrium savings behavior exhibits the partial-adjustment-to-target properties postulated in the monetary approach literature. In particular, there exists in each country a unique target stock of money balances at which planned savings is zero and expenditure equals expected nominal income. We show that risk-averse households strive to accumulate money balances in excess of those required to finance a spending flow equal to expected nominal income. Such buffer stocks of liquidity are held in equilibrium, even though households discount future utility, for self-insurance against unexpected productivity shocks and allow households to attain a less variable profile of consumption relative to output. However, in equilibrium, self-insurance is not complete: real expenditure is shown to be positively correlated with unanticipated productivity shocks and is in fact Markovian even though shocks are assumed to be i.i.d. A key result is that, in equilibrium, planned saving in each country is strictly less than the gap between actual and target money balances. These properties of equilibrium saving behavior are used to place restrictions on the non linear stochastic difference equation which is shown to govern the time path of the balance of payments in each country. In particular, payments imbalances which result from an unanticipated, transitory productivity shock are shown to exhibit persistence which results from each household's attempt to restore its cash balance position to target only partially in any given period. Section 4 provides some concluding remarks.

## 2. The Model

We study a world economy comprised of a continuum of countries, indexed  $i \in [0,1]$ , each with its own central bank and a representative household. Households have an infinite planning horizon, are risk averse, and have access to a stochastic, Ricardian technology which transforms labor into the single, tradable consumption good. We assume that labor is not mobile internationally and that no utility is attached to leisure. Time is discrete, and in each period  $t = 0, 1, 2, \dots$  the labor input requirement in each country is randomly shocked. These shocks are assumed to be independently and identically distributed both across countries and time so that, with a continuum of countries, there is no fluctuation in the per capita world supply of output. Normalizing the labor endowment in each country to unity, let  $\varepsilon_t^i$  denote the output of country  $i$  at date  $t$ . Formally,  $\varepsilon_t^i$  is a drawing from a finite set of non-negative random variables from the fixed c.d.f.  $G$  such that  $G(0) > 0$  and  $G(\bar{\varepsilon}) = 1$ . The only asset available to each household is domestic money which is the liability of the domestic central bank and is backed entirely by fiat international reserves. Denote by  $H_t^i$  and  $R_t^i$  the monetary liabilities and international reserve holdings of the central bank in  $i$  at date  $t$ , and let  $1/\theta^i$  be the rate at which that bank stands ready to exchange reserves for balances of its monetary liabilities acquired by central banks in other countries as the asset of foreign purchases by domestic residents. The balance sheet of the central bank in country  $i$  is given in Figure 1.

Trading arrangements are as follows. At the beginning of each period, before the realization of the productivity shock is known, each household makes a spending-savings decision. Nominal spending is constrained by beginning-of-period money balances. One member of each household, the shopper,

takes a portion of the beginning-of-period money balances and visits other countries to purchase the consumption good. Purchases must be made in the seller's currency. Upon arriving in a foreign country with goods to sell,<sup>2</sup> the shopper exchanges domestic for foreign balances with that country's central bank at the rate of  $\theta^f/\theta^h$  units of foreign currency per unit of domestic currency. The shopper then uses the foreign currency acquired to purchase the consumption good in that country. The process continues until each shopper exhausts the money balances with which he began. While the shopper is away, the worker in each country learns the productivity shock and produces output which is sold to shoppers from other countries who exchange their home currencies for domestic money balances with the domestic central bank. At the end of the period, shoppers return home, divide their purchases with workers, and consume, while central banks settle claims among themselves. Central banks in countries in which sales (exports) exceed purchases (imports) gain reserves since foreign currencies acquired through trade are convertible to reserves at fixed rate. Given our assumption that monetary liabilities are backed entirely by foreign exchange reserves, the fixed exchange rate regime is sustainable with certainty.

Denote by  $p_t^i$  the money price of goods in  $i$  at  $t$ . We shall assume sufficient goods market arbitrage so that national price levels are linked by the purchasing power parity condition

$$p_t^i = (\theta^i/\theta^j)p_t^j \quad (1)$$

for all  $i, j \in [0,1]$  and  $t = 0, 1, 2, \dots$ . An equivalent interpretation of (1) is that the implicit rate of exchange between reserves and the consumption good

$$\kappa_t = p_t^i / \theta_t^i \quad (2)$$

is equal across all countries  $i \in [0,1]$  in each period  $t = 1, 2, \dots$ . Following Lucas (1980), we shall study a world economy with a constant per capita supply of international reserves  $\bar{R}$  and seek an equilibrium in which national price levels are constant. We note, however, that the equilibrium time paths of spending and the balance of payments will fluctuate in equilibrium as countries are continually shocked by new drawings of  $\epsilon$ . In Section 3, we investigate the persistence of these fluctuations. However, we must first establish the existence of a stationary rational expectations equilibrium. We begin with the intertemporal optimization problem of a representative household.

Each household acts so as to<sup>3</sup>

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (3)$$

$$\text{s.t. } 0 \leq pc_t \leq H_t ; \quad (4)$$

$$H_{t+1} = H_t - pc_t + p\epsilon_t . \quad (5)$$

where  $0 < \beta < 1$  is the rate at which future utility is discounted,  $u : \mathbb{R}^+ \rightarrow \mathbb{R}$  is a bounded, strictly concave, and twice differentiable utility function with  $u'(0) = \infty$ , and  $c_t$  is consumption of the household at time  $t$ . Denote by  $\alpha_t = pc_t$  nominal spending at time  $t$ .

Let  $v(H; p; G)$  be the value of (2) for a household which begins the period with nominal balances  $H$ , faces the constant domestic price level  $p$  in the current and all future periods, and behaves optimally. The household's current period decision problem is

$$v(H;p;G) = \max_{0 < \alpha < H} u(\alpha/p) + \beta \int V(H - \alpha + p\varepsilon; p; G) dG(\varepsilon) . \quad (7)$$

The following proposition, which is proven in the appendix, establishes the relevant properties of optimal spending behavior implied by (7).

Proposition 2.1: Under the assumptions on  $u$  and  $G$ , there exists a unique, continuous, strictly increasing, and strictly concave value function and a unique, continuous, linearly homogeneous expenditure function  $\alpha(H;p;G)$  which solves (2). The marginal propensity to spend is strictly positive and less than unity. That is, for all  $\lambda > 0$

$$0 < \alpha(H+\lambda; p; G) - \alpha(H;p;G) < \lambda . \quad (8)$$

We note that expenditure depends upon money balances, the price level, and the probability distribution of productivity shocks, since actual productivity and thus output is unknown at the time the spending decision is made.

Let  $s(H;p; ;G) \equiv p\mu - \alpha(H;p;G)$  denote the household's planned accumulation of money balances over the period where  $\mu \equiv E\varepsilon$ ,<sup>4</sup> and denote by  $s(H;p; ;G) \equiv p\varepsilon - \alpha(H;p;G)$  realized accumulation of money balances. Planned and actual saving will differ whenever actual productivity differs from mean productivity. The following corollary follows immediately from Proposition 2.1.

Corollary 2.2: There exists a unique, continuous savings function  $s(H;p; ;G)$  which is linearly homogeneous in  $H$  and  $p$ . The marginal propensity to save is strictly negative and less than unity in absolute value. That is, for all  $\lambda > 0$

$$0 < s(H;p;\varepsilon;G) - s(H+\lambda; p; G) < \lambda . \quad (9)$$

As demonstrated in the appendix, the result that the marginal propensity to spend is strictly less than unity follows from the Inada condition on preferences and the assumption that there is a strictly positive probability that the household will have no output to sell at the end of the period. Under these conditions, it is never optimal to allow the stock of money balances to be drawn down to zero, or to consume all of any investment to money balances. If  $u'(0) < \infty$  or  $G(0) = 0$ , it will be optimal for a household to spend all of its beginning-of-period money balances when the latter falls below the threshold level  $\hat{H}$ , where  $\hat{H}$  is the unique solution to  $u'(\hat{H}/p) = \beta \int v'(p\varepsilon) dG(\varepsilon)$ .

The international reserve holdings of each central bank evolve according to the Markov process

$$R_{t+1} = R_t - \kappa c(R_t/\kappa; G) + \kappa \varepsilon_t ; \quad (10)$$

where we have used the fact that  $H/p = R/\kappa$  and the result, established in the appendix, that optimal consumption--real expenditure--is a time invariant function of real balances. In equilibrium, the per capita demand for reserves by all central banks must equal the given supply  $\bar{R}$  at the assumed constant national price levels and implicit rate of exchange between reserves and the consumption good. The per capita end of period demand for reserves depends upon the beginning-of-period distribution of reserves among central banks,  $J(R)$ . Given  $J$ , the per capita demand is

$$\iint (R - uc(R/\kappa; G) + \kappa\varepsilon) dJ(R) dG(\varepsilon) . \quad (11)$$

Substituting  $m \equiv H/p = R/\kappa$  into (11) and letting  $F(m) \equiv J(\kappa m)$  denote the beginning-of-period distribution of real balances among countries, we obtain the following equilibrium condition

$$\iint (m - c(m; G) + \varepsilon) dF(m) dG(\varepsilon) = \bar{R}/\kappa . \quad (12)$$

From the goods-market arbitrage relationship (2), we know that national price levels are proportional to  $\kappa$ , the implicit reserves price of goods. The value of  $\kappa$  which satisfies (12) will depend upon the distribution of real reserves  $F(m)$  over countries. For households' expectations that national price levels be constant over time to be rational, we require that  $F(m)$  be a stationary distribution for the Markov process which governs the evolution of real balances in each country

$$m_{t+1} = m_t - c(m_t; G) + \varepsilon_t . \quad (13)$$

The existence of a stationary rational expectations equilibrium is established in the following proposition.

Proposition 2.3: There exists a unique, stationary rational expectations equilibrium. The equilibrium distribution of international reserves among countries does not change over time and is given by  $J(R) = F(R/\kappa)$  where  $F$  is the unique stationary distribution for (13) and  $\kappa$  is the equilibrium (implicit) rate of exchange between reserves and the consumption good

$$\kappa = \bar{R} / \iint (m - c(m; G) + \varepsilon) dF(m) dG(\varepsilon) . \quad (15)$$

In equilibrium, national price levels are constant and satisfy, for all  $i \in [0, 1]$

$$p^i = \theta^i \bar{R} / \int (m - c(m; G) + \varepsilon) dF(m) dG(\varepsilon) . \quad (16)$$

Proof: Foley and Hellwig (1975), Proposition 4, pp. 335-336, establish the existence of a unique stationary  $F$  for the Markov process (13) in a two-state model in which  $\text{Prob}(\varepsilon = 0) = q$  ;  $\text{Prob}(\varepsilon = \bar{\varepsilon}) = 1-q$  . The extension to the case in which  $\varepsilon$  takes on more than two values is trivial and will be omitted. Given  $F$  ,  $\kappa$  is uniquely determined by (15). Given the equilibrium  $\kappa$  , the unique stationary distribution of international reserves among countries is defined by  $J(R) = F(R/\kappa)$  . Since  $F$  is a stationary distribution, we have

$$\int m_{t+1} dF(m) - \int m_t dF(m) = \mu - \int c(m_t; G) dF(m) = 0 ; \quad (17)$$

for all  $t = 0, 1, 2, \dots$  so that the goods market clears in every period at the constant national price levels  $p^i = \theta^i \kappa$  and exchange rates  $\theta^i / \theta^j$  for all  $i, j \in [0,1]$  . Q.E.D.

The interpretation of the stationary equilibrium established in Proposition 2.3 is as follows. If international reserves are initially distributed among countries according to  $J$  , then national price levels  $p^i = \theta^i \kappa$  ,  $i \in [0,1]$  will clear the goods and asset markets in each period  $t = 0, 1, 2, \dots$  and the distribution of international reserves among countries will not change. In this equilibrium, the balance of payments in each country will be shown to fluctuate in a persistent manner as national productivities are continually shocked. These payments imbalances will induce reserve flows among countries, but in the aggregate, the fraction of countries with international reserves less than or equal to any particular value will remain constant. Alternatively, if international reserves are initially distributed according to some other  $J_0(R) \neq J(R)$  ,

the system will converge to the stationary rational expectations equilibrium if households have static expectations. Of course, in this case, static expectations will only be rational in the limit.<sup>5</sup> This convergence under static expectations is exactly analogous to the properties of the two-country model studied by Dornbusch ((1973); 1980, Ch. 8) based upon postulated expenditure and savings functions and static expectations. However, in Dornbusch's analysis the long-run equilibrium in which static expectations are rational is characterized by a zero balance of payments in each country. We now investigate the balance of payments adjustment process exhibited in the rational expectations equilibrium of our model.

### 3. The Balance of Payments Adjustment Process

In the stationary, rational expectations equilibrium, the time path of the balance of payments in each country is completely characterized by the following non-linear stochastic difference equation

$$s_{t+1} = s(\phi(s_t; \epsilon_t) + s_t; \epsilon_{t+1}) ; \quad (18)$$

where  $\phi(s; \epsilon) \equiv s^{-1}(s(H; p; \epsilon; G) + p(\mu - \epsilon)) = s^{-1}(s(H; p; \mu; G) - H)$ . The balance of payments is not Markov: knowledge of  $s_t$  alone is not sufficient to determine the probability distribution of  $s_{t+1}$ . However given  $\{s_t = s; \epsilon_t = \epsilon\}$  we can calculate

$$\text{Prob}\{s_{t+1} \leq s' | s_t = s; \epsilon_t = \epsilon\} = G((s^{-1}(s') - s - \phi(s; \epsilon))/p). \quad (19)$$

In order to obtain sharper results, we must first establish additional properties of optimal precautionary savings behavior. We now show that saving in each country exhibits the precise set of partial-adjustment-to-

target behavior typically postulated in the monetary approach literature (Dornbusch (1973)). We begin with the following proposition.

Proposition 3.1: There exists a unique target stock of money balances,  $H^T$ , at which spending equals expected nominal income  $\alpha(H^T; p; G) = p\mu$ , and planned saving is zero,  $s(H^T; p; \mu; G) = 0$ . (See Figure 2.)

Proof: We seek a fixed point to the transition equation

$$H_{t+1} = H_t - \alpha(H_t; p; G) + p\mu. \quad (20)$$

From Proposition 2.1, the rhs of (20) is a continuous, strictly increasing function of  $H$  with slope less than unity. It follows that there is exactly one  $H$  such that (see Figure 3)

$$s(H^T; p; \mu; G) = 0. \quad (21)$$

Furthermore, since saving is a strictly decreasing function of money balances, planned saving is strictly positive (negative) when  $H < (>) H^T$ .

Q.E.D.

Corollary 3.2: Target money balances strictly exceed target expenditure:  $H^T > \alpha(H^T; p; G)$ .

In the economies studied in this paper, money serves both as a medium of exchange and as a buffer stock against future unanticipated productivity disturbances. For this reason, risk averse households strive to accumulate money balances which are more than sufficient to finance a spending flow equal to expected nominal income. In the event of an unanticipated, transitory decline in productivity, households are able to draw down their inventories of cash balances in excess of target expenditure

to finance a less variable profile of consumption relative to productivity. However, in equilibrium, self-insurance is not complete: expenditure is strictly positively correlated with unanticipated productivity shocks. This is most easily seen by expressing expenditure at time  $t$  as a function of time  $t-1$  desired end-of-period money balances and the unanticipated productivity shock

$$\alpha(H_t; p; G) = \alpha(H_{t-1} + s(H_{t-1}; p; \mu; G) + p(\epsilon_{t-1} - \mu); p; G) . \quad (22)$$

The observation follows from the Proposition 2.1 result that expenditure is a strictly increasing function of  $H_t$  and thus  $p(\epsilon_{t-1} - \mu)$ .

We now investigate the extent to which households attempt to close the gap between actual and target money balances via planned saving. The following proposition establishes that planned saving (in absolute value) is strictly less than the difference between actual and target money balances.

Proposition 3.3: Planned saving is strictly less (in absolute value) than the gap between actual and target money balances. That is, for all  $H \neq H^T$  (see Figure 4)

$$0 < \frac{s(H;p;\mu;G)}{H^T - H} < 1 . \quad (23)$$

Proof:  $H + s(H;p;\mu;G)$  is strictly increasing in  $H$  with slope less than unity. Thus, for  $H < H^T$ ,  $H^1 = H + s(H;p;\mu;G) > H$  and  $H^1 + s(H^1;p;\mu;G) > H^1$ . Exactly similar argument applies for  $H > H^T$ .

Q.E.D.

The derivation of a partial-adjustment accumulation policy from the intertemporal optimization problem of a risk-averse agent who receives no direct utility from asset stocks and who faces no cost of adjusting these stocks is, to our knowledge, new. Dornbusch and Mussa (1975) obtain an optimal partial-adjustment policy in a deterministic, intertemporal maximization framework in which money balances yield positive utility and show that saving is actually proportional to the gap between actual and target money balances. We again note that the individual saving behavior derived above is consistent with a general equilibrium in which aggregate saving is zero in each period at the constant national price levels given by (16).

These results place restrictions on the non linear stochastic difference equation (18) which characterizes the equilibrium time path of the balance of payments in each country. These restrictions are now summarized in the following propositions.

Proposition 3.4.1: The expected current-period balance of payments surplus is of the same sign and of smaller absolute magnitude than last period's payments surplus if, last period, there was no unanticipated productivity shock ,

$$0 < E(s_{t+1} | \varepsilon_t = u; s_t = s) / s_t < 1 . \quad (24)$$

Proposition 3.4.2: The expected current-period balance of payments surplus is of the opposite sign and of smaller absolute magnitude than last period's payments surplus if, last period, the balance of payments was equal to the innovation in nominal income (planned savings was zero);

$$-1 < E(s_{t+1} | \varepsilon_t = \varepsilon; s_t = p(\varepsilon - u)) / s_t < 0 . \quad (25)$$

Proposition 3.4.3: The expected current-period balance of payments surplus is zero if, last period, there was no unanticipated productivity stock and the balance of payments was zero;

$$E(s_{t+1} | \epsilon_t = \mu; s_t = 0) = 0 . \quad (26)$$

Proposition 3.4.4: The expected current-period balance of payments surplus is of opposite sign and of smaller absolute magnitude than last period's nominal income innovation if, last period, the balance of payments was zero;

$$0 < E(s_{t+1} | \epsilon_t = \epsilon; s_t = 0) / p(\mu - \epsilon_t) < 1 . \quad (27)$$

The intuition behind the non-Markovian behavior of the balance of payments is now apparent. If last period's payments surplus was positive because the household was attempting to replenish its money balances to target, this period's surplus will, on average, be positive (but smaller) since only a partial adjustment to target is planned in any given period. If however, last period's surplus was positive due to an unanticipated increase in exports, this period's payments surplus will, on average, be negative as the household temporarily raises spending above expected nominal income, attempting to draw down only gradually its excess cash balances to target (c.f. Frenkel and Mussa (1984), p. 6). Thus, it is necessary to know both  $\epsilon_t$  and  $s_t$  to determine the distribution of  $s_{t+1}$ .

Although the conditional distribution of  $s_{t+1}$  depends upon the realizations of  $\epsilon_t$  and  $s_t$ , we now show that the expected asymptotic balance of payments surplus in each country is zero.

Proposition 3.5: The expected asymptotic balance of payments surplus is zero in each country:

$$\lim_{n \rightarrow \infty} E(s_{t+n} | s_t = s; \varepsilon_t = \varepsilon) = 0 . \quad (28)$$

Proof:  $E(s_{t+n} | s_t = s; \varepsilon_t = \varepsilon) = pE(m_{t+n+1} | m_t) - pE(m_{t+n} | m_t)$  where  $m_t = \phi(s; \varepsilon)/p + s/p$ . But, from Proposition 2.3 and Feller (1971) Theorem 1, p. 249

$$\lim_{n \rightarrow \infty} p(E(m_{t+n+1} | m_t) - E(m_{t+n} | m_t)) = 0 ; \quad (29)$$

for any initial distribution of real balances including

$$\text{Prob}(m_t = \phi/p + s/p) = 1 .$$

Q.E.D.

#### 4. Concluding Remarks

This paper has provided a choice theoretic account of the balance of payments adjustment process in a stochastic, general equilibrium. Clearly, the model abstracts from much which is relevant to the functioning of actual fixed exchange rate regimes: capital mobility, the possibility of exchange crises, the choice of a reserve currency to name just three. Yet, we would argue that our approach provides a useful first step towards more refined general equilibrium models which capture these and other features.

## APPENDIX

In this appendix, we establish the properties of the optimal expenditure function stated in the text.

Consider the intertemporal optimization problem

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

$$\text{s.t. } 0 \leq c_t \leq m_t, \quad (2)$$

$$m_{t+1} = m_t - c_t + \varepsilon_t \quad (3)$$

where  $m = H/p$ . Facing a constant price level, household consumption in each period is constrained by real balances which evolve according to the stochastic difference equation (3). Schechtman ((1976); ) and Foley and Hellwig ((1975; Proposition 1, p. 330) prove that there is a unique value function

$$v(m) = \max_{0 \leq c \leq m} \{u(c) + \beta \int v(m-c+\varepsilon) dv(\varepsilon)\} \quad (4)$$

which solves (4).  $v$  inherits all the properties of  $u$  except that it is not necessarily twice differentiable; it is strictly concave. It follows that the optimal consumption policy is uniquely defined by

$$u'(c) = \beta \int u'(m-c+\varepsilon) dv(\varepsilon) . \quad (5)$$

From strict concavity,

$$0 < c(m+\lambda) - c(m) < \lambda . \quad (6)$$

Furthermore,  $c(m) < m$ ,  $\forall m > 0$  (Foley and Hellwig (1975); Proposition 2(a), p. 330). Since  $\alpha = pc(H/p)$ , spending is linearly homogeneous in  $H$  and  $p$ . The results in Section 3 on optimal partial adjustment savings behavior are new since Foley and Hellwig consider a two state model in which it is never optimal to hold the existing stock of cash balances.

## NOTES

<sup>1</sup>c.f. Samuelson (1968, p. 8)

[Putting money in the utility function] is not the only way of introducing the real convenience of cash balances. An even better way would be to let [the utility function] depend directly only on time stream of [consumptions], and then to show that holding an inventory of [cash balances] does contribute to a more stable and greatly preferable stream of consumptions.

<sup>2</sup>Recall that a fraction  $G(0)$  of countries are not productive in any given period.

<sup>3</sup>To ease notation, we drop the "i" superscripts when no confusion will result.

<sup>4</sup>We shall assume that  $\epsilon = \mu$  is one of the possible drawings from  $G$ .

<sup>5</sup>This argument is made rigorously by Lucas (1980; p. 140).

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Assets	Liabilities
$\theta^i R_t^i$	$H_t^i$

Figure 1: Balance sheet of central bank in country  $i$ .

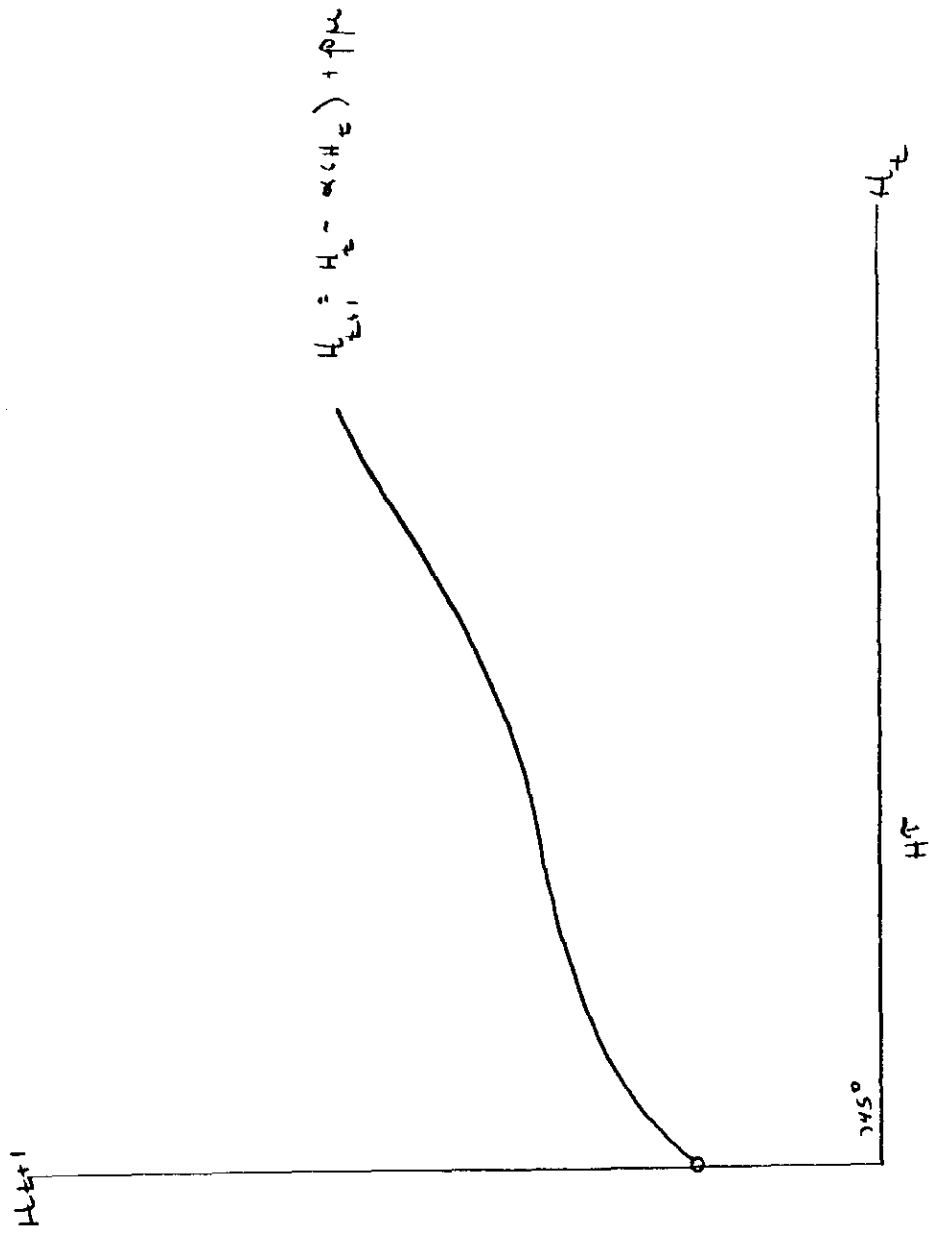


Figure 2: Determination of unique stable target money balances.

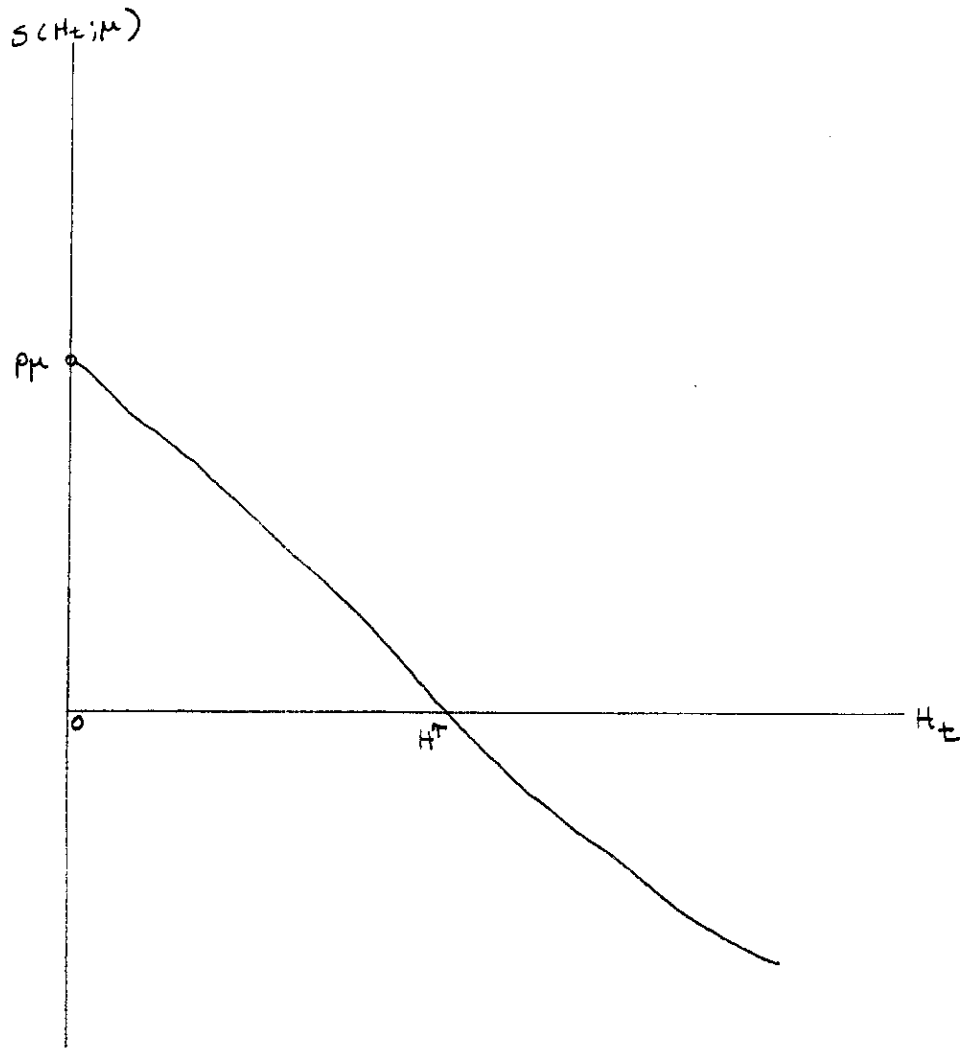


Figure 3: Planned saving.

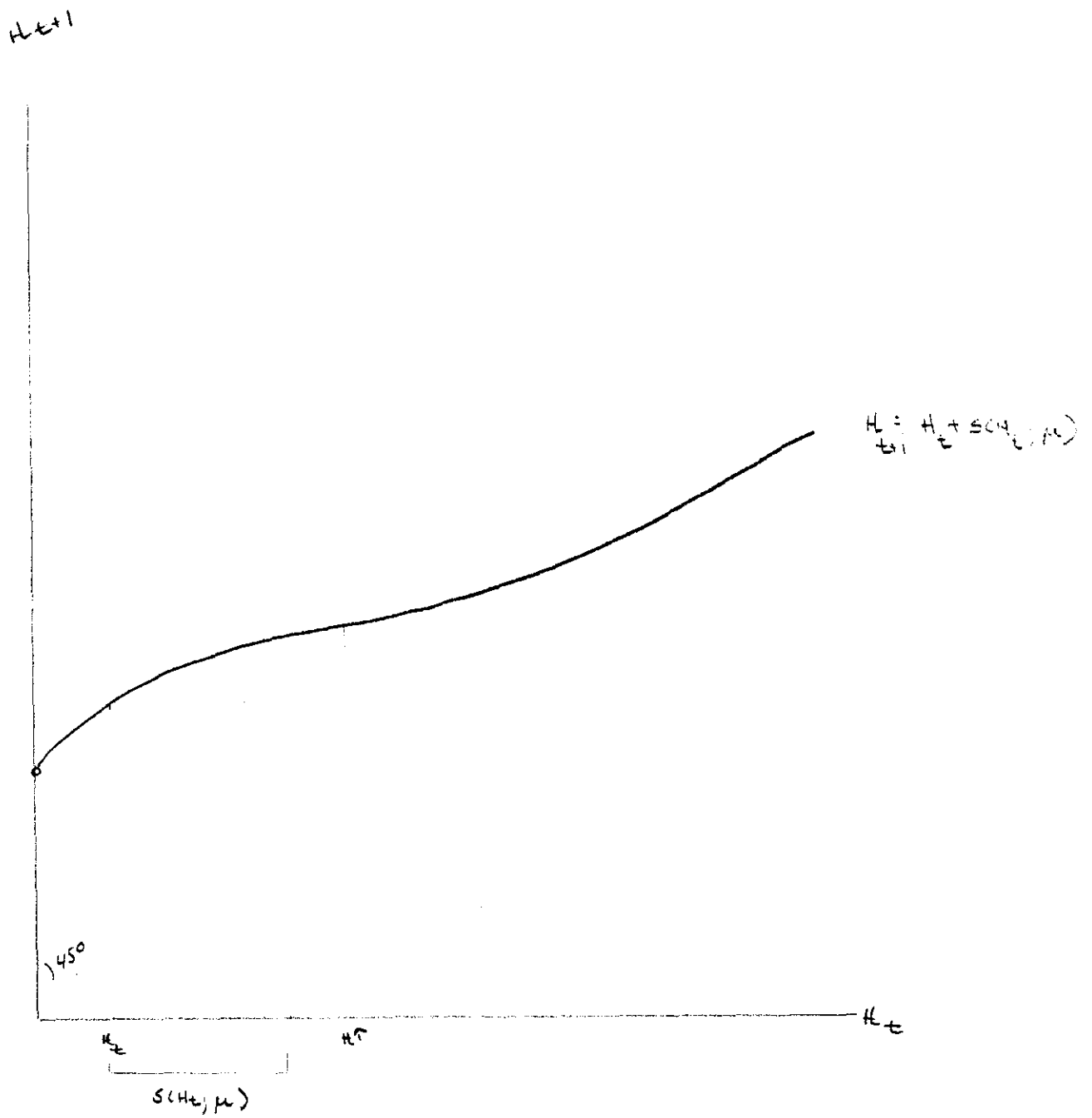


Figure 4: Planned saving less than the gap between actual and target balances