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ABSTRACT

In this paper we approach the concept of logrolling by examining a voting system where choices are made among sets of competing projects as a game in characteristic function form. We translate the question: "Will there be prices for votes on different projects which clear the market?" into a different, but equivalent question: "Is the formal game we have described a market game?" We show that in general the answer is no, unless all voters have virtually the same preferences.

LOGROLLING AND BUDGET ALLOCATION GAMES

by

M. Shubik* and L. Van der Heyden**

1. ON LOGROLLING AND BUDGET ALLOCATION GAMES

1.1. Budget Allocation Games

There is a considerable literature on simple games and voting [5, 10]. The most elementary game of this form is the simple majority voting game where, if there are n voters in total any coalition of size greater than*** $[n/2]$ is winning and all others are losing. An extremely simple characteristic function [14] of the following form is associated with a game of this type:

$$(1) \qquad v(S) = 0 \quad \text{for} \quad |S| < [(n+1)/2]$$
$$= 1 \quad \text{for} \quad |S| \geq [(n+1)/2] \; .$$

In this paper a broader class of games involving voting is considered

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***The notation $[x]$ stands for the largest integer less than or equal to x. Thus $[3/2] = 1$.

both in a sidepayment and nosidepayment version. The type of game to be described has as its basic counterpart in everyday life the voting on how to spend a budget or the decisions on appropriations by a committee. For this reason the class of games constructed is called Budget Allocation Games or Appropriation Games. We will describe them more formally in Section 3.

For purposes of exposition we begin with a simple version of the sidepayment game. Consider n individuals each having a utility function and m bills or items of appropriation to be voted upon. Suppose that each individual i places a value of a_{ij} on item j. Furthermore suppose that item j has a total cost of c_j and that individual i (or his district) will be required to pay a sum c_{ij} for j if it is approved.

We assume that

(2) $$\sum_{i=1}^{n} c_{ij} = c_j .$$

Suppose further that there is an upper limit to spending. In particular not more than some bound B can be spent on all appropriations.

Let W be the set of successful bills. Then we require that

(3) $$\sum_{j \in W} c_j \leq B .$$

In the sidepayment games which we consider first we may assume that the costs are measured in a money which can be treated as a linear utility. Thus the net worth to i of having bill j pass is given by

(4) $$a'_{ij} = a_{ij} - c_{ij} .$$

Without any further loss of generality we can limit our investigation to the a'_{ij} , c_j and B without further need for considering the a_{ij} and c_{ij} .

We assume that all bills are to be voted on in sequence until the upper bound on appropriations is reached. All individuals are completely informed and the game can be played cooperatively.

A simple majority is required to pass any bill. The voters are not required to spend all appropriations. In particular if they wished they could spend nothing.

The mechanism described above is sufficient to generate the characteristic functions of this set of games. It is easy to see that the simple games form a special subset of these games. In general the calculation of the characteristic function for a game of any size will be a large tedious combinatoric problem. An example is given below for a five person game, mainly for illustration, however the goal in this paper is to consider the properties of the core of this whole class of games without necessarily having to calculate the characteristic functions explicitly. An examination of the cores will enable us to consider when the possibility for logrolling or an economic market for votes exists and when it does not.

<u>A Five Person Example and Its Characteristic Function</u>:

Consider projects and 5 individuals with the following preferences, adjusted for costs (rows of the tableau correspond to players):

1	3/4	1/2	1/4	0
0	1	3/4	1/2	1/4
1/4	0	1	3/4	1/2
1/2	1/4	0	1	3/4
3/4	1/2	1/4	0	1

Suppose $c_j = 1$ for $j = 1, \ldots, 5$ and $B = 3$. This means that the voters have enough money to accept any 3 bills.

If we use a maxmin calculation to evaluate the amounts obtainable by the 1 and 2 person coalitions then $v(\overline{i}) = v(\overline{ij}) = 0$ for all i and j (where the notation $\overline{i}$ and $\overline{ij}$ stand respectively for the set consisting of player i and the set consisting of players i and j).

If the group as a whole were required to spend all appropriations then:

$$v(\overline{i}) = \frac{3}{4}$$

$$v(\overline{12}) = v(\overline{23}) = v(\overline{34}) = v(\overline{45}) = v(\overline{51}) = 2 \quad \text{otherwise} \quad v(\overline{ij}) = 2\frac{1}{4}$$

$$v(\overline{123}) = v(\overline{234}) = v(\overline{345}) = v(\overline{451}) = v(\overline{512}) = 5\frac{1}{2} \quad \text{otherwise} \quad v(\overline{ijk}) = 5\frac{1}{4}$$

$$v(\overline{ijk\ell}) = 6\frac{3}{4}$$

$$v(\overline{12345}) = 7\frac{1}{2} .$$

We note that although the structure of preferences appears to be completely symmetric the characteristic function is not.

1.2. Prices for Votes, Cores and Market Games

In the literature in political science and economics there have been discussions of the process of logrolling and the possibility of selling votes [2,6,7,8,17]. Especially when appropriations are concerned it seems to be likely that representative A may make a deal to vote on representative B's favorite bill in return for a vote on his own favorite.

A way in which we are able to address the question of does there exist an efficient price system for the exchange of votes is to see if it is possible to formalize the process of voting on many bills as a market game. A market game [13] has the property that it and every subgame which can be formed by removing one or more players will have a nonempty core. Shapley and Shubik [13, 15] have shown that if one is given as a datum a game in characteristic function form which has the property that it is a market game (or equivalently a totally balanced game) it is possible to find at least one trading economy which will give rise to this characteristic function. This tells us that we can work backwards from the characteristic functions to an underlying structure described in terms of economic trade. In particular if we can start with a description of a voting mechanism, calculate the characteristic function of the game it leads to and then establish that the game is a market game we will then be able to show that a price system for the exchange of votes exists. If the resultant game is not a market game then this establishes that no price system for votes can exist.

2. GAMES, CORES AND BALANCE

In this and the subsequent sections we give a formal analysis of the voting games we are considering.

Let N denote the set $\{1, 2, ..., n\}$ and $\mathcal{N}$ the set of all nonempty subsets of N. Let E^N denote the n-dimensional Euclidean space with coordinates indexed by the elements of N. For $S \in \mathcal{N}$, E^S is defined as the $|S|$-dimensional subspace of E^N obtained by requiring all the coordinates corresponding to the elements of $N-S$ to be zero. We refer to E^S_+ as the nonnegative orthant of E^S. Given two vectors x and y in E^N, $x \cdot y$ denotes their ordinary inner product $\Sigma_{i \in N} x^i y^i$. We further define e^S to be the vector in E^S defined by

$$(5) \qquad (e^S)^i = 1 \quad \text{for} \quad i \in S,$$
$$= 0 \quad \text{for} \quad i \in N-S.$$

We will sometimes write $x(S)$ for the inner product $x \cdot e^S$.

A game G is defined as a pair (N,V), where N is the above-mentioned finite set and V is a function from $\mathcal{N}$ to the subsets of E^N, satisfying for each $S \in \mathcal{N}$:

$$(6) \qquad V(S) \text{ is a closed, non-empty subset of } E^S;$$

$$(7) \qquad V(S) = V(S) - E^S_+ \text{ (algebraic subtraction).}$$

The elements of N are called the <u>players</u>, the elements of $\mathcal{N}$ the coalitions and V is called the characteristic function of the game G. The elements of $V(S)$ represent payoff vectors that the members of the coalition S can attain through their coordinated actions.

A game G is said to be compact (resp. convex) if there exists a function F, mapping the coalitions into subsets of E^N such that for each coalition S, $F(S)$ is compact (resp. convex) and $V(S) = F(S) - E^S_+$. The pair (N,F) will be called a representation of the game (N,V). A game with sidepayments is a game G for which there exists a representation (N,F) such that

$$(8) \qquad F(S) = \{x \in E^S | x \cdot e^S = v(S)\} ,$$

where v is a real-valued set function with domain $\mathcal{N}$. Games with sidepayments have as an implicit assumption the existence of a "u-money" or mechanism for the linear transfer of utility. This allows for the evaluation of every coalition at its aggregate monetary worth. These games are easily seen to be convex. We will alternatively refer to sidepayments and nosidepayment games by (N,v) and (N,V), where both v and V will be called characteristic functions. This should cause no confusion in reading the further argument.

The core of a game $G = (N,V)$ is defined as the set of payoff vectors $x \in V(N)$ such that for each $S \in \mathcal{N}$, the projection of x onto E^S is not in the interior of $V(S)$. If no such payoff vector exists, we say that the game has no core. For a game (N,v) with sidepayments, the core can be defined as the set $\{x \in E^N : x \cdot e^S \geq v(S)$ for all $S \in \mathcal{N}$, $x \cdot e^N = v(N)\}$.

An important problem in game theory is to identify games having cores. The most successful way until now has been through the introduction of balanced games. Letting τ be a subset of $\mathcal{N}$ and τ_i be the set $\{S \in \tau : i \in S\}$, we say that τ is <u>balanced</u> if there exist nonnegative <u>weights</u> $\{f_S : S \in \mathcal{N}\}$ satisfying:

(9) $$\sum_{S \in \tau_i} f_S = 1 , \quad \text{for all } i \in N ;$$

(10) $$f_S = 0 , \quad \text{for } S \notin \tau .$$

Define Γ as the set of all nonnegative vectors $f = \{f_S : S \in \mathcal{N}\}$ verifying (9) and (10) for some balanced set τ. Restricting ourselves to convex games, a game G is <u>balanced if</u>:

(11) $$\sum_{S \in \mathcal{N}} f_S V(S) \subset V(N) , \quad \text{for all } f \in \Gamma .$$

Shapley [11] has proven that balanced games characterize the sidepayment games having a core, while Scarf [9], introducing a more general definition of balance, but equivalent to (11) for convex games, established that every balanced game has a core. Unfortunately, there are nosidepayment games having cores without being balanced.

Given a compact game G and a vector $\lambda \in E_+^N$, Shapley [12] defines a real-valued set function v_λ by

(12) $$v_\lambda(S) = \max\{\lambda \cdot x | x \in V(S)\} .$$

After scaling player i's payoffs by λ^i and allowing sidepayments with the scaled payoffs, the λ-<u>transfer game</u> $G_\lambda = (N, V_\lambda)$ originates where

(13) $$V_\lambda(S) = \{x \in E^S | \lambda \cdot x \leqq v_\lambda(S)\} .$$

Shapley then proves the following proposition, which will be most useful in applying results of sidepayment theory to the nosidepayment games.

Proposition 2.1. A compact, convex game G is balanced if and only if, for each $\lambda > 0$, the λ-transfer game G_λ of G is balanced.

To complete this introductory section on game theory, the notions of subgames and totally balanced games still have to be introduced. A v subgame of a game (N,V) is a game (R,V) with $R \in \mathcal{N}$ and having as its characteristic function the function V restricted to the domain $\mathcal{N} \cap R$, consisting of all the nonempty subsets of R. A game is said to be totally balanced if all its subgames are balanced.

3. INTRODUCING SIMPLE BUDGET ALLOCATION GAMES

We define a simple budget allocation game as a triple $(N,\mathcal{W},U)$ where $(N,\mathcal{W})$ is a simple game and where the utility matrix U gives the preferences of the players (or voters) of N over a finite set of outcomes.

Following Shapley [10], we recall that a simple game is a pair $(N,\mathcal{W})$, where $\mathcal{W}$ is a subset of $\mathcal{N}$, called the set of winning coalitions. We can define the set of losing coalitions $\mathcal{L}$ by

(14) $$\mathcal{L} = \mathcal{N} - \mathcal{W} .$$

Blocking coalitions are the losing coalitions whose complement in N is also losing. Defining $\mathcal{W}^+$ as the set of supersets of the elements of $\mathcal{W}$, the collection $\mathcal{W}$ satisfies

(15) $$\mathcal{W} = \mathcal{W}^+ ,$$

(16) $$\mathcal{W} \neq \emptyset, \mathcal{N} .$$

Condition (15) asserts that any superset of a winning coalition is winning, while (16) eliminates the trivial cases where all coalitions are winning or none are. We assume further for this paper that all coalitions of size n-1 are winning, or more formally that

(17) $N - \{i\} \in \mathcal{W}$ for all $i \in N$.

A player i violating (17) is called a veto-player.

The players are presented with a finite set $P = \{1, 2, \ldots, p\}$ of projects (or outcomes), out of which a single one must be selected.* A project is selected as soon as there is a winning coalition in favor of it. In order to completely define how the game is played, we specify that if no winning coalition is formed, project 1 is selected. This project is really a "dummy" project needed to reveal the voters' preferences for the status-quo, where none of the "real" projects, labelled 2 to p are selected. The preferences of the players are, as already mentioned, given by the utility matrix $U = \{u_j^i | i \in N, j \in P\}$, where u_j^i is a real number representing the utility of player i for project j .

We recall that the question we want to answer is: when does there exist an underlying economic market where voters trade votes at equilibrium prices? Given previous work by Shapley and Shubik for the sidepayment case, and by Billera [3], Billera and Bixby [4] for the nosidepayment case, this question can be reformulated as: when are our simple voting games totally balanced?

*This formulation includes the one given in Section 1.

4. THE SIDEPAYMENT CASE

The characteristic function v of a simple budget allocation game $(N, \mathcal{W}, U)$ can be defined in the following way:

$$(18) \qquad \begin{aligned} v(S) &= \max_{j \in P} u_j(S) && \text{for } S \in \mathcal{W}, \\ &= u_1(S) && \text{for } S \in \mathcal{L},\ N-S \in \mathcal{L}, \\ &= \min_{j \in P} u_j(S) && \text{for } S \in \mathcal{L},\ N-S \in \mathcal{W}, \end{aligned}$$

where we recall that we write $u_j(S)$ for the sum $\Sigma_{i \in S} u_j^i$. The definitions (18) describe the fact that a winning coalition seeks to maximize its joint value, while a blocking coalition can impose the status quo. Losing coalitions which are not blocking can insure themselves only of the worst outcome.

The property that in the absence of veto-players, any payoff vector in the core must coincide with an outcome will be easily established, after which we derive a necessary condition for the game to have a core.

Proposition 4.1. A simple budget allocation game without veto-players has at most a one-point core coinciding with one of the outcomes.

Proof. If the payoff vector x belongs to the core $x(N) = u_k(N)$ for some project k . If x is not identical to some project in P , we can find two players i and h verifying $x^i > u_k^i$ and $x^h < u_k^h$. Hence it follows, since $N - \{i\} \in \mathcal{W}$, that

$$(19) \qquad x(N - \{i\}) < u_k(N - \{i\}) \leqq v(N - \{i\}) ,$$

contradicting the assumption that x belongs to the core.

Assume now that the one-point core is given by outcome k . If a winning coalition S strictly prefers an outcome j different from k , we have that

$$(20) \qquad u_k(S) < u_j(S) \leqq v(S) ,$$

contradicting that the payoff vector $u_k = \{u_k^i | i \in N\}$ is in the core. Similarly, if a blocking coalition S can do better for itself by imposing the status quo, then the core must also be empty. We summarize both facts in the following proposition.

Proposition 4.2. For a simple budget allocation game $(N, \mathcal{W}, U)$ with blocking coalitions given by $\mathcal{B}$, to have a core it is necessary that there exists a project k such that:

$$(21) \qquad u_k(S) = \max_{j \in P} u_j(S) \quad \text{for all} \quad S \in \mathcal{W} ,$$

$$(22) \qquad u_k(S) \geqq u_1(S) \quad \text{for all} \quad S \in \mathcal{B} .$$

If there is a project k satisfying conditions (21) and (22), one immediately can write that $v(T) = u_k(T)$ if $T \in \mathcal{W}$ and $v(T) \leqq u_k(T)$ otherwise. Hence if R is a winning coalition,

$$(23) \qquad v(R) = u_k(R)$$

and

$$(24) \qquad v(T) \leqq u_k(T) \quad \text{for all} \quad T \subset R .$$

If R is a blocking coalition, any subset T of R is either blocking or losing so that (23) and (24) hold with k replaced by 1 . If R is losing without being blocking, then by definition of $v(R)$,

$v(R) = u_j(R)$ for some $j \in P$ and any subset T of R verifies $v(T) \leqq u_j(T)$. So we have shown that also in this case, (23) and (24) are satisfied with j instead of k.

The above discussion has shown that given any subgame (R,v), there is a project k satisfying (23) and (24). Considering any non-negative vector of weights $f \in \Gamma$ associated with a balanced collection τ of subsets of R, we are now in a position to establish that

$$\text{(25)} \qquad \sum_{T\in\tau} f_T v(T) \leqq \sum_{T\in\tau} f_T u_k(T) = \sum_{i\in R} u_k^i \left(\sum_{T\in\tau_i} f_T \right)$$

$$= u_k(R) = v(R) .$$

Hence any subgame (R,v) is balanced, so that our simple budget allocation game has been shown to be totally balanced.

Proposition 4.3. A simple budget allocation game without veto-players is totally balanced if and only if there is a project preferred by all winning coalitions and preferred over the status-quo by all blocking coalitions.

5. THE NOSIDEPAYMENT CASE

With all the foundations laid down in Sections 2 and 4, this case, also the more interesting one, will be easy to analyze.

Given a project j in P, we define $F_j(S)$ as follows:

$$\text{(26)} \qquad F_j(S) = \{x \in E^S : x^i \leqq u_j^i \ \text{ for } \ i \in S\} .$$

The characteristic function $\widetilde{V}$, mapping the subsets of N into subsets of E^N, is given by:

$$(27)\qquad \tilde{V}(S) = \bigcup_{j\in P} F_j(S) \quad \text{if} \quad S \in \mathcal{W} ,$$

$$= F_1(S) \quad \text{if} \quad S \in \mathcal{L} , \quad N\text{-}S \in \mathcal{L} ,$$

$$= \bigcap_{j\in P} F_j(S) \quad \text{if} \quad S \in \mathcal{L} , \quad N\text{-}S \in \mathcal{W} .$$

These definitions can be interpreted similarly as the corresponding definitions in the sidepayment case. Since winning coalitions can select any project they want, there is no reason why they should not be allowed to randomize between projects. It is thus legitimate to convexify their set of attainable payoff vectors by redefining the characteristic function V of the game as follows:

$$(28)\qquad V(S) = \text{conv}(\tilde{V}(S)) \quad \text{if} \quad S \in \mathcal{W} ,$$

$$= \tilde{V}(S) \quad \text{if} \quad S \notin \mathcal{W} ,$$

where $\text{conv}(A)$ denotes the convex hull of A .

We now have a convex compact game $G = (N,V)$ which by Proposition 2.1 is balanced only if all the λ-transfer games with $\lambda > 0$ are balanced. Considering the λ^0-transfer game with $\lambda^0 = e^N$, we obtain the sidepayment game analyzed in Section 4. If this game is balanced, then Proposition 4.3 tells us that there is a project, say k , which is preferred by all winning coalitions, implying

$$(29)\qquad u_k(N - \{i\}) \geqq u_j(N - \{i\}) \quad \text{for all} \quad j \in P , \quad i \in N .$$

Summing (24) over the players, one notes that

$$(30)\qquad u_k(N) \geqq u_j(N) \quad \text{for all} \quad j \in P .$$

Suppose now that player h is not happy with project k, more specifically that $u_\ell^h = \max_{j \in P} u_j^h > u_k^h$. From (30) it then follows that

(31) $$u_k(N - \{h\}) > u_\ell(N - \{h\}) .$$

Defining λ^h by

(32) $$(\lambda^h)^i = 1/\mu \quad \text{for} \quad i \neq h ,$$

$$= \mu \quad \text{for} \quad i = h ,$$

it is easy to see that in the λ^h-transfer game $\mathcal{G}_{\lambda^h}$, the grand coalition follows player h's choice ℓ when μ is large enough, while $N - \{h\}$ selects project $k \neq \ell$. Proposition 4.3 enables us to conclude that, since G_{λ^h} is not balanced, G also can not be balanced, so that a necessary condition for balancedness is the existence of a project k verifying

(33) $$u_k^i \geqq u_j^i \quad \text{for all} \quad j \in P , \quad i \in N .$$

It is now easily seen that if condition (33) is satisfied by some project k in game G, then by Proposition 4.3, all λ-transfer games G_λ are balanced. The same obviously holds for all subgames, so that (33) is revealed to be a sufficient condition (or total balancedness of game G as well.

Proposition 5.1. For a simple voting game without veto-players and side-payments to be totally balanced, it is necessary and sufficient that there exists a project k preferred by all the individual players.

We note to conclude that Proposition 5.1 is really an ordinal result, since condition (34) is invariant under independent and strictly monotonic transformations of the players utilities.

6. SOME FURTHER PROBLEMS AND COMMENTS

6.1. Simple Budget Allocation Games with Veto Players

We now comment on simple budget allocation games with veto-players. While the condition formulated in Proposition 4.3 is sufficient for this proposition to hold, it is no longer necessary as the following example shows.

Consider a 3-person simple budget allocation game with the set of players given by $N = \{1, 2, 3\}$ and winning coalitions given by $\mathcal{W} = \{123, 12, 13\}$. Let the utility matrix U be

$$U = \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & c \end{pmatrix}$$

and assume that $a, b, c > 0$. The characteristic function v is defined by

$$v(\overline{123}) = \max(a, b+c) , \quad v(\overline{12}) = \max(a,b) ,$$

$$v(\overline{23}) = 0 , \quad v(\overline{13}) = \max(a,c) ,$$

$$v(\overline{1}) = a , \quad v(\overline{2}) = v(\overline{3}) = 0 .$$

This game has a core for all positive values of a , b and c , since the allocation giving the payoff of the grand coalition to player 1 can not be blocked by any coalition. By super additivity of v it is clear that also all 2-person subgames are balanced. If $b+c < a$, player 1 prefers the status-quo to the project favored by the grand coalition, violating the condition of Proposition 4.3.

The same example can be used to show that the condition of Proposition 5.1 is also no longer necessary when one allows veto-players. A λ-transfer game of this nosidepayment game can be thought of as a simple budget allocation game with sidepayments and utility matrix U_λ given by

$$U_\lambda = \begin{pmatrix} \lambda_1 a & 0 \\ 0 & \lambda_2 b \\ 0 & \lambda_3 c \end{pmatrix}$$

All these λ-transfer games are totally balanced as shown above, and hence we may again invoke Proposition 2.1 to conclude total balancedness for the original non-sidepayment simple voting game, although there is not a unique project preferred by all players as required by Proposition 5.1.

6.2. Constructing the Market Game When Possible

We leave the task of constructing the market game and hence actually calculating the price of votes when a simple voting game is totally balanced, as an open problem.

We note that our prime concern in this paper is with the conditions for logrolling to be feasible and we have established that in the context presented here it requires the unlikely condition that there exists a project most favored by all.

6.3. Cardinal or Ordinal Utility?

In our discussion we have utilized cardinal utility scales. Scarf [9] in his discussion of balance conditions made use of preference orderings only. Billera [3] uses cardinal utility. Shapley and Shubik [16] in their discussion of the inner core of nosidepayment games and its relation to the core of associated sidepayment games also use cardinal utility.

6.4. Logrolling as a Process in Politics

A look at the actual political process in any legislature is enough to persuade most individuals that vote trading as part of political quid pro quo is usually present. But vote trading does not imply a market price for votes in the same sense as there is a price for goods in a mass market economy.

It is possible that the exchange rates for votes might be characterized by a different solution concept other than the core, for example the Aumann Maschler bargaining set [1]. It is also possible that the intuitive idea of vote trading can only be analyzed adequately by a model in extensive form, i.e. by a model which is directed more towards the analysis of process, rather than one cast in an essentially static strategic mode.

Our major purpose in this paper was to examine in a rather literal manner the possibility of a connection between static economic markets and a market for votes. Our essentially negative conclusion is that the highly attractive analogy between economic markets and the trading of votes in a legislature almost never holds true.

Acknowledgments

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