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A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

PART 21

FIAT MONEY, BANK MONEY, THE FLOAT AND THE MONEY RATE OF INTEREST

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# A THEORY OF MONEY AND FINANCIAL INSTITUTIONS

## PART 21

### FIAT MONEY, BANK MONEY, THE FLOAT AND THE MONEY RATE OF INTEREST\*

Martin Shubik

#### 1. INTRODUCTION

In this paper it is shown that for any trading economy satisfying the conditions required to guarantee the existence of a competitive equilibrium price system<sup>1</sup> there exist two associated trading economies one using bank money and the other using bank money and fiat money such that in the first competitive equilibrium is achieved with a zero rate of interest and in the second competitive equilibrium is achieved with a positive rate of interest. The fiat money issue is used to cover the float costlessly.

#### 2. COMPETITIVE EQUILIBRIUM

Consider an economy with  $m$  traders,  $n$  goods and  $T$  time periods. Each trader  $i$  has a utility function of the form:

$$\varphi_i(x_{j,1}^i, x_{j,2}^i, \dots, x_{j,T}^i) \quad \text{for } j = 1, \dots, n.$$

where  $x_{j,t}^i$  is the amount of commodity  $j$  consumed by  $i$  during  $t$ .

Each trader  $i$  owns a bundle of resources  $(A_{11}^i, \dots, A_{nT}^i)$  where  $A_t^i = (A_{1,t}^i, \dots, A_{n,t}^i)$  for  $t = 1, \dots, T$ . Although this model involves  $T$  time periods we may consider for the general equilibrium analysis that there are  $nT$  goods to be traded immediately. This gives us a one period, static or strategic form model of trade.

A competitive equilibrium is defined by a set of prices

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$p_{1,1}, p_{2,1}, \dots, p_{n,T}$  and distributions  $x^i$  such that

$$(1) \quad \max_{x^i, t} \varphi_i(x_{1,1}^i, \dots, x_{nT}^i) \quad \text{subject to}$$

$$(2) \quad \sum_{j=1}^n \sum_{t=1}^T p_{j,t} (x_{jt}^i - A_{jt}^i) = 0$$

and

$$(3) \quad \sum_{j=1}^m A_{j,t}^i = \sum_{j=1}^m x_{j,t}^i$$

yields a Pareto optimal outcome.

### 2.1. A Step towards Dynamics

Suppose that all of the commodities have a duration of only one period. We may reinterpret the general equilibrium model as involving a sequence of  $T$  markets. The institutional arrangements governing the path of trade and the handling of credit are where our consideration of money appears.

Although general equilibrium theory gives the appearance of being institution free it can best be considered as an extreme case with institutions so advanced; information communication and trust so complete that one mass market which trades in the first period's spot goods and in futures for all other periods carries out all economic activity in one trading session. All trade is made in terms of a money of account which has no corporeal existence and is neither fiat nor bank money.

There are two considerably more realistic models which can be constructed which involve banking, bank money and fiat directly. They are described below.

### 3. A COMPETITIVE ECONOMY WITH A PERFECT FORECASTING BANK AND BANK MONEY

#### 3.1. The Float

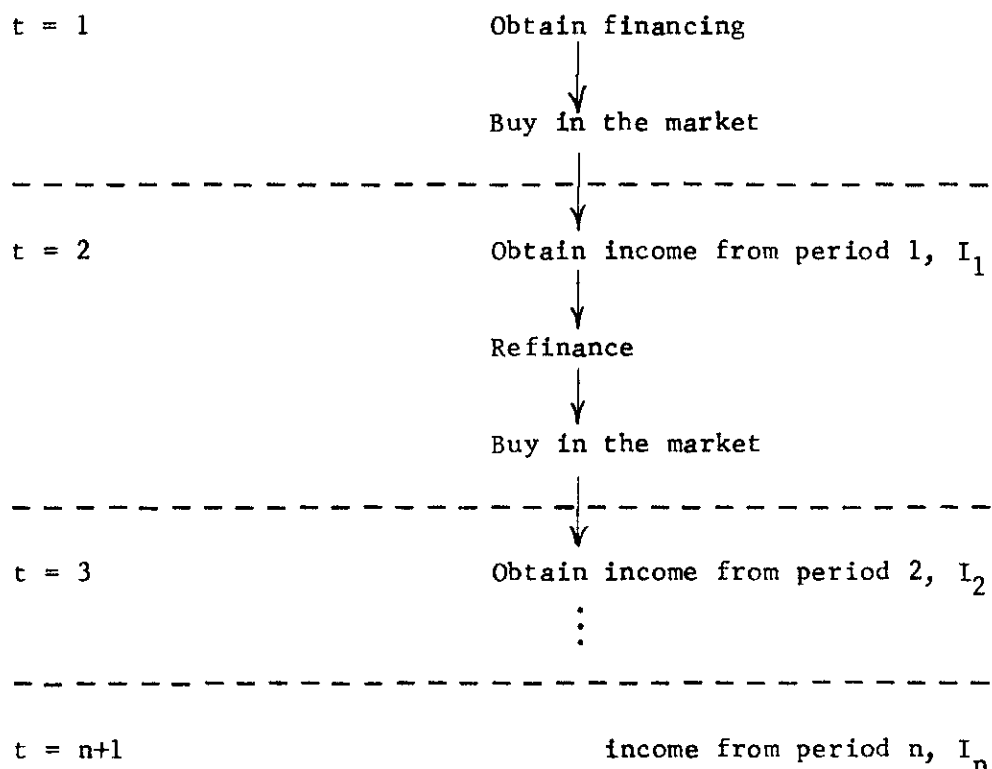
A key element in trade which causes the need for financing or the extension of credit or trust is the lack of synchronization of payments and receipts in trade. An easy way to introduce this into a trading model is to consider that at each period all individuals are required to sell all of their goods but they do not obtain the income from this sale until the start of the next period.

It is as though all goods were consigned to a central market each period where they are sold, the money from their sale is gathered during trade and paid out to its owners after trading for that period has been completed. In this scheme the size of the float or the amount of money in limbo will be precisely the total value of trade during that period.\*

Suppose that there are no futures markets, there is also no fiat money in existence, however a banking system exists which can create bank debt or bank money. We may view trade as follows:

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\*It is fairly straightforward to introduce a distributed lag on payments, in which case the float will be smaller and more erratic, however conceptually nothing will have changed.



#### FINAL BALANCING OF BOOKS

Even if all individuals had the same preferences and endowments in this model there would still be a need for one period "bridging finance" to cover the float. We return to this in Section 4.

### 3.2. Avoiding Bankruptcy or Restrictive Finance

A true dynamics would dispense with the almost mystical concept of perfect foresight. If looked at correctly the assumption of perfect foresight makes sense by recasting the problem in static terms. The competitive equilibrium is that set of prices which would exist if all individuals had the appropriate set of self-fulfilling prophecies. The chances that individuals would have prior knowledge of these prices are slim even if the economic dynamics has strong equilibrating tendencies.

If we accept a static view of trade then the search for a general equilibrium amounts to finding those prices which, if they were supplied in advance would serve to coordinate individual economic activity.

When we consider the presence of a bank to provide financing of trade, in order to keep the system well defined we must also specify the bank's state of information. In particular if the bank wishes to avoid having any of its borrowers becoming bankrupt or failing to obtain efficient financing the bank must have perfect information concerning the prices and financial strength of each individual at all times. Suppose that the bank has perfect information at every time  $t$ . It may calculate the present value of the future income of every individual. Let  $W_t^i$  be the present value of future income to individual  $i$  at time  $t$ .

Let  $b_t^i$  = the net financial position of individual  $i$  at the start of period  $t$  after receipt of income from  $t-1$ , but before refinancing. Income from period 0 can be regarded as a free injection of fiat money by an outside controlling agency which we may regard as the government.

$$(4a) \quad \sum_{i=1}^m b_1^i = 0 \quad \text{if there is no fiat issued}$$

$$(4b) \quad = B > 0 \quad \text{if there is a net amount of fiat issued.}$$

We may assume that individual  $i$  starts with an amount of fiat of  $B^i$ . Here we define:

$$(5) \quad \sum_{i=1}^m B^i = B .$$

At this point we assume that  $B^i \geq 0$ . Later it is possible to consider negative values in terms of taxation.

Let  $u_t^i$  = the amount of bank money borrowed or money deposited by individual  $i$  at time  $t$ . If the bank wishes to be as "liberal" as possible yet capable of avoiding bad loans it will have the following policy:

$$(6) \quad \begin{aligned} u_t^i &\leq W_t^i && \text{if } b_t^i \geq 0 \\ &\leq W_t^i + b_t^i && \text{if } b_t^i \leq 0 . \end{aligned}$$

If the bank does not have a completely accurate forecast of individuals' incomes then in order to avoid bad debts it cannot lend more than the minimum estimate of income. If its minimum estimate is higher than actual then the danger of bankruptcy is not avoided, if it is lower than actual then the credit constraint will be binding in some market. We do not deal with imperfect forecasting here.

The bank considered here is an "outside bank"; it is a mechanism which costlessly produces the credit that it issues and "sterilizes" all payments which it receives. We assume that the bank pays the same money rate of interest,  $\rho$  on deposits as it charges on loans.

### 3.3. Normalizing Prices and Financing the Float

As the price system is homogeneous of order zero in prices we have a degree of freedom that can be tied down in the three models presented here. The first, and important feature to observe is that there are only two "quantities" of fiat money. They are zero or some. The size of any positive amount does not matter. What counts from the viewpoint of distribution is the ratios of the distribution of money.

With complete generality we may set  $p_{1,1} = 1$  and set

$$(7) \quad B = \sum_{j=1}^n p_{j1} A_{j1} .$$

In two of the three models discussed here; the general equilibrium economy and the economy without fiat but with banking money, equation (7) is not relevant; however for purposes of comparison we have  $p_{1,1} = 1$  in all models.

In the model with fiat money and banking we have not yet specified what the  $B^i$  should be. If we wish to obtain a monetary model which gives the same physical outcomes as the general equilibrium we need to have  $B^1 : B^2 : \dots : B^m$  as  $W^1 : W^2 : \dots : W^m$ . This is discussed in Section 4. Other ratios are the equivalent of tax transfers.

#### 4. THE RATE OF INTEREST

##### 4.1. Bank Money Only

Let  $W^i$  = the total present value of the income of the  $i^{\text{th}}$  trader at some competitive equilibrium point. In the straight general equilibrium model the value of the final holdings equals the value of the initial holdings. In the context of a model where all goods must be bought through a market we must therefore have the present value of income equal to expenditure; but owing to the time lag in the receipt of income, if the books are to balance at the start of period  $n+1$  then we must have

$$(8) \quad W^i - \frac{W^i}{1+\rho} = 0$$

which implies  $\rho = 0$ .

We note that even though the money rate of interest is zero, credit is nevertheless restricted by the conservative lending policy and that which is borrowed must be paid back.

#### 4.2. Fiat and Bank Money: General Equilibrium Proportions

The total amount of fiat is  $B$ , it is issued in the ratio of  $W^1 : W^2 : \dots : W^m$ , hence the issue is

$$\frac{W^1}{W-B}, \frac{W^2}{W-B}, \dots, \frac{W^m}{W-B} .$$

The accounting relationship needed to balance the books must now include the issue of fiat. As all money is worthless in the  $n+1^{\text{st}}$  or settlement period, any maximizing trader will attempt to be in debt for at least the amount of the money income he receives in the  $n+1^{\text{st}}$  period. The perfect forecasting bank will have restricted him so that he cannot be in debt for more than his final income (see (6)). Thus we have:

$$(9) \quad \frac{W^i}{W-B} + \frac{W^i}{1+\rho} - W^i = 0 \quad \text{for } i = 1, \dots, m$$

hence

$$(10) \quad (1+\rho) = \frac{W}{W-B} \quad \text{or} \quad \rho = \frac{B}{W-B} .$$

An interpretation of the roles of the fiat money, the bank money and the float and rate of interest is that the initial injection of fiat provides the traders with an interest free financing of the first period's float.\* The banking system provides the means for the intertemporal financing. The rate of interest is precisely the rate required for the banking system to consume the issue of fiat money.

It must be stressed that the banking system is an "outside bank" and is a mechanism which may costlessly create and reabsorb credit. All banking income is sterilized and neither the government nor the banking system consume real goods.

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\* See Example 2 in Section 5 for what happens if the float is not fully covered by the fiat issue.

#### 4.3. Taxation

Even if  $B > 0$  this does not necessarily call for the  $B^i > 0$ . We consider any distribution of the  $B^i$  which can be written as:

$$(11) \quad B^i = T^i + \frac{W^i}{W} B,$$

where

$$(12) \quad \sum_{i=1}^m T^i = 0.$$

Clearly the nontaxation or subsidy solution will no longer hold. This problem is similar to one on taxation to show that any point on the (individually rational part of the) Pareto optimal set can be achieved by lump sum money taxes and subsidies.

#### 4.4. The Infinite Horizon

Even though the economy has an infinite horizon as long as the payoffs to each trader are bounded then we will be able to have a positive money rate of interest given by the ratio of the first term to the sum of the series. An example in Section 5 illustrates this.

At this point it is probably more reasonable to specialize the form of the utility function to:

$$(13) \quad U^i = \sum_{t=1}^{\infty} \beta_i^{t-1} \varphi_i(x_{jt}^i), \quad i = 1, \dots, m.$$

This introduces linear separability in the valuation of intertemporal consumption. With production the complementarities are naturally introduced by time links in capital goods. Similarly with consumption it is more reasonable to

consider high interlinkages where some form of consumer durable is involved.

An example of modeling difficulties in a dynamic interpretation of a utility function is provided by an extended Cobb-Douglas function. Consider:

$$(14) \quad \varphi_1(x_1, \dots, x_T) = T \sqrt[T]{\prod_{t=1}^T x_t} .$$

This is bounded but has the unreasonable feature that failure to obtain one good at any time wipes out value for all time.

#### 4.5. Multiple Equilibria and the Money Rate of Interest

One needs to consider only an example as simple as trade among two individuals with two commodities treated as exchange through time to see that a different rate of interest may be associated with each competitive equilibrium point and hence an exogenous selection of the rate of interest combined with the initial issue of fiat selects the equilibrium point.

Consider two individuals with utility functions  $\varphi_1(x^1, y^1)$  and  $\varphi_2(x^2, y^2)$  where the first individual owns all of the resource to be traded in period 1; call this  $A_1$ . The second individual owns all of the commodity in the second period; call this  $A_2$ .

Suppose that the general equilibrium yields three equilibrium points. In each instance we normalize the first price to be 1. We denote the three price systems by  $(1, p_a)$ ,  $(1, p_b)$  and  $(1, p_c)$ . We calculate the wealth of the individuals at each equilibrium. We have  $(W_a^1, W_a^2)$ ,  $(W_b^1, W_b^2)$  and  $(W_c^1, W_c^2)$ .

In the models with fiat and bank money the exogenous fiat issuing agency, or the "referee for these games" issues  $A_1$  units of fiat in proportions of

$W_a^1 : W_a^2$  ;  $W_b^1 : W_b^2$  and  $W_c^1 : W_c^2$  respectively. Our accounting equations give us:

$$(15) \quad \frac{W_a^1 A_1}{W_a^1 + W_a^2} + \frac{W_a^1}{1 + p_a} = W_a^1$$

or

$$(16) \quad p_a = \frac{A_1}{W_a^1 + W_a^2 - 1} = \frac{A_1}{p_a A_2}$$

and similarly for the two other equilibrium points. A specific example is calculated for illustration in Section 5.

## 5. SOME EXAMPLES

### EXAMPLE 1

Given  $m$  individuals each with a utility function of the form:

$$(17) \quad U^i = \sum_{t=1}^T \beta^{t-1} x_t^i \quad \text{where } 0 < \beta < 1 .$$

Assume that there is a supply of  $m$  units of a single nondurable good supplied each period. Thus total resources can be indicated by  $(m, m, \dots, m)$ . Further assume that each individual has an ownership claim of  $1/m$  each period. In the general equilibrium model this would mean each owns a vector of resources of  $(1, 1, \dots, 1)$ , each would consume his own initial resources and obtain:

$$(18) \quad U^i = \frac{1 - \beta^T}{1 - \beta}$$

and the prices would be  $1, \beta, \beta^2, \dots, \beta^{T-1}$ .

Suppose we start off each individual with one unit of fiat money. Spot prices in this model will be

$$1, \beta(1+\rho), \beta^2(1+\rho)^2, \dots$$

where  $\rho$  is the money rate of interest. From (10)

$$(19) \quad (1+\rho) = \frac{1}{\beta} \left\{ \frac{1 - \beta^T}{1 - \beta^{T-1}} \right\} \quad \text{for } T > 1 .$$

for  $t \rightarrow \infty$ ,  $(1+\rho) = 1/\beta$  and the spot prices are constant over time. When  $t$  is finite prices are somewhat higher than for the infinite economy as the banking system has less time to pull out the issue of fiat.

#### EXAMPLE 2: AN INFINITE ECONOMY WITH A PARTIALLY COVERED FLOAT

It has been asserted in Section 3 that the issue of fiat money is used to totally finance the float. This is not necessarily so. However if it does then for the infinite economy we obtain a money rate of interest equal to the "natural rate" otherwise we do not.

Consider an economy as in Example 1, except that instead of a per capita issue of 1 unit of fiat we normalize the financial system so that the fiat covers only a percentage  $\alpha$  of the first period float. In order to maintain prices of:

$$1, \beta(1+\rho), \beta^2(1+\rho)^2, \dots, \beta^{n-1}(1+\rho)^{n-1}$$

the traders would have to borrow  $1-\alpha$  each in the first period as well as further sums subsequently. The interest payments will equal the income at settlement hence

$$(1-\alpha) + \frac{[\beta(1+\rho) - 1]}{(1+\rho)} \left[ \frac{1 - \beta^{n-1}}{1 - \beta} \right] = \frac{\beta^{n-1}}{1+\rho},$$

for  $n \rightarrow \infty$  this gives

$$1+\rho = \frac{1}{(1-\alpha)(1-\beta) + \beta}$$

where  $\alpha$  is the percentage of the float financed free. If  $\alpha = 0$ ,  $\rho = 0$ ; if  $\alpha = 1$  then  $1+\rho = 1/\beta$  which gives the "natural rate" of interest.

EXAMPLE 3: A SEASONAL ECONOMY

Given  $m$  individuals each with a utility function of the form:

$$(18) \quad U^i = \sum_{t=1}^T \beta^{t-1} \sqrt{x_t^i}$$

consider equal ownership claims each period with  $4m$  units available in the first period and  $9m$  in the second period. In general equilibrium prices will be:

$$\frac{1}{2}, \frac{1}{3}\beta, \frac{1}{2}\beta^2, \frac{1}{3}\beta^3, \dots \quad \text{the fiat issue is } 2$$

and

$$(19) \quad w^1 = w^2 = \frac{2 + 3\beta}{1 - \beta}.$$

From (10) 
$$1+p = \frac{2 + 3\beta}{5\beta} .$$

Let us reverse the sequence so that there are  $9m$  units in the first period and all subsequent odd periods and  $4m$  in the second and future even periods. Prices will be:

$$\frac{1}{3}, \frac{1}{2}\beta, \frac{1}{3}\beta^2, \frac{1}{2}\beta^3, \dots \quad \text{the fiat issue is } 3$$

and

$$(20) \quad W^1 = W^2 = \frac{3 + 2\beta}{1 - \beta}$$

which gives

$$(21) \quad 1+p = \frac{3 + 2\beta}{5\beta} .$$

Hence we see that for  $0 < \beta < 1$  the rate of interest will be larger if the economy starts at a point of high economic activity in a seasonal economy than if it starts at the low point in the seasonal. The reason appears to be that the quantity of the float relative to future time linked economic activity is larger for the bigger start hence the banking system has more float to pull out.

#### EXAMPLE 4: AN EXPONENTIALLY DECAYING ECONOMY

Consider the same model as Example 1 with the following modification. Suppose that instead of a steady stream of "manna" or the one consumer good, i.e.  $(1, 1, 1, \dots)$  there is an exponentially decreasing supply  $(1, \pi, \pi^2, \dots)$ . Relative to the previous economy in Example 1 the future cash needs would be shrinking if the interest rates and prices were the same. However:

$$(22) \quad 1 + \rho = \frac{1}{\beta\eta} \quad \text{where} \quad 0 < \eta, \beta < 1 .$$

From (22) it can be seen that the money rate of interest must be much higher in the decaying economy than in the stationary one. For example with  $\beta = \eta = 1/2$  we obtain

$$\rho = 3 \quad \text{in decay and} \quad \rho = 1 \quad \text{in a stationary economy.}$$

A more realistic model might have  $\beta = .94$  and  $\eta = .99$ ; this gives  $\rho = 7.46\%$  compared with  $\rho = 6.38\%$ . The price level in the decaying economy inflates.

It must be noted that in these simple models, once the government has issued fiat it plays no further role. In a modern economy it clearly has many means for injecting and removing fiat hence has considerable control. For example a constant governmental income tax of a percentage  $(1-\eta)$  of income will change the bank rate to  $1/\beta$ . This however requires the government to sterilize the money, i.e. it must not be used for purchases of real goods.

Both governments and banks consume real resources with tax money and bank profits. The modeling of these phenomena will be dealt with in a further paper.

If we set  $1/\beta > \eta > 1$  we have a growing (but bounded income) economy. This has a lower rate of interest than the stationary economy and has falling spot prices. For example suppose  $\beta = .94$  and  $\eta = 1.01$ ; this gives  $\rho = 5.33\%$  compared with  $6.38\%$  and instead of a stationary spot price of 1 each period, prices fall at the rate of  $1/\eta = .9901$ .

#### EXAMPLE 5: MULTIPLE EQUILIBRIA

Consider two traders with the following utility functions\* and initial

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\*This example is a slight modification of an example from an unpublished manuscript by L. S. Shapley and M. Shubik.

bundles

$$U^1(x,y) = x + 100(1 - e^{-y/10}) \quad (40,0)$$

$$U^2(x,y) = y + 110(1 - e^{-x/10}) \quad (0,50)$$

The conditions for the competitive equilibria given by:

$$x(1 + 11e^{-x/10}) = 10 \log 110 .$$

There are three solutions to this equation within the zone of the contract curve delineated by giving the first trader an ownership claims to all of the 40 units to be sold in the first period and the second of all of the 50 units to be sold in the second period. Table 1 shows allocations, prices and payoffs to the two traders at the three equilibria. The last two columns show the spot value of trade each period based on the assumption that all goods go through the market.

TABLE 1

	Allocations (to trader 2)	Future Prices		Payoffs		Discounted Trade Volumes
		$P_1$	$P_2$	$U^1$	$U^2$	
Origin	0    50	-	-	40	50	—
Contract Curve $C^1$	40   44.89	-	-	40	154.89	—
$C^2$	4.83   7.83	-	-	133.69	50	—
Comp Eq.						
$E_a$	7.74   10.74	1	.1972	130.29	70.01	40   9.86
$E_b$	26.82   29.82	1	1.3333	99.87	132.30	40   66.665
$E_c$	36.78   39.77	1	3.5714	67.26	146.99	40   178.57

We may calculate the three different interest rates directly from (16)

$$\rho_a = \frac{4}{5} \left( \frac{1}{.1972} \right) = 633.8\%$$

and similarly

$$\rho_b = 93.75\% \quad \text{and} \quad \rho_c = 35\%$$

Thus in the second period the actual spot prices are 1.447, 2.5833 and 4.8214 and the spot volumes of trading in money are 72.35, 129.165 and 241.07 respectively.

## 6. CONCLUDING REMARKS

There are many separate but related problems to be dealt with in the construction of a satisfactory theory of money and financial institutions. The guiding principle behind this series of papers is to try to isolate various of the phenomenon and to deal with them separately.<sup>2</sup> The models dealing with the isolated phenomena will almost always be "unrealistic" but a premature attempt at realism may frequently result in complications which defy analysis or a confounding of explanation by the many different forces at play.

In particular there are two closely allied phenomena which have not been dealt with in this paper but will be dealt with subsequently. The first is the relationship between the competitive equilibria of a nonmonetary economy and the stationary states in an economy with an initial issue of fiat followed by a banking system which adjusts the rate of interest until a stationary state is reached.

The second problem which is not dealt with here, but in the next paper is the effect of uncertainty on the rate of interest in an economy which has

an outside bank but no insurance. It will be shown that money takes on an insurance component and the size of non interest earning balances increases from merely being the initial float to including money held as a "masse de manoeuvre" or for precautionary purposes.

It must also be noted that this analysis is not an artifact of the choice of time period. If we consider the model with continuous time but still require a sequencing of expenditures and receipts, then there is still a need for an issue of fiat to cover the (limiting instantaneous) float.

#### REFERENCES

<sup>1</sup>Debreu, G. (1959) The Theory of Value. John Wiley, New York.

<sup>2</sup>Shubik, M. (1972-1975) "A Theory of Money and Financial Institutions," Parts 1-20, Cowles Foundation Discussion Papers and Pre-Numbered Papers.