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ON FLEXIBILITY OF FUTURE PREFERENCE

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# ON FLEXIBILITY OF FUTURE PREFERENCE\*

by

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## 1. Preference and Opportunity

The economist's traditional model of choice -- whether consumer's choice or planner's choice -- is based on an analytical separation of preference and opportunity.

Preference is represented by an ordering on an all-inclusive set of alternatives, between some of which choice might conceivably arise. The only qualification for inclusion of an alternative in this choice space  $\mathcal{X}$ , say, is that it is sufficiently meaningful to the chooser so that if called upon to make choices between any two or more alternatives he is capable of making such choices in a manner consistent with an ordering of all of the elements (alternatives)  $x$  of  $\mathcal{X}$ . The ordering is assumed to indicate whether or not any two given elements  $x, x'$  are equivalent ( $x \sim x'$ ) and if not, which is preferred\*\* ( $x \succ x'$  or  $x \prec x'$ ).

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\*\* The simplest definition of such an ordering is through a relation  $\succsim$  ("preferred or equivalent to") which is complete ( $x \succsim x'$  or  $x' \succsim x$ ) and transitive ( $x \succsim x', x' \succsim x''$  implies  $x \succsim x''$ ).

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Opportunity is, in turn, represented by a subset  $A$  of  $\mathcal{B}$  consisting of those alternatives actually available at the time of choice. In the case of a consumer dependent on the market, this would be the set of all those commodity bundles (i.e., vectors of which the components are amounts of stated commodities) that can be bought on a given budget at market prices. For the subsistence farmer or the economic planner, it would be a set representing production possibilities under the control of the chooser. Or it might represent what can be achieved by a combination of production and trading.

To apply this model, one need not assume that the chooser works out for himself the ordering of all elements of the choice space before he learns what his opportunity is. One merely assumes that he would reveal such an ordering if presented with enough different opportunities, by responding to each opportunity with the choice of a best element from it. By a best element is meant an element which is preferred or equivalent to all other elements in the opportunity set  $A$ .

Such a model gives perhaps a tolerable approximation to reality in discussing choices of which the consequences are limited to the immediate future. In any case, economists have derived useful insights from the use of this model in analyzing, say, the effects of excise taxes on consumption, prices, welfare, and the distribution of income. When we turn to choices between alternatives of which the desired effects extend for a long period into the future, however, revision or refinement of the choice model is needed.

## 2. Sequential Choices for an Extended Future Period.

A straightforward extension of the choice space to such cases would seem to be obtained by regarding as the element of the choice space the program, that is, a sequence of successively dated commodity bundles. However, for long enough periods ahead, choice alternatives don't present themselves in just that way in reality. The individual choosing an education for a profession does not spell out sequences of commodity bundles that he expects to consume for each choice of profession. He is guided instead, among other things, by the real income expectations associated with each choice, leaving the detailed consumption choices to be worked out later. Likewise the planner will decide on the scale of a hydroelectric development without specifying, to himself or to others, to what precise bundle of uses the energy generating capacity to be created will be put in each future year. One can confidently assert that all choices occurring in real life are sequential, "piece-meal," choices between alternative ways of narrowing down presently existing opportunities, rather than "once-and-for-all" choices between specific individual programs. The mere passage of time cuts down opportunities even in the case of inaction on the part of a decision-maker. Incorporating inaction among available alternatives, one can therefore look upon economic choice at any one time as an inevitable choice between several or many specific subsets of the opportunity set available at that time. As time proceeds, a sequence of such choices need to be made.

### 3. Sequential Choice According to an Ordering of Programs.

In principle a sequential model of economic choice is compatible with the representation of preference by an ordering of completely spelled-out programs. If one limits the analysis to opportunity sets or opportunity subsets that contain a best program, one can order the opportunity subsets in the same way in which their respective best programs are ordered. The choice predicted by such a model could be described as follows. The decision maker selects one particular best program from the present opportunity set. Whenever the need for choice between alternative opportunity subsets arises, then or later, he invariably chooses that opportunity subset (or one such) that contains the not as yet realized segment of the preselected best program.

This model of sequential choice over time ignores at least two kinds of uncertainty that affect real-life choices. In representing the alternatives as sharply defined sets of completely spelled-out programs, the model ignores unforeseeable future enlargements of production possibilities by improvement of existing methods or by the discovery of new methods or resources — or for that matter future contractions of production possibilities by calamity or loss of knowledge. On the other hand, in deriving choice between opportunities from an ordering of programs adopted once-and-for-all, the model ignores the possibility, much used in real life, of having choices arising in a later future be guided by new preference orderings modified in the light of experience up to the time of choice.

The purpose of the present remarks is to draw attention to the latter kind of uncertainty, which concerns the chooser's own future preferences. In order to concentrate discussion on it, we shall continue the fiction of complete certainty with regard to future production possibilities.

#### 4. Flexibility of Future Preference

There are several reasons -- besides the uncertainty about future production possibilities here ruled out -- why a decision maker should wish to retain some flexibility with regard to his future preference. In the first place, to be able to make consistent responses to hypothetical choice situations involving the details of preference with regard to consumption in a distant future would require a considerable effort of visualizing and evaluating alternative programs. Secondly, taste evolves with experience. A model that freezes preferences by the adoption at an initial point in time of an ordering of programs for an indefinite future period is likely to become an unacceptable straight-jacket as time proceeds. At what age would the individual consumer be supposed to embrace the ordering that is to guide all his consumption choices for the remainder of his life? And the economic planner, who presumably attempts to aggregate the preferences of the population -- perhaps with some admixture of his own values -- wouldn't he wish to retain flexibility so that he can respond in the future to newly perceived currents of taste and desire?

One might be tempted to ignore these objections if technological alternatives were to have the trait that much higher levels of desired outputs could be achieved if the commodity composition of output were to be fixed long in advance. It would seem, however, that the remarkable adaptability of modern plant and equipment causes the cost of flexibility of preference, in terms of amounts of output foregone, to be greatly outweighed by its value to the chooser, in terms of outputs being of the kinds most desired at the time these outputs become available. It therefore seems worthwhile to explore the idea of flexible preference further.

In order to formalize this idea we must give up the separation of opportunity and preference. Let a program  $x$  be denoted  $x = (x_1, x_2, x_3, \dots)$ , where the  $x_t$  form a sequence of (finitely or infinitely many) successive consumption vectors, all belonging to the same commodity space  $X$ . The set of all programs, denoted  $\mathcal{C}$ , then is a cartesian product of identical spaces  $X$ . Let  $\mathcal{A}, \mathcal{A}', \mathcal{A}''$  denote the subsets of  $\mathcal{C}$  representing opportunities between which choices may arise. In particular, we shall denote by  $\mathcal{A}_t, \mathcal{A}'_t, \dots$  sets of programs with the property that all programs  $x, x', \dots$  of any  $\mathcal{A}_t$  have the same values for the first  $t$  vectors,

$$x_s = x'_s = \dots = x_s(\mathcal{A}_t), \text{ say, for } s = 1, \dots, t.$$

Such sets will be called  $t$ -uniform. Clearly, if  $\mathcal{A}_t$  is  $t$ -uniform, it is also  $t'$ -uniform for all  $t' < t$ .

##### 5. Partitioning of Opportunities as Time Proceeds.

Perhaps the simplest model of the gradual narrowing of opportunities would be as follows. Let  $\mathcal{A}$  be the opportunity set existing at the time  $t = 1$ . If  $\mathcal{A}$  happens to be at least 1-uniform, no immediate choice arises. If  $\mathcal{A}$  is not 1-uniform,  $\mathcal{A}$  partitions into finitely or infinitely many 1-uniform subsets  $\mathcal{A}_1, \mathcal{A}'_1, \mathcal{A}''_1, \dots$ , that can be labeled by the different values

$$x_1(\mathcal{A}_1), x_1(\mathcal{A}'_1), x_1(\mathcal{A}''_1), \dots,$$

assumed by  $x_1$  in all programs of the corresponding subset  $A_1, A'_1, A''_1, \dots$ , respectively. We shall assume that a choice between the subopportunities  $A_1, A'_1, A''_1, \dots$ , is then needed at time  $t = 1$ . If  $A_1$  is chosen, say, and if  $A_1$  happens to be at least 2-uniform, no further choice is as yet required at time  $t = 2$ . But if  $A_1$  is not 2-uniform, it partitions in turn into 2-uniform subsets  $A_2, A'_2, \dots$  between which a choice is to be made at  $t = 2$ . In this manner choice proceeds sequentially over time.

It is undoubtedly an oversimplification that this model puts the time of choice between subopportunities at the last possible moment before the consumption flows for the subopportunities begin to differ. Actually, investment choices occur frequently where the effects on consumption are first felt several periods after the time of choice. It will not be difficult to introduce a refinement of the rules on the timing of required choices that takes this circumstance into account. What matters is that these rules be fully specified, and that they are understood to be part of the description of the alternatives between which choice is to be made at any given time. For the present discussion, we shall adopt the above simplified model.

## 6. Preference Ordering of Opportunities

It is now proposed that a preference ordering be applied not to all programs, but to all sets of programs that may arise as opportunities. If two opportunities are compared in this ordering, it is understood that in each case

further choices between subopportunities will be needed at the times called for by the above description. The outcome of these further choices is not prejudged by the giving ordering. To illustrate the point, suppose that a chooser is confronted on Monday morning with opportunities *A* and *B* below regarding the dinners to be served to him during the coming week,

Opportunity	Monday	Tuesday	each remaining day
<i>A</i>	steak	fish	choice from a given menu M
<i>B</i>	steak	omelette	choice from menu M
<i>C</i>	steak	choice of fish or omelette	choice from menu M

and suppose he prefers *A* to *B*. After this preference is revealed he is again asked to choose between *A* and *C*. It is then quite consistent with his revealed preference of *A* over *B* if he regards *C* as preferred over, rather than equivalent to, *A*. The point is that in choosing between *A* and *B*, he has to decide about the Tuesday dinner on Monday morning. In choosing *C* he obtains a postponement of the determination of his preference for the Tuesday dinner until Tuesday morning.

#### 7. Attitudes Toward Postponement of Choice

It is of course equally possible that he declares himself indifferent between *A* and *C*, thus manifesting indifference to the timing of the choice of his Tuesday dinner. If he takes such a view in all possible choices, he indeed follows the preselected strategy described in section 3 above.

His ordering of opportunities is derived from a once-and-for-all ordering of programs by representing each opportunity by the best program it contains.

It is finally conceivable that he prefers  $A$  to  $C$ , thus expressing that he would prefer to choose the Tuesday dinner on Monday instead of on Tuesday.

The view has been expressed above that, in choices affecting a more distant future, it is likely that there is at least neutrality toward, and probably a certain preference for, postponement of choice. The following postulates are presented as exercises in giving formal expression to some of these possibilities. All postulates apply to the preference ordering  $\succeq$  applicable at time  $t = 1$ . This time will also be referred to as "now."

(P) If  $A$  is not 1-uniform, and if  $A_1$  is a best 1-uniform subset among all 1-uniform subsets  $A_1, A'_1, A''_1, \dots$ , into which  $A$  partitions, then  $A_1 \sim A$ .

This postulate says, quite naturally, that augmentation of an opportunity  $A_1$  by the addition of one or more alternatives  $A'_1, A''_1, \dots$  such that choice between  $A_1, A'_1, A''_1, \dots$  is immediately needed has no value to the chooser if none of the added alternatives  $A'_1, A''_1, \dots$  is by itself preferred to  $A_1$  at that time.

The remaining postulates are concerned only with choice between  $t$ -uniform opportunities for  $t \geq 1$ . They represent alternative supplementations of (P).

(P') If  $A_t \cup B_t$  is t-uniform with  $t \geq 1$ , then

$$A_t \cup B_t \succeq A_t.$$

This postulate says, quite cautiously, that a widening of the range of alternatives between which choice is not called for prior to some future time  $t$  cannot make an opportunity less desirable.

(P'') If  $A_t \cup B_t$  is t-uniform with  $t \geq 1$ , and

if  $A_t \sim B_t$ ,  $A_t \cap B_t$  is empty, then

$$A_t \cup B_t \succ A_t.$$

This says, somewhat more positively, that if an opportunity  $A_t \cup B_t$  can be subdivided into two non-overlapping and equivalent opportunities  $A_t$  and  $B_t$  between which, and within which, choice is not required prior to some future time  $t + 1$ , then the original opportunity is preferred to either of its parts. It thus expresses a definite preference for postponement of choice between alternatives now deemed equivalent.

The requirement in (P'') that  $A_t \sim B_t$  is somewhat restrictive.

One feels that enlargement of future choice by an additional opportunity  $B_t$  not now deemed equivalent to that already available ( $A_t$ ) may now be welcomed because of the possibility that at such a time  $t' \geq t + 1$  at

which further choice is called for,  $B_t$  will be found to contain a  $t'$ -uniform subset  $B_{t'}$  then deemed superior to the then best  $t'$ -uniform subset  $A_{t'}$  of  $A_t$ . The two remaining postulates go further than (P'') in this respect.

(P''') If  $A_t \cup B_t$  is  $t$ -uniform with  $t \geq 1$ , if  $A_t \cap B_t$  is empty, and if for some  $t' \geq t + 1$  the best  $t'$ -uniform subsets  $A_{t'}$ ,  $B_{t'}$  of  $A_t$  and  $B_t$ , respectively, satisfy  $A_{t'} \sim B_{t'}$ , then  $A_t \cup B_t \succ A_t$ .

This postulate accepts  $B_t$  as a welcome enlargement of  $A_t$  on some sufficient evidence that  $B_t$  is a possible runner-up to  $A_t$ . The sufficient evidence is that, at some future time  $t' \geq t + 1$ , the now deemed best of the alternatives  $B_{t'}$ ,  $B'_{t'}$ , ... then available within  $B_t$  is now deemed equivalent to the now deemed best of the alternatives  $A_{t'}$ ,  $A'_{t'}$ , ... then available within  $A_t$ .

(P<sup>IV</sup>) If  $A_t \cup B_t$  is  $t$ -uniform with  $t \geq 1$ , and if  $A_t \dot{\sim} B_t$ ,  $A_t \cap B_t$  is empty, then  $A_t \cup B_t \succ A_t$ .

This says, much more strongly, that any enlargement of a  $t$ -uniform opportunity  $A_t$  that conserves its  $t$ -uniformity is desirable, no matter how poorly the added opportunity  $B_t$  would be rated if it had to be compared now with  $A_t$ .

A chooser who adopts such a postulate is telling himself that one can never know how drastically one's preferences may have changed when the time of choice arrives. Obviously,  $(P^{IV})$  implies  $(P')$ ,  $(P'')$  and  $(P''')$ . A postulate intermediate between  $(P^{IV})$  and the conjunction of  $(P')$ ,  $(P'')$ ,  $(P''')$  could be formulated if some measure of the strength of preference of  $A_t$  over  $B_t$  were available, or were introduced by postulate.

#### 8. The Role of Judgment in Preference Ordering.

Two views can be found in the literature concerning the relation of the idea of a preference ordering to actual choice. One of these expresses the descriptive interpretation of a preference ordering as a device for describing and predicting choices made by an individual with consistent tastes. In the other, normative interpretation, a preference ordering satisfying certain postulates is recommended to a decision-maker as a device that can help him achieve greater consistency in choices made at different occasions than his own instinct unaided by analysis would be capable of. As more of the subtler aspects of real-life choice situations are incorporated in our analyses, the descriptive interpretation becomes less convincing, while the normative interpretation gains in importance. A set of postulates imposed on a preference ordering then becomes a format in which the indispensable judgments that determine the ordering itself can be entered.

While there are many programs in any real-life opportunity, there are, in the appropriate mathematical sense, a great many more sets of programs, i.e., sub-opportunities, within any one opportunity. It may therefore be felt that the extension of the idea of a preference ordering from programs to sets of programs is going to be too unwieldy to be useful. Without prejudging the answer, it may be suggested that an ordering of opportunities be defined, not on the set of all mathematically conceivable opportunities, but on some suitably restricted set of opportunities, defined in some manner to correspond to the kind of opportunities between which actual choices arise. Such a restriction of the set of opportunities being ordered would facilitate the expression of the judgments underlying a particular ordering. Since preference itself is the fruit of experience, one cannot expect to obtain useful judgments of preference between alternatives too dissimilar from those involved in choice experience.

#### 9. Assets Viewed as Opportunities

Assets have been difficult to fit into the theory of consumer's preference. In the case of durable consumers' goods, one can of course replace such a good by the flow of its services. However, a decision to acquire an automobile does not presuppose a decision about the number of passenger-miles to be consumed in a year through the use of that car. Nor does a decision to improve land by drainage imply a simultaneous choice as to the crops to be grown on that land as improved.

A point in favor of applying the idea of a preference ordering to opportunities is that physical assets fit very naturally into such a scheme.

An automobile, a record player, a tract of land, a machine shop, and the entire capital equipment and natural resources of an economy, all represent opportunities in the precise sense here developed. Each of these assets defines a set of consumption flows obtainable through its use, with sequential decisions about that use being called for continually to narrow down or determine the actual use as the future draws nearer.

Even non-physical assets, claims to income from productive property, or to interest and repayment from money loans made to governments or individuals, can be looked upon as opportunities requiring sequential decisions about the uses of the income, and possibly of the proceeds from sale of the asset itself at or prior to its maturity. However, if we wish to include such assets as well, we shall have to generalize the notion of an opportunity to that of a set of programs each available subject to some uncertainty that will gradually diminish as time of choice draws near. This is so even if we are maintaining the fiction of complete certainty about future production possibilities. The reason is that our admission of uncertainty with regard to future consumers' preferences introduces uncertainty into the market valuations that in turn affect the opportunities represented by stocks, bonds, good will and other non-physical assets.

#### 10. The Planning Horizon.

Viewing physical assets as opportunities resolves one awkward problem in the application of the theory of economic choice: the problem of choosing the length of the planning period. If assets are not represented in the preference ordering, a planning model is likely to recommend maximizing

consumers' satisfaction during the planning period by failing to ensure an appropriate capital stock at the end of the period. Specifying a "terminal" capital stock as a constraint in such a maximization avoids this gross error. It is still open to the objection that the opportunity represented by the terminal capital stock is predetermined, rather than allowed to find its proper magnitude by being weighed against consumers' satisfactions during the planning period. Once terminal assets are themselves entered into the preference ordering as opportunities, the choice of the length of the planning period becomes a matter of degree of approximation or coverage of detail. In principle, the choices predicted or recommended by the model no longer depend on the length of the planning period. In practice, they might be somewhat improved by the greater effort of observation and evaluation required if a longer planning period is selected.

#### 11. References to Flexibility in the Economic Literature

The idea of flexibility has been discussed in the economic literature in connection with investment choices that enable the firm to profit from the satisfaction of uncertain future consumers' preferences. An early example is Hart's discussion [1940, Chapter IV] emphasizing that it may increase profit expectation to choose capital equipment permitting relatively low-cost production over a wider range of outputs, as compared with capital equipment permitting absolutely minimum cost production at one specific anticipated level of output, but relatively higher cost production at some other levels that could come about due to the unpredictability of consumers' preferences, or of other factors.

I do not know of earlier attempts to incorporate uncertainty about future preferences into the model of preference itself, except for one brief discussion of some of the ideas of this paper [Koopmans, 1950].

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