

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

Box 2125, Yale Station
New Haven, Connecticut

COWLES FOUNDATION DISCUSSION PAPER NO. 96

Note: Cowles Foundation Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. Requests for single copies of a Paper will be filled by the Cowles Foundation within the limits of the supply. References in publications to Discussion Papers (other than mere acknowledgment by a writer that he has access to such unpublished material) should be cleared with the author to protect the tentative character of these papers.

Programming of Economic Development*

Alan S. Manne

October 26, 1960

* Research undertaken by the Cowles Commission for Research in Economics under Task NR 047-006 with the Office of Naval Research.

Programming of Economic Development

Alan S. Manne*

October 26, 1960

Introduction

Unlike most of the studies described within this volume,** the usual

** This paper is intended to appear within a forthcoming Cowles Foundation Monograph, A. S. Manne and H. M. Markowitz (eds.), Studies in Process Analysis: Economy-Wide Production Capabilities.

case of economic development is inherently a dynamic process - one involving interrelated variables at distinct points in time. Without a dynamic framework, it is particularly difficult to make sense out of investment flows. At any given point in time, such flows are much more closely related to future increases in output than to current absolute levels. There is an all-too-obvious way to handle the dynamics of investment: working with a multi-period model, and, e.g., forecasting the evolution of an economy over a 100-year horizon, subdivided into 20 five-year periods, each with its own appropriate set of technology and of consumption preferences!

However, in the more modest kind of development programming work designed for practical implementation, the model has usually been phrased in terms of

* T. N. Srinivasan has contributed much to this paper - both in model formulation and in detailed criticisms of an earlier draft.

the changes occurring within a single specific time period. For example, the Chenery-Kretschmer (1956) model of Southern Italy is concerned with planning capital investment so as to bring about specified levels of "final demand" corresponding to a 75% increase in regional income over a decade. Similarly, Sandee's model of Indian development (1959) is phrased in terms of a single time period, and with maximizing national income in the target year of 1970, i.e., at the end of India's Third and Fourth Five Year Plans.

We will start off by reviewing a small-scale example similar to that employed by Chenery and Kretschmer -- one in which investment flows during the target year are regarded as an exogenous component along with consumption, the main element in final demand. We will then go on to modify their analysis in such a way as to let the model itself determine the investment component of final demand during the target year. This particular endogenous treatment of investment is in much the same spirit as Sandee's, but is obtained by assuming balanced exponential economic growth subsequent to the target date.

The proposal represents little more than a variant of the dynamic models suggested by Leontief (1953) and von Neumann (1945-46). Unlike these formulations, this one lends itself to straightforward linear programming computational techniques. The simplification is achieved through abandoning the attempt to calculate a maximum growth rate, but instead posing the problem as one of how to achieve any specified growth rate at a minimum investment cost -- i.e., with a minimum reliance upon foreign aid and upon sacrifices in current consumption.

A small-scale example

In order to introduce a model of Southern Italian development, Chenery and Clark present a small-scale linear programming tableau illustrating the

types of activities and constraints they regard as significant. Like the larger model of Southern Italy, this one is phrased in terms of minimizing the total capital investment needed in order to bring about specific levels of final demand (private and public consumption plus investment) in a target year. The model is of a "make-or-buy" variety; it is intended for the purpose of selecting out those commodities in which the region has a comparative advantage, and those which the region would do well to import. In the authors' own words:

The most important allocation problem for an underdeveloped country is to choose the sectors in which it should try to increase its exports and those in which it should expand production for the domestic market, usually as a substitute for imports. p. 284.

To provide a simple numerical example, just four sectors are distinguished: (1) finished goods, (2) agriculture, (3) basic industry, and (4) services. (See Table 1.) The level of domestic production of finished goods is denoted by x_{11} , domestic production of agricultural goods by x_{21} , etc. As an alternative to domestic production, it is possible to import each of the three categories of physical products. The fourth category, services, cannot be exported or imported. The levels of the import activities are denoted by x_{12} , x_{22} , and x_{32} ; the export activities by x_{13} , x_{23} , and x_{33} .

Each column of coefficients within Table 1 lists the inputs and outputs corresponding to one unit's worth of gross output of the commodity associated with that column.* Input flow coefficients are distinguished by negative signs and output flows by positive signs. For example, the leftmost column, to be

* One minor difference between this and the original version of the Chenery-Clark model: Labor availability is neglected as a non-limitational constraint.

Table 1. An Illustrative Model of Economic Development

	Industries										Constants q_i	Optimal implicit prices u_i
	(1) Finished goods			(2) Agriculture			(3) Basic industry			(4) Services		
Activity level identifi- cation, x_j	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	x_{41}		
(1) Finished goods, a_{1j}	.8	1.0	-1.0								320	2.26
(2) Agricul- ture, a_{2j}	-.3			.9	1.0	-1.0					105	2.54
(3) Basic industry, a_{3j}	-.1			-.1			.7	1.0	-1.0		40	2.26
(4) Services, a_{4j}	-.2			-.1			-.1			.9	60	.61
(5) Foreign exchange, a_{5j}		-1.0	.9		-1.1	1.0		-.9	.8		-20	2.51
Capital coeffi- cients, c_j	.7			2.0			1.8			.55		
Optimal activity levels, x_j	524	0	99	290	0	0	0	122	0	215	$\sum_j c_j x_j = 1,068$	
$c_j - z_j$	0	.25	0	0	.17	.03	.28	0	.25	0		

operated at intensity x_{11} , lists the inputs and outputs corresponding to one unit's worth of gross annual production of finished goods. For each unit of gross annual production, there will be .2 unit of finished goods consumed and therefore a net output of .8 shown in row 1. In addition, one unit of gross annual output takes .3 unit of agricultural products, .1 unit of basic industrial materials, and .2 unit of services. The capital to be invested in industry 1 must consist of .7 unit of stock per unit of gross annual output flow.

The import and export activities do not require any capital investment. They are closely linked to each other through restriction (5) - the foreign exchange constraint. For example, in the case of finished goods, activity 12 specifies that for each unit of finished goods imported annually (positive sign in row 1), the economy will have to spend one unit of foreign exchange (negative sign in row 5). For each unit of finished goods exported annually, activity 13 indicates that the economy can expect to earn .9 unit of foreign exchange. (Due to transportation costs, tariff barriers, and market imperfections, there is apparently a 10% differential here between the export and the import prices.)

Within each of the categories of goods and services, Chenery and Clark follow Leontief's procedure of specifying just one activity for domestic production. Unlike a process analysis, this model does not contain sufficient technological detail to distinguish, for example, between highly capital-intensive methods of producing electricity (hydroelectric installations) and less capital-intensive methods (thermal plants). All such distinctions between domestic processes are neglected.

The first four constraints of Table 1 specify that the gross availability of each good - including imports as well as domestic production - must be at

least sufficient to cover the final demand plus the interindustry demand.

Thus, in the case of row 2 (agricultural products), the condition reads

as follows:

$$\begin{array}{rcll}
 \text{gross annual} & & & \\
 \text{domestic} & + & \text{imports} & \geq & \text{final demand} \\
 \text{availability} & & & & \text{(consumption plus} & + & \text{exports} & + & \text{interindustry} \\
 & & & & \text{investment)} & & & & \text{demand} \\
 \\
 1.0 x_{21} & + & 1.0 x_{22} & \geq & 105 & + & 1.0 x_{23} & + & .3 x_{11} + .1 x_{21}
 \end{array}$$

$$\text{Or:} \quad -.3 x_{11} + .9 x_{21} + 1.0 x_{22} - 1.0 x_{23} \geq 105$$

Or in more
general

$$\text{notation:} \quad a_{2,11} x_{11} + a_{2,21} x_{21} + a_{2,22} x_{22} + a_{2,23} x_{23} \geq q_2$$

$$\text{Or:} \quad \sum_j a_{ij} x_j \geq q_i \quad (i = 2)$$

The fifth row is concerned with the availability of foreign exchange during the target year. It spells out the requirement that the cost of imports should not exceed export earnings plus anticipated net loans and gifts:

$$\begin{array}{rcll}
 \text{import costs} & \leq & \text{net loans} & & \text{export earnings} \\
 & & \text{and gifts} & & \\
 1.0 x_{12} + 1.1 x_{22} + .9 x_{32} & \leq & 20 & + & .9 x_{13} + 1.0 x_{23} + .8 x_{33}
 \end{array}$$

Reversing the direction of the inequality and rearranging terms, this constraint may be rewritten so as to correspond to the detached coefficients form in Table 1:

$$-1.0 x_{12} + .9 x_{13} - 1.1 x_{22} + 1.0 x_{23} - .9 x_{32} + .8 x_{33} \geq -20$$

The minimand -- total capital to be invested domestically -- is a linear function of the domestic production activities, and the coefficients c_j represent capital stocks per unit of gross annual output flow:

$$\begin{array}{l} \text{minimand} = \text{total capital} = .7 x_{11} + 2.0 x_{21} + 1.8 x_{31} + .55 x_{41} \\ \text{requirements} \end{array}$$

The solution to this linear programming model may be computed via the simplex method, and -- to slide-rule accuracy -- is shown in the line labeled "optimal activity levels, x_j ." Apparently, the country's comparative advantage lies in finished products (commodity 1). Enough of this item is to be produced so as to yield an export surplus. In agricultural products -- despite the high capital-output ratio -- it is optimal for the country to be self-sufficient. All basic industrial products (commodity 3) are to be imported; and all services (commodity 4) must be produced domestically. The ratio of total capital requirements to total annual final demand is of a reasonable order of magnitude: $1,068 / (320 + 105 + 40 + 60) = 2.0$.

Investment as an endogenous element

Now let's turn to the possibility of handling investment as an endogenous element within our model -- leaving consumption as the only exogenous element in "final demand." Because of the durability of capital goods, the investment activity planned toward the target date can only make sense in relation to the developments anticipated during the post-target period. Our treatment of investment is based upon one crucial assumption -- that subsequent to the target date (the model's cutoff date), all flows within the economy will expand

exponentially at an identical annual rate, to be termed \underline{r} . Thus, if x_{11} is the rate of domestic production of finished goods at the target date, then \underline{t} years later, the rate of production of finished goods, $x_{11}(t) = x_{11} e^{rt}$. Similarly, if $q_1 = 320$ is the annual rate of final demand for finished goods at the cutoff date, the final demand for finished goods \underline{t} years later will be $320 e^{rt}$. Note that exponential balanced growth is not necessarily the most desirable of all possible modes. It does, however, have one virtue possessed by no other kind of time path. If investment flows are to be proportional to the time derivative of output, then exponential growth is the only kind of trajectory in which investment and output can be constantly proportional to each other. Exponential growth for the post-horizon period seems like a convenient way to explain investment flows and to avoid "edge effects" within a finite horizon model. Nothing more.

Now for some specific details. The Chenery-Clark model, treating investment as an exogenous activity, may be written as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_j c_j x_j \\ \text{subject to:} \quad & \sum_j a_{ij} x_j \geq q_i \quad (\text{all } i) \\ & x_j \geq 0 \end{aligned} \quad (1)$$

Suppose that subsequent to the target date, the a_{ij} and c_j coefficients remain unchanged,* and that the constants $q_1(t)$ and activity

* If the a_{ij} coefficients are to remain constant, this implies that we are ignoring the effects of any technological change during the post-cutoff phase. Such change is bound to occur, but by how much and in what directions is th's factor likely to distort the investment flows programmed for the target year?

levels $x_j(t)$ are to be respectively, $q_i e^{rt}$ and $x_j e^{rt}$. (The parameter t denotes the number of years elapsed since the target date.) If the amount of the i^{th} commodity stock invested in the j^{th} activity is to be maintained constantly in proportion to the output rate of the j^{th} activity:**

** The assumption of constant proportions between capital and output represents a convenient simplification in a long-term model of economic growth. In order to take advantage of economies of scale, individual plants will actually be constructed in sizeable increments, even though demand is growing continuously. From the overall viewpoint, however, these increments in capacity are supposed to be small enough to justify the approximation of constant proportions.

$$\begin{array}{l} \text{total capital stock} \\ \text{of } i^{\text{th}} \text{ commodity} \\ \text{utilized in } j^{\text{th}} \\ \text{activity at time } t \end{array} = b_{ij} x_j(t) = b_{ij} x_j e^{rt}$$

$$\therefore \begin{array}{l} \text{investment rate} \\ \text{in } j^{\text{th}} \text{ activity} \\ \text{at time } t \end{array} = \frac{d [b_{ij} x_j e^{rt}]}{dt} = r b_{ij} x_j e^{rt}$$

The model indicated within (1) may now be rewritten, treating investment as an endogenous element:

$$\begin{array}{ll} \text{minimize} & \sum_j c_j x_j \\ \text{subject to:} & \sum_j a_{ij} x_j e^{rt} \geq q_i e^{rt} + \sum_j r b_{ij} x_j e^{rt} \quad (2) \\ & x_j \geq 0 \quad (\text{all } i ; t \geq 0) \end{array}$$

Since t is a continuous time parameter, there are an infinite number of equations in (2) -- one commodity balance equation for each point in time over the infinite future. However, we are entitled to divide these all by e^{rt} . Then, transposing the terms involving the capital coefficients b_{ij} , we find ourselves back with an ordinary linear program involving just a finite number of constraints:

$$\sum_j (a_{ij} - rb_{ij}) x_j \geq q_i \quad (\text{all } i)$$

Table 1 has been reconstructed into Table 2 to demonstrate this idea. For illustrative purposes, it is assumed that finished goods (commodity 1) represent the only item involved in capital formation. Hence $b_{1j} = c_j$, and all other $b_{ij} = 0$. Table 2 is based upon two other specific numerical conditions: (1) that the growth rate is 10% per annum compounded continuously ($r = .10$); and (2) that since gifts and loans of foreign exchange are unlikely to expand indefinitely at 10% yearly, the value of q_5 has been set at zero. The other right-hand side constants, q_1, \dots, q_4 are shown at the same values as in Table 1. Now, however, these elements are intended to reflect consumption only, and not the investment component of final demand.

The optimal solution does not differ greatly from the previous one in Table 1. The same set of activities is employed: domestic production of commodities 1, 2, and 4; exports of 1; and imports of 3. There is, however, a significant increase in the capital investment required to equip the economy for continuously sustained 10% growth. The total capital to be invested prior to the target date was previously 1,068, and now rises to 1,460, an increase

Table 2. An Illustrative Model of Economic Development; Exponential Growth

	Industries										Constants q_i	Optimal implicit prices u_i
	(1) Finished goods			(2) Agriculture			(3) Basic industry			(4) Services		
Activity Level identification, x_j	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}	x_{41}		
(1) Finished goods, $a_{1j} - rb_{1j}$.73	1.0	-1.0	-.2			-.18			-.055	320	2.90
(2) Agriculture, $a_{2j} - rb_{2j}$	-.3			.9	1.0	-1.0					105	3.27
(3) Basic industry, $a_{3j} - rb_{3j}$	-.1			-.1			.7	1.0	-1.0		40	2.90
(4) Services, $a_{4j} - rb_{4j}$	-.2			-.1			-.1			.9	60	.79
(5) Foreign exchange, $a_{5j} - rb_{5j}$		-1.0	.9	-1.1	1.0		-.9	.8			0	3.22
Capital coefficients, c_j	.7			2.0			1.8			.55		
Optimal activity levels, x_j	780	0	160	380	0	0	0	160	0	280	$\sum_j c_j x_j = 1,460$	
$c_j - z_j$	0	.32	0	0	.27	.05	.37	0	.32	0		

of over one-third.*

* Since just 20 units of foreign exchange were lost, and since this item now has an implicit price of 3.22, only a small portion of the difference in capital requirements can be attributed to the elimination of foreign loans and gifts during the post-target period.

Several further refinements could readily be introduced into this model of exponential growth during the post-cutoff period: (1) Some investment activity may be directed toward equipment replacement, rather than toward capacity expansion. The replacement component will grow at the same exponential pace as the balance of the economy -- provided that there is a constant distribution of equipment replacement ages, and provided also that there is an exponential age distribution of equipment at the cutoff date. (2) Several distinct time periods could be distinguished prior to the cutoff phase. If the cutoff date is, say, 10 years off, it may be worth the effort involved in distinguishing between average flows during three individual time periods: the first five years, the second five years, and the target year itself. (3) Suppose that there are certain elements within the economy that are incapable of growing as rapidly as the remainder. For example, the foreign market for coffee may be expanding at the rate of only 2% per annum, whereas there is a portion of the economy that is capable of growing at the rate of 8%. In this case, we may suppose that the objective is one of setting up several independent sub-economies at the cutoff date: one dependent upon the foreign exchange earnings from coffee and expanding at 2%, and another completely autonomous one capable of

8% sustained growth.* Alternatively, instead of independent sub-economies,

* For more on the possibility of independent sub-economies, see Kemeny, Morgenstern, and Thompson (1956).

one could specify an arbitrary independent percentage growth rate for each activity at the target date, and neglect the problem of economic balance thereafter.

Quadratic time paths

Sandee's demonstration model of India's growth (1959) is one of the most impressive works in this area, and deserves careful study by anyone concerned with the programming of economic development. The author has devoted much thought to balancing against each other the often conflicting elements of realism, of data scarcity, and of computing limitations. The following comments should be regarded as critical of just one aspect of the overall model: that dealing with investment behavior during the cutoff phase. Sandee has suggested handling this element through assuming a quadratic time path for each commodity flow. If, for example, the symbol $Y(t)$ denotes national income in year t , with 1960's national income being $Y(0)$ and 1959's being $[Y(0) - g]$, then he would approximate national income at all subsequent dates by the following time path, where t denotes years elapsed since 1960:

$$Y(t) = Y(0) + gt + ht^2 \quad (3)$$

Or

$$h = \frac{Y(t) - Y(0) - gt}{t^2} \quad (4)$$

Once a specific target date T has been chosen, relation (4) conveniently indicates the acceleration constant h as a linear function of $Y(T)$. Now if we are interested in investment behavior during the target year only, the quadratic time path turns out to be a convenient choice of function. With a capital-output coefficient of b , the simple acceleration principle tells us that the rate of investment $I(T)$ during the target year is proportional to the rate of change of output at that time, and is therefore a linear function of the absolute level of output at T :

If capital stock at target date T = $b Y(T)$

net investment rate at target date T = $I(T) = b \left[\frac{dY(T)}{dt} \right] = b [g + 2 h T]$

Or: $I(T) = b \left[g + \frac{2(Y(T) - Y(0) - gT)}{T} \right]$

Suppose, however, that we are concerned with economic balance both during the target year itself and also during the preceding and following periods. Then the quadratic time path (3) clearly implies that the invested fraction of national income varies from year to year, and that ultimately it approaches zero. For example, in the slightly pathological case of g being zero and $Y(0)$ approaching zero, then regardless of what specific value is chosen for the target date, or for income at the target date, or therefore for the parameter h :

$$\lim_{Y(0) \rightarrow 0} \left[\frac{I(t)}{Y(t)} \right] = \frac{2 b h t}{ht^2} = \frac{2b}{t} \quad (5)$$

Thus, in the fifth year of a given program, there would be twice as high a fraction of the national income devoted to investment as during the tenth year! Furthermore, with the plausible value of 2.0 for the capital-output ratio \underline{b} , relation (5) would say that 400% of the national income should be devoted to investment at year 1. No doubt, this numerical example is a contrived one, and actual Indian figures would turn out to be more reasonable. However, the case points to a difficulty that is inherent in quadratic time paths of output: an ultimately diminishing ratio of investment to national income. Exponential growth is the only mode in which it is possible for output and its time derivative to remain in a constant proportion to each other.

References

- Chenery, H., and P. Clark, Interindustry Economics, John Wiley and Sons, Inc., New York, 1959.
- Chenery, H., and K. S. Kretschmer, "Resource Allocation for Economic Development," Econometrica, October, 1956.
- Kemeny, J. G., O. Morgenstern, and G. L. Thompson, "A Generalization of the von Neumann Model of an Expanding Economy," Econometrica, April, 1956.
- Leontief, W., Studies in the Structure of the American Economy, Oxford University Press, New York, 1953, Chapters 3 and 4.
- von Neumann, J., "A Model of General Economic Equilibrium," Review of Economic Studies, Vol. 13, No. 33, (1945-46).
- Sandee, J., "A Long-Term Planning Model for India," United Nations, TAO/IND/22/Rev. 1, 9 June 1959.