COWLES FOUNDATION DISCUSSION PAPER NO. 7

Suggested Experiments on Tastes and Beliefs

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October 14, 1955

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I. Types of Consistency

By consistency of some aspects of behavior is meant its constancy (which permits prediction). Table I indicates one possible classification of the notion of consistency of behavior. Such a classification is not meant to be exhaustive but is presented merely as a matter of convenience.

	static	dynamic
exact		
stochastic		

Table I

Exact consistency refers to the presence of some constants (other than distribution parameters) in behavior, whereas stochastic consistency means the presence of some probability distributions with constant parameters. Static consistency concerns the constancy of behavior parameters over time (other than parameters characterizing the pattern of change), whereas dynamic consistency concerns the constancy of parameters that describe the change of behavior over time. The process of learning illustrates well the notion of dynamic consistency when seen as a subject's attainment of a stable equilibrium at predictable rates of change. In general, the learning model will be a stochastic one; it predicts the pattern of change of the probability of correct response.

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* Paper given at the Seminar in the Application of Mathematics to the Social Sciences, University of Michigan, April 14, 1955. Research undertaken by the Cowles Commission for Research in Economics under Contract Nonr-358(01), NR 047-006 with the Office of Naval Research.

The notion of consistency is particularly relevant to tastes and beliefs. In the following example dealing with economics of choice, we shall assume stochastic static consistency. We shall deal with tastes, not with beliefs, by assuming certainty (or known probabilities).

Given x units of commodity X
y units of commodity Y
x' units of commodity X'
y' unit of commodity Y'

let " $\succ$" denote a preference ordering among x, y, x', and y' and write Pr ( ) for "probability that." Then we should like to assume a mild "law of comparative judgment": there exist four numerical functions $u(x)$, $v(y)$, $w(x')$, and $t(y')$ (utility functions) such that

(1) $$Pr(x \succ y) \geq Pr(x' \succ y')$$

if and only if

(2) $$u(x) - v(y) \geq w(x') - t(y').$$

(This idea can easily be generalized to interpret X, Y etc., as bundles of goods or as bets with known probabilities.) Now, let X and X' be the same commodity, and let Y and Y' be the same commodity; then, in (2), w and t are replaced by u and v. It follows that

(3) $$Pr(x \succ y) = Pr(y \succ x)$$

if and only if

(4) $$u(x) - v(y) = v(y) - u(x) = 0$$

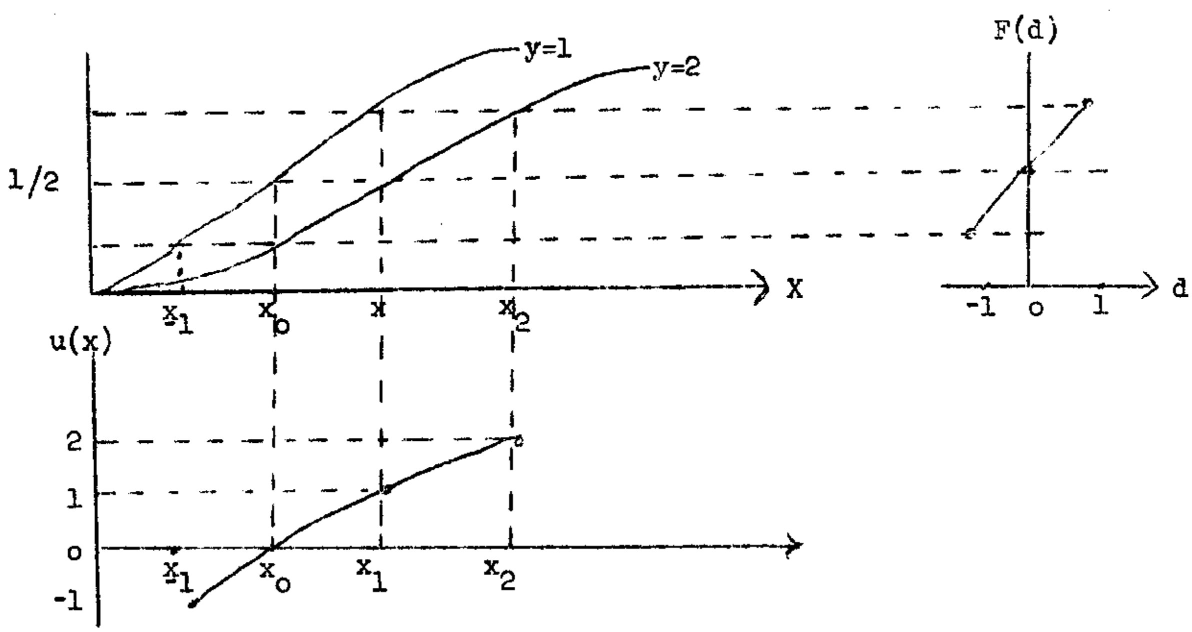
If we interpret $\succ$ as weak preference ordering (preferred or indifferent to) or if the subject has always to pick X or Y, then

(3) and (4) would in fact define

(5) $$Pr(x \succ y) = Pr(y \succ x) = 1/2$$

Suppose we observe a subject's repeated choices for varying pairs (x,y), and interpret the frequencies as probabilities. This is summarized in figure I where the contour lines of $Pr(x > y)$ for different values of y have been plotted.

Figure I



Assume $v(1) = 0$, $v(2) = 1$, to fix the scale and origin. We have from observations (see Figure): $Pr(x_o > 1) = 1/2 = Pr(x_1 > 2)$. Then $u(x_o) = v(1) = 0$; $u(x_1) = v(2) = 1$. Moreover, $u(x_2) = 1 = u(x_1) - 0$, or $u(x_2) = 2$, since (see Figure) $Pr(x_2 > 2) = Pr(x_1 > 1)$. Hence the curve $u(x)$ can be traced out by varying x. We can also trace the probability function $F(d) = Pr(x > y)$, where $d = u(x) - v(y)$. This method illustrates what may be called the non-parametric estimation of utilities, and does not require the assumption of normality of F as contrasted with Thurstone's "Case V" of paired comparisons. To obtain transitivity, we have merely to assume F symmetrical. Broadly speaking the problem is the following: given a finite sample of choices, to fit functions F, u and v such that the prediction error will be a minimum. But perhaps consistency is dynamic, i.e., the subject "learns" his utility functions; or perhaps he becomes less and less "hesitant" so that F tends to become a step function: F = 1 or 0 as $d > \text{or} < 0$.

II. Consistency of Beliefs and Tastes

Table 2 offers a convenient way of classifying the various schools in this area.

utility \ probability	objective	subjective
objective	Pascal	Bayes, de Finetti
subjective	D. Bernoulli v. Neumann	Ramsey, de Finetti, Savage

Table 2

However, we can extract that which all these schools have in common:

$a \in A$, a set of actions

$s \in S$, a set of states of the world

$\phi(a,s)$, the payoffs or physical results

In addition, it is postulated that there exist numerical functions

$$\mu[\phi(a,s)] \qquad \text{(utility)}$$

$$p(s) \geqslant 0, \quad \sum_{s \in S} p(s) = 1 \qquad \text{(probability)}$$

satisfying the following general criterion called maximization of expectation):

$$\text{(6)} \qquad \text{If } \max_{a \in A} \sum_{s \in S} \mu[\phi(a,s)]\ p(s) = \sum_{s \in S} [\phi(\hat{a},s)]\ p(s)$$

then choose $\hat{a} \in A$.

For the objective utility school, $u[\phi\,(a,s)]$ can be replaced by $\phi\,(a,s)$ itself, this being a real number (money). This, however, is open to a variety of criticisms. Most results of action cannot be expressed as a number; for example, a social position, a bundle of goods, a life history. The subjective schools present a more interesting and more general point of view.

One may regard the problem as one of prediction: that is, given ϕ and A, predict $\hat{a}$. This might be obtained empirically, from a large number of observations on ϕ, A and $\hat{a}$. However, the behavior rule (6) would suggest to go beyond this surface and explore the consistency of tastes (u) and beliefs (p) that allegedly underlie the choice. If (6) is valid, better predictions can be made.

III. Separation of Tastes and Beliefs

In this section we shall follow Ramsey's thought very closely, by suggesting the following experiment. Given a payoff table as shown in table 3, (with x, y a pair of money amounts or of other rewards), a person is asked to judge whether the distance between two fixed points A and B is less than a prescribed number by choosing either option I or option II (also called actions a and $\bar{a}$, respectively).

actions / states	option I	option II
	a	$\bar{a}$ (not a)
AB s	x	y
AB > s	y	x

Table 3

Suppose that corresponding to Table 3, s = 5, 6, 7, 8, ..., inches, he gives the sequence of options I, I, II, II, II, ..., then Ramsey would define

$$(7) \qquad Pr(\overline{AB} \lessdot s^*) = 1/2 = Pr(\overline{AB} \gtrdot s^*)$$

if, for s = s*, the individual is indifferent between options I and II (i.e., $a \gtrdot \bar{a}$ and $\bar{a} \gtrdot a$, or stochastically, if $Pr(a > \bar{a}) = 1/2$. In the above example s* would be somewhere between 6 and 7 inches.

Postulate 1: Given s, the choice (a or $\bar{a}$) is independent of x, y provided that they are not indifferent.

It follows that if "AB $\lessdot$ s*"has probability 1/2, then it retains this probability for any x, y.

This postulate, or norm, requires <u>beliefs</u> <u>to</u> <u>be</u> <u>independent</u> <u>of</u> <u>rewards</u>. That is, decisions should not be based on wishful thinking. It can be verified whether the subject obeys this norm, or at least learns to obey it, as the experiment progresses.

Now consider the following payoff table (table 4).

states \ actions	a	$\bar{a}$
AB< s	x	y
AB $\gtrdot$ s	x	z

Table 4

Ramsey defined <u>equidistant utilities</u> as follows: if for s = s*, a person is indifferent to a and $\bar{a}$ (i.e., if he assigns probabilities of 1/2 to "AB<s*" and "AB $\gtrdot$ s*"), then

$$(8) \qquad u(y) - u(x) = u(x) - u(z).$$

[This would be clearly true if $u(x) = 1/2\ u(y) + 1/2\ u(z)$, i.e., if in accordance with criterion (6), the subject computed the expected utilities resulting from a and $\bar{a}$, respectively.]

Postulate 2: If y, x, z have equidistant utilities for $s = s^*$, then they have equidistant utilities for any other value of s. That is utilities are independent of beliefs.

Now fix $u(x) = 0$, $u(y) = 1$. Then $u(z) = -1$; and by repeatedly applying the postulate, one finds rewards with utilities 1/2, 1/4, ..., -1/2, -1/4, ..., 2, 4, ..., etc.

Thus far, only probability = 1/2 is defined. By applying criterion (6) to various observed choices, we can define other values of probability.

IV. Decision and Learning.

The above discussion suggests that decision can profitably be studied from the point of view of consistency. The subject's decisions reveal a consistent function u and a consistent function p, satisfying criterion (6) or the postulates of Ramsey (or similar ones of Savage) that underlie that criterion. Does the subject at least "learn," i.e., does he converge to consistent behavior in this sense? Are there any regularities in this learning process? Can it be accelerated by proper training? In this context, a variety of interesting experiments may be directed.