COWLES COMMISSION DISCUSSION PAPER: STATISTICS NO. 359



# Comments on Causality and Identification

by T.C. Koopmans

March 21, 1951

The following comments on H.A. Simon's paper Stat. 353, prepared in rather limited time, fall under two headings:

I. The description of causal structure

II. The relationships between causal structure and the problem of identification.

I shall follow Simon in using "linear terminology" for convenience, although the proper generalizations to non-linear systems should not be too difficult to carry through.

## I

The discussion centers on properties of the coefficient matrix invariant under premultiplication with a non-singular diagonal matrix. The question needs to be considered why this mathematical topic is associated with the notion of causal structure. I believe that this association has a good reason.

A critic may argue that if all the (linear) equation system does is to explain the determination of the variables by a natural law, this law remains just as valid or invalid after linear transformation of the system

with a general non-singular matrix, and diagonal transformation matrices have no special commendation.

However, the notion of a chain or structure of causality would probably not have arisen if there were not at least conceptually the possibility of active intervention at definite places in the chain. The notion of causal structure becomes an operational one if we regard it as a relationship between the point at which intervention applies and the set of variables affected by intervention, rather than as a relationship between variables only.

In the language of the linear example, this possibility of intervention can be illustrated by the assumption that we have the power to vary the constant terms of all equations. It is such a power which permits us to think of the individual equations as separate laws rather than considering the entire equation system as one non-decomposable law.

I conjecture that the classification of causal structures in linear systems developed by Simon can be formulated entirely in terms of a study of the sets of variables affected by variations in individual "constant" terms.

It may be that the foregoing remarks are superfluous in the light of Simon's sections 4 and 8.1. However, I do not fully understand the role played by his "experimenters" who include nature, and who also set the coefficients which have prescribed values (zero). These experimenters seem to wield larger powers than human research workers do, and also larger powers than needed to give content to the notion of causal structure. A one-dimensional power of intervention in each equation is sufficient for the latter purpose.

## II

The following tentative and loosely connected observations are offered at this time in the hope that they may help us to visualize and further unravel the relationships between causal structure and identifiability uncovered by Simon.

The description of causal structure depends almost exclusively on information as to which coefficients are zero, which are not (the only further condition being that the values of the non-zero coefficients permit the ranks of certain matrices to attain maximal values). Its main mathematical apparatus is the theory of finite sets. What is established with certainty is the absence of an effect of certain types of intervention on certain variables.

The identification problem arises in preparing the further attempt to assess numerically the magnitude of the effects of a given intervention on a given variable where such effect is not absent. The problem whether such a magnitude is identifiable (whether sufficient observations contain information on it) is a problem in infinite sets.

The information that enters into the determination of identifiability may take the form of prescribed zero coefficients, but this is by no means the only possible form. Even in the linear example, linear functions of the coefficients in one equation may be given prescribed values. The parallels that have been uncovered refer therefore in first instance to the case where identifying information takes the form of prescribed zeros of single coefficients only.

I believe that, as a result of Simon's analysis, further thought should be given to the meaning of identifiability (as we have used it) in systems involving exogenous variables.