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Note on Computation of Sampling Variances and Covariances in the Case of Reduced Form Estimation

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The following will iddicate how to compute the sampling variances and covariances after "limited information"* estimation has been completed. It is based on Anderson and Rubin's, "Estimation of the Parameters of a Single Stochastic Difference Equation in a Complete System." We shall use the same notation, and all references are to this mimeographed article.

According to footnote 17, we may in 7.96-98 omit any column or row, instead of only the N-th. It is convenient to omit the 1st when we normalize

(1) $\quad \beta = (1, \beta_2 \dots \beta_{+1})$

as is usually the case. 7.96 then becomes

(2) $\quad \sigma(\hat{\beta}, \hat{\beta}) = c(\zeta - \beta'\psi)_{).;1(} \left[\Theta_{)1;1(}\right]^{-1} (\zeta - \psi'\beta)_{)1;.(}$

where the subscript).;1(to any matrix A indicates that no row and the first column of A is to be omitted;)1;1(means that both first column and row are omitted. In (2) we have (from 3.48)

(3) $\quad c = \hat{\beta}\Omega\hat{\beta}' = (1 + v)\ \hat{\beta}\ W\ \hat{\beta}'$

where v is obtained from 4.17 and W from 4.7, while $\hat{\beta}$ is the result of the estimation after normalization.

In the usual case (1) the matrix ζ which describes the normalization rule is

$$\zeta = \begin{pmatrix} 1 & 0 & . & . & . & 0 \\ 0 & . & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & . & . & . & . & 0 \end{pmatrix}$$

* This method was formerly called the "reduced form" method.

so that it follows from 7.48 that

$$\psi = (1 \ldots \beta_{+1}) \begin{pmatrix} 1 & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{pmatrix} = (1, 0, \ldots 0).$$

Therefore we have in (2)

$$(I - \beta'\psi)_{).;1(} = \left[I - \begin{pmatrix} 1 & 0 & \cdot & \cdot & 0 \\ & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & 0 \end{pmatrix} \right]_{).;1(} = \begin{pmatrix} 0 & \cdot & \cdot & \cdot & 0 \\ 1 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 1 \end{pmatrix}$$

This reduces (2) to

$$(4) \quad \sigma(\hat{\beta}' \hat{\beta}) = c \begin{pmatrix} 0 \\ I \end{pmatrix} \left[\Theta_{)1;1(} \right]^{-1} (0 \; I\,)) = c \begin{pmatrix} 0 & 0 \\ 0 & (\Theta_{)1;1(})^{-1} \end{pmatrix}$$

where Θ is estimated as follows.

From 7.40* (corrected):

$$(5) \quad \hat{\Theta} = P^{**} M_{z^o z^o} P^{**\prime} - \frac{1}{c} \frac{v}{1+v} \Omega \hat{\beta}' \hat{\beta} \Omega$$

Using (3) and 3.41 (in which $\frac{1}{\Sigma\mu} = 1 + v$) we write for the last term

$$\frac{1}{(1+v)\hat{\beta} W \hat{\beta}'} \cdot \frac{v}{1+v} (1+v)^2 W \hat{\beta}' \hat{\beta} W$$ so that we compute $\hat{\Theta}$ from

$$(6) \quad \hat{\Theta} = P^{**} M_{z^o z^o} P^{**\prime} - v \frac{W \hat{\beta}' \hat{\beta} W}{\hat{\beta} W \hat{\beta}'}$$

which can be done directly from the intermediate steps in the "limited information" estimation. Having $\sigma(\hat{\beta}' \hat{\beta})$ we obtain the remaining variances and covariances from

$$(7) \quad \sigma(\hat{\beta}' \hat{\gamma}^{*}) = \sigma(\hat{\beta}' \hat{\beta}) \, P^{\neq}$$

* In T.W. Anderson and H. Rubin, "Outline of Computational Procedure for the Reduced Form Method," the matrix $M_{z^o z^o}$ is called N. In (7) we have $P^{\neq}$, for which Anderson and Rubin use the notation $\tilde{P}^{*}$.

(8) $\sigma(\hat{\gamma}^{*\prime}\,\hat{\gamma}^{*}) = P^{\prime}\,\sigma(\hat{\beta}^{\prime}\,\hat{\gamma}^{*}) + c(M_{z^{*}z^{*}})^{-1}$

Once the "limited information" estimation is completed, the outline for the computation of sampling variances and covariances is therefore as follows:

(a) compute c from (3)

(b) compute θ from (6); omit the first row and column [to get $\hat{\theta}_{)1;1(}$ and invert $\hat{\theta}_{)1;1(}$]

(c) estimate $\sigma(\hat{\beta}^{\prime}\,\hat{\beta})$ from (4)

(d) estimate $\sigma(\hat{\beta}^{\prime}\,\hat{\gamma}^{*})$ and $\sigma(\hat{\gamma}^{*\prime}\,\hat{\gamma}^{*})$ from (7) and (8)