COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 2038

Some Considerations on the Expectation and Planning Horizon Relevant to Entrepreneurial Decisions

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April 11, 1952

I The Setting of the Problem.

I.1 Purpose. In the last two years, as part of the research program on the Project on "Expectations and Business Fluctuations", we have conducted a number of interviews with selected manufacturing firms for the purpose of exploring the nature, goals and methods of their forward planning and the relation between anticipations and economic decisions. In the course of these case studies it was generally found that, at least *prima facie*, observed behaviour did not agree with the behaviour suggested by much of the existing theory based on the assumption of "rational behaviour". In the present note we intend to explore the rational of certain aspects of the observed behaviour. To the extent that observed behaviour can be broadly reconciled with "rational behaviour", such an exploration might suggest modifications or refinements of our existing theories.

I.2 The Hicksian Theory of Entrepreneurial Planning and Observed Behaviour.

We may usefully start out with a brief summary of the essential features of the analysis of planning under certainty, as elaborated by such authors as Hicks, Mosak and Lange.

At any given point of time the firm is supposed to act so as to achieve certain goals over a certain span of time called the horizon. These goals are assumed to be the maximization of a certain function whose arguments are the future history of some or of all the variables with which the firm deals. Let us, for the sake of clarity, suppose that there are K such variables in every one of the T periods which compose the horizon. The values of these K variables in the $t^{\underline{th}}$ future interval will be denoted by x_{tk} $k = 1, 2, \ldots, K$. These K variables will be referred to as the components of the $t^{\underline{th}}$ move. We shall assume that the activity of the firm in the $s^{\underline{th}}$ period, or $s^{\underline{th}}$ move, is fully described by the value of these K variables.

The goal of the firm is then supposed to be the maximization of a function

$$(1) \qquad G_0 \; (x_{11} \ldots x_{1k} \ldots x_{T1} \ldots x_{TK}) .$$

The function G_0 will be referred to hereafter for brevity as the Pay-off Function as of the initial point of time O. In the Hicksian approach, this pay-off function reduces to the sum of discounted net receipts over the horizon.

Now, to quote Hicks in Value and Capital (page 193), "The 'decision' that confronts any particular entrepreneur at any date" (-e.g., our point O) "may be regarded as the establishment of a production plan", i.e., the determination of the specific values of the KT variables which will maximize the pay-off function. The firm, however, is not in a position to choose just any set of values of these variables because these variables are themselves subject to various types of constraints which can be broadly classified as demand constraints, supply constraints, and transformation constraints.

In order to reach its "decision" the firm must, therefore, have anticipations as to the values of the parameters of the future constraints. We shall call these anticipated values the "expected parameters" of future constraints. In the Hicksian analysis, the demand and supply constraints degenerate into one-parameter constraints stating that the purchase or selling price is equal to a certain constant which is completely out of the firm's control or, as we shall say, fully determined by the "environment". Thus, the expected parameters of demand and supply constraints reduce to expected future prices. This is, of course, a special case. In general, we may say that the components of the pay-off function are subject to various kinds of constraints, the parameters of which are outside the control of the firm, being determined, either by initial conditions (i.e., by the history preceding 0, including the values of x_{tk} for $t \leqq 0$) or, by future moves of the environment.

The problem of maximizing the pay-off function subject to these expected constraints can be logically reduced to the problem of solving a system of simultaneous equations. At point 0 the firm is supposed to solve this system somehow, and to come up with a solution which will consist of a specific value for each of the KT variables of the pay-off function. These values, which we will denote by $\widetilde{x}_{tk}$ have the property of yielding the greatest value of the pay-off function among all values that are "achievable" in view of the constraints. (There may, of course, be more than one solution, but this may be ignored for the moment, as it is irrelevant to our argument.) The firm proceeds then to implement this solution with respect to the K components of the first move, i.e., to set $x_{1k} = \widetilde{x}_{1k}$. The remaining part of the solution or "decision", $\widetilde{x}_{tk}$, $t > 1$, represents the plan for future operations.

Thus, at point 0 the firm is supposed to reach a "decision" not only with respect to its first move, but also with respect to every component of every future move over the horizon; although, of all these moves, only the first move can be implemented currently; and in order to reach this "decision" it must have expectations about the parameters of every future constraint.

This conclusion leaves with us an uneasy feeling, for introspection and casual observation strongly suggest that firms and other economic agents seldom, if ever, behave this way. The information we have secured so far through our interviews and case studies suggests that while firms, by and large, do set up plans with respect to various phases of their operations on the basis of anticipated demand, supply, and production conditions; nevertheless, the horizon of the plans and of the anticipations may well be different for different phases of the operations and for different types of anticipations. Furthermore, one phenomenon that has been found to occur rather frequently is that the length of the horizon, for at least certain types of anticipations and plans, is a periodic function of time. It is longest at a certain point (or points) of the year (in one of the industries, for instance, it reached a length of approximately 16 months at this point); thereafter the terminal date of the horizon remains unchanged so that the length of the horizon gradually shrinks to a minimum (about 4 months in the above instance) at which point it grows again, suddenly, to its maximum length. The occurrence of this behaviour appears to be associated with seasonality in the demand for the output (or in the supply of some essential input); in fact, when sales exhibit more than one seasonal peak there tends to be as many points within a year at which the length of the horizon suddenly expands.

It was also frequently stressed by the respondents that their plan did not represent a decision as to the actual level of future activity; in fact on occasions the respondents almost seemed to claim that the plan had no implications at all with respect to future operations.

One possible explanation as to why we do not find in reality the type of planning suggested by the Hicksian analysis is that firms do not operate under conditions of certainty. However, this explanation, which is undoubtedly relevant, does not seem to be sufficient or even necessary to account for all of the phenomena we have mentioned earlier. A re-examination of the role of expectations and plans in the economy of the firm suggests that these phenomena can be accounted for, at least partly, without giving explicit recognition to the effect of uncertainty provided we recognize that the formation of reliable estimates about the parameters of future constraints and the setting up of detailed plans can be done only at a cost, i.e., only by allocating scarce resources to this task.

Specifically, we shall argue that even if the firm were in a position to estimate with complete accuracy the parameters of future constraints by devoting enough of its resources to this task, it may be sufficient to form anticipations and make plans with respect to only certain aspects of the future; and that, in fact, the anticipations that need be formed and the moves that may usefully be planned may be but a small subset of all future parameters and moves which will finally determine the outcome of the entire "game".

The argument that we shall develop more rigorously below involves, essentially, the following considerations:

(a) What the firm must really worry about at point 0, is not the establishment of the best production plan, as Hicks suggests, but the solution for the best first move, i.e., the determination of $\tilde{x}_{1k}$ and its implementation, obtained by setting $x_{1k} = \tilde{x}_{1k}$. For, clearly, the first move is the only one that can, and must, be implemented immediately.

(b) In order to determine the value $\tilde{x}_{1k}$ of the K components of the first move which, together with the optimum value of the components of future moves, will maximize the pay-off function, it may not at all be necessary to have information on all the parameters of future constraints or to plan all components of all future moves.

(c) If information on some parameters is not necessary in determining $\tilde{x}_{1k}$, then it will not be necessary, at point 0, to devote resources to secure such information.

(d) Finally, if we add the reasonable assumption that information about conditions at a future point of time will increase as we approach that point, and that the cost of ascertaining such future conditions decreases in time, then rational behaviour will actually involve <u>not</u> devoting scarce resources at point zero to ascertaining parameters which are irrelevant at this point of time.

The notion that the problem of the firm at point 0 is not that of deciding on a single valued plan, is essentially not a new one. It has already been pointed out more or less explicitly by various authors that as soon as we recognize the existence of uncertainty about the possible value of the parameters of future constraints it is, in general, not desirable for the firm to "decide" on a single valued plan except with respect to the first move itself.[1] Marschak, in particular, states explicitly[2] that the firm must decide the first move but, as for the future, it should decide not any specific move but rather a "strategy", i.e., a rule for choosing future actions in response to later information. What we are concerned with in this note, however, is to explore certain consequences of the fact that at point 0 only the first move is to be decided.

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[1] See for instance: A. G. Hart, "Anticipations, Uncertainty and Dynamic Planning", August M. Kelley, New York, 1951; and "Risk, Uncertainty and The Unprofitability of Compounding Probabilities" in Studies in Mathematical Economics and Econometrics in memory of Henry Schultz, 1942.

H. Markowitz, "On The Certainty Equivalence and Risk Discount Hypothesis", Cowles Commission discussion paper: Economics, No. 295.

J. Marschak: "Role of Liquidity under Complete and Incomplete Information", American Economic Review, Vol. XXXIX No. 3 (May, 1949), pp. 182-195.

G. Tintner: "The Theory of Choice Under Subjective Risk and Uncertainty", Econometrica Vol. 9, July-October, 1941, pp. 298-304.

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[2] Op. cit., p. 188

II Irrelevant Future Parameters and Future Moves.

II.1 The nature of the pay-off function and the constraints. As in the previous section, we can assume that the firm acts so as to maximize a function which involves all of its future moves x_{tk}. The value of this function (outcome of the game) will depend, however, not only on its own moves but also on certain parameters which the firm does not control. We may usefully divide parameters into two groups; (i) those that are fully determined, and known as of point 0, these may be labeled "initial conditions" and will be denoted by a^i_{00}; and (ii) those which will be determined by future moves of the environment; these will be denoted as $a^i_{\ell 0}$ where the subscript $\ell = 1, 2, \ldots, T$ characterizes the point of time at which the environment's move will take place. The pay-off function to be maximized can then be written as

$$G_0 \; (x_{tk}, \;\; a^i_{00}, \;\; a^i_{\ell 0}) \; .$$

This function is to be maximized with respect to x_{tk} subject however to certain constraints. In the Hicksian model the only constraints are transformation constraints, and they are described by a single transformation function. This seems highly unrealistic even for the transformation constraints. Furthermore we need not assume perfect competition in the buying and selling market and we must therefore give explicit recognition to demand and supply constraints. Under these conditions it appears that the restrictions on the x_{tk}'s can be described usefully by a set of "dated" constraints, i.e., by a set of constraints relating to specific intervals of time. Each of the constraints relating to the first period will involve subsets of the variables x_{1k} and certain parameters. We shall denote these parameters by the symbols a^i_{01} and a^i_{11}. The first subscript indicates whether this parameter is already determined at the beginning of period 1 (subscript 0) or whether it will be determined within period 1 (subscript 1); the second subscript indicates the date of the (earliest) constraint in which this parameter enters. With complete generality we can then describe the first period constraints by a set of functions:

$$2.1 \quad f_{1j}\,(x_{1k}, a^i_{01}, a^i_{11}) = 0 \qquad j = 1, \ldots, J_1$$

when J_1 denotes the number of first period constraints. Of course not all the x_{1k}, a^i_{01}, and a^i_{11} need appear in every one of the constraints. Similarly, some of x_{tk} may appear in the constraints but not explicitly in G_0. Some of these moves may denote "intermediate products" or moves which do not appear as such in the pay-off function but are nonetheless relevant to the final outcome through the fact that they appear in certain constraints.

Some of the constraints (2.1) will typically take the form of inequalities rather than equalities; this fact will play an important role in some later developments of our argument.

Let us consider now the second period constraints; these will, in general, involve the second period moves x_{2k}, and certain parameters a^i_{02}, a^i_{12}, a^i_{22} (and, possibly, some of the parameters a^i_{01} and a^i_{11}). In addition, however, they will generally involve at least some of the components of the first move x_{1k}. For, generally, the specific limitations within which the x_{2k} can be chosen will depend on moves made (or not made) in the first period. Thus, the form of the Marshallian demand schedules for the second period, might depend among other things on prices charged in the first period, first period advertising outlays, etc. Similarly the conditions under which second period inputs can be transformed into output might depend on such first period actions as changes in production facilities and production of parts or components. The second period constraints can thus be described by a set of functions:

$$(2.2) \qquad f_{2j}\,(x_{2k}, x_{1k}, a^i_{01}, a^i_{11}, a^i_{02}, a^i_{12}, a^i_{22}) = 0 \qquad j = 1, \ldots, J_2.$$

Similarly the constraints for any other period s will be described by a set of functions

$$f_{sj}\,(x_{tk}, a^i_{\ell\tau}) = 0$$

$$s = 1, \ldots, T; \quad j = 1, \ldots, J_t;$$

$$t, \tau = 1, \ldots, s; \quad \ell = 0, 1, \ldots \tau.$$

The date of a constraint is thus characterized by the largest t subscript of any of the variables x_{tk} appearing in the constraint. We might note in passing that the Hicksian assumption that all the limitations on the choice of the x_{tk} can be expressed by a single "production function" relation, amounts to assuming that $\frac{\partial f_{sj}}{\partial x_{tk}} \equiv 0$ for every f_{sj} except f_{T1}, i.e., that there exists a single constraint with date T. It implies that all variables x_{tk}, $t < T$, can be chosen with complete freedom, an assumption very hard to defend on realistic grounds.

Note that while we are giving explicit recognition to the effect of every move within the horizon on later constraints, the effect on these constraints of moves preceeding period 1 is incorporated in the parameters a^1_{0t}. The reason, of course, is that as of the decision point 0, these moves are by-gones and therefore as much outside the firm's control as all other moves of the environment. If the j^{th} constraint for the second period were looked at from the decision point 1, (i.e., at the end of period 1), then it might be written as:

$$f'_{2j}\ (x_{2k},\ b^1_{12},\ b^1_{22}) = 0\ .$$

The parameters b^1_{12}, b^1_{22} however will be uniquely related to the quantities x_{1k} and $a^1_{\ell\tau}$ ($\ell = 0, 1;\ \ \tau = 1, 2$); and similarly the function f'_{2j} will be related to f_{2j}. This relation must be such that the constraint imposed on x_{2j} by f_{2j} for the given values of x_{1k} and $a^1_{\ell\tau}$ is identical with the constraint imposed on x_{2j} by f'_{2j}. Thus the effect of the passage of time on any future constraint is that a growing number of variables entering into the constraint is transformed into constants and freedom of choice is narrowed down.

The conditions under which certain parameters of future constraints and certain later moves are irrelevant at point 0, may be stated in different ways; it will be useful to examine several alternative formulations since this will help to bring out the operational meaning of "irrelevance".

II.2 Irrelevance: as a partitioning of the pay-off function. A first useful formulation can be arrived at along the following lines. Each of the constraints f_{tj} implies that one of the x_{tk}'s can be expressed as a function of the remaining variables (components of moves) in the constraint. The entire set of constraints, say F in number ($F = \sum_{t=1}^{T} J_t$), implies that F of the components x_{tk} can be expressed as functions of the remaining TK - F components. Accordingly, the problem of maximizing G_0 subject to the system of constraints can be reduced to the problem of maximizing unconditionally some new function, G_0^* of these remaining variables; this function, G_0^* will involve in addition to these variables the parameters of the original function G_0 as well as the parameters of the various constraints. Let us denote by $Z_{tr}, r = 1, 2, \ldots, R_t$; $R_t = K - J_t$, the first, second ... last of the components, x_{tk}, of the t^{th} move which have not been eliminated by this process. The function to be maximized unconditionally can then be written as:

$$(II.2.1) \qquad G_0^* = G_0^* \; (Z_{tr}, a^i_{\ell t}) .$$

We will say that there exist irrelevant parameters of future constraints and irrelevant components of future moves with respect to the planning point 0 if the function G_0^* satisfies the following two conditions:

Condition I G_0^* can be expressed identically as the sum of two functions:

$$G_0^* = G_{0I}^* + G_{0II}^*$$

in such a way that G_{0I}^* depends on:

(i) all the components of the first move Z_{1r}; $r = 1, \ldots, R_1$

(ii) only a proper subset of components of later moves, Z_{tr}, $t = 2, \ldots, T$,

(iii) only a proper subset of the parameters $a^i_{\ell\tau}$ for $\ell > 0$, $\tau > 1$.

If G_0^* and G_{0I}^* are differentiable the conditions listed so far imply that there exist values of t and r for $t \geq 2$ such that

$$\text{(a)}\quad \frac{\partial G^*_{OI}}{\partial Z_{tr}} \equiv 0 \quad \text{while} \quad \frac{\partial G^*_O}{\partial Z_{tr}} \not\equiv 0$$

and values of i, $\tau > 1$, $\ell > 0$ such that

$$\text{(b)}\quad \frac{\partial G^*_{OI}}{\partial a^i_{\ell\tau}} \equiv 0 \quad \text{while} \quad \frac{\partial G^*_O}{\partial a^i_{\ell\tau}} \not\equiv 0 .$$

We will denote by Y_{tk}, $k = 1, \ldots, K_t$, the subset of the R_t variables Z_{tr} which enter into G^*_{OI}, for given t. These variables may or may not appear also in G^*_{OII}. The subset of the variables Z_{tr}, $t > 1$, entering into G^*_{OII} but not into G^*_{OI} will be denoted by

$$Y'_{tk}, \quad k = 1, 2, \ldots, K'_t \quad (K'_t = R_t - K_t).$$

We will similarly denote by $b^i_{\ell\tau}$, $i = 1, \ldots, I'_{\ell\tau}$, the subset of the $I_{\ell\tau}$ parameters $a^i_{\ell\tau}$ for given ℓ and τ which appear in G^*_{OI}. These parameters may also appear in G^*_{OII}. Finally we will denote by $b^{i'}_{\ell\tau}$, $i = 1, 2, \ldots, I''_{\ell\tau} = I_{\ell\tau} - I'_{\ell\tau}$, the subset of $a^i_{\ell\tau}$ which appear in G^*_{OII} but not in G^*_{OI}. Making use of this notation we can restate condition I thus

$$\text{I}' \qquad G^*_O \; (Z_{1r}, Y_{tk}, Y'_{tk}, b^i_{\ell\tau}, b^{i'}_{\ell\tau}) \equiv$$

$$G^*_{OI} \; (Z_{1r}, Y_{tk}, b^i_{\ell\tau}) + G^*_{OII} \; (Z_{1r}, Y_{tk}, Y'_{tk}, b^i_{\ell\tau}, b^{i'}_{\ell\tau}).$$

In order to state our second condition we will need the following additional notations:

(i) Let $\tilde{Z}_{1r}$, $\tilde{Y}_{tk}$ denote the values of the variables entering into G^*_{OI} which maximize G^*_{OI}, so that

$$G^*_{OI} \; (\tilde{Z}_{1r}, \tilde{Y}_{tk}, b^i_{\ell\tau}) = \tilde{G}^*_{OI} \geq G^*_{OI} \; (Z_{1r}, Y_{tk}, b^i_{\ell\tau}).$$

(ii) If, in G^*_{OII}, we replace Z_{1r}, Y_{tk} by $\tilde{Z}_{1r}$, $\tilde{Y}_{tk}$, then G^*_{OII} becomes a function of Y'_{tk} only. Denote then by $\tilde{Y}'_{tk}$ the value of the variables Y'_{tk} which maximize G^*_{OII} subject to the conditions $Z_{1r} = \tilde{Z}_{1r}$, $Y_{tk} = \tilde{Y}_{tk}$.

Then

$$G^*_{OII}(\tilde{Z}_{1r}, \tilde{Y}_{tk}, \breve{Y}'_{tk}; b^i_{\ell\gamma}, b^{i'}_{\ell\gamma}) \equiv \breve{G}^*_{OII} \geq$$

$$G^*_{OII}(\tilde{Z}_{1r}, \tilde{Y}_{tk}, Y'_{tk}; b^i_{\ell\gamma}, b^{i'}_{\ell\gamma})$$

(iii) Finally let $\tilde{\tilde{Z}}_{1r}, \tilde{\tilde{Y}}_{tk}, \tilde{\tilde{Y}}'_{tk}$, denote the values of the variables which maximize G^*_0, given the parameters $b^i_{\ell\gamma}$, $b^{i'}_{\ell\gamma}$, i.e., such that

$$G^*_0(\tilde{\tilde{Z}}_{1r}, \tilde{\tilde{Y}}_{tk}, \tilde{\tilde{Y}}'_{tk}; b^i_{\ell\gamma}, b^{i'}_{\ell\gamma}) \equiv \tilde{G}^*_0 \geq$$

$$G^*_0(Z_{1r}, Y_{tk}, Y'_{tk}; b^i_{\ell\gamma}, b^{i'}_{\ell\gamma})$$

Our second condition is then:

<u>Condition II</u>

$$\tilde{G}^*_0 = \tilde{G}^*_{OI} + \breve{G}^*_{OII} .$$

In other words, the two functions G^*_{OI} and G^*_{OII} which satisfy condition I should be such that the maximum of G^*_{OI} plus the "conditional" maximum of G^*_{OII} should add up to the unconditional maximum of the original function G^*_0.

If the problem of maximizing G^*_{OI} should not yield a unique solution, then condition II should hold for every one of the solutions. In this case, of course, the maximization of G^*_0 must itself admit of more than one solution.

Suppose that conditions I and II hold simultaneously, and the solution $\tilde{\tilde{Z}}_{1r}, \tilde{\tilde{Y}}_{tk}$ is unique, then, necessarily:

$$\tilde{Z}_{1r} = \tilde{\tilde{Z}}_{1r} \text{ and } \tilde{Y}_{tk} = \tilde{\tilde{Y}}_{tk} .$$

If the maximization of G^*_0 does not have a unique solution it must still be true that every solution $\tilde{Z}_{1r}, \tilde{Y}_{tk}$ to the maximization of G^*_{OI} must correspond to one of the possible solutions to the maximization of G^*_0. Since the consideration of multiple solutions lengthens the argument without changing the conclusion, in what follows we will ignore this additional complexity.

If the pay-off function G^*_0 can be expressed as the sum of two functions satisfying conditions I and II we will say that G^*_0 <u>partitions</u>. Such a partitioning will be called non-trivial if the set $b^{i'}_{\ell\gamma}$ and Y'_{tk} are non-empty. In what follows the word partitioning will be used to denote non-trivial partitioning.

We are now ready to state the implications of partitioning. We have indicated that the decision problem of the firm at point 0, might be regarded as that of determining the best value of the components Z_{1r} which, by definition, is represented by $\tilde{\tilde{Z}}_{1r}$. But, if G_0^* partitions, the value $\tilde{\tilde{Z}}_{1r}$ coincides with $\tilde{Z}_{1r}$; and $\tilde{Z}_{1r}$ can be found by maximizing G_{0I}^*, without regard to G_{0II}^*. It follows that the parameters $b_{\ell\gamma}^{i'}$ and the components Y_{tk} which appear in G_0^* but not in G_{0I}^*, are irrelevant with respect to the decision problem at point 0. We can therefore say that if G_0^* partitions there will exist, at point 0, irrelevant parameters, namely the parameters $b_{\ell\gamma}^{i'}$, and irrelevant components of future moves, namely the components Y_{tk}'.

Suppose that there is a cost attached to estimating, at point 0, parameters of future constraints. Then, especially if this cost is higher than the cost of estimating these parameters later, rational decision-making will not call for devoting scarce resources to the estimation of irrelevant parameters. Similarly it will not call for determining the best value of irrelevant future moves if there are costs attached to doing this. Hence, if partitioning exists, we might expect to find anticipations and plans relating to only some aspects of future conditions and future operations.

The problem we must face next is that of examining whether there exist significant examples of functions which partition, especially among those functions which occur in economic analysis.

III Examples of Partitioning When the Functions Involved are Continuous and Differentiable.

III.1 Segmentation of the pay-off function and constraints. A very simple example of partitioning is represented by the case in which the function G_0 to be maximized consists of a sum of functions, say:

$$G_0 = \Phi_{01} + \Phi_{02} \cdots\cdots + \Phi_{0T}$$

such that each Φ_{0t} involves only components with date t; and when each of the J_t constraints for period t involves only components dated t. The first condition implies that the outcome of the entire game can be represented as the sum of the outcomes for successive periods. This situation is not too unrealistic with respect to economic problems; for instance, the Hicksian pay-off function representing the sum of discounted net receipts belongs to this class. The second condition implies that one period's actions in no way restrict or modify the choices open in every successive period. Under these conditions the function G_0^*, determined by G_0 and the constraints, will itself represent the sum of functions, say $G_0^* = \Phi_{01}^* \cdot \cdot \cdot \Phi_{0T}^*$, such that the function Φ_{0t}^* will involve only a subset Z_{tr} of the period t variables, x_{tk}. In this case we have immediately

$$G_{01}^* = \Phi_{01}^*, \qquad G_{02}^* = \sum_{2}^{T} \Phi_{0t}^* .$$

It follows that all parameters except those of Φ_{01}^* and of the first period constraints and all components except those of the first move are irrelevant at point 0.

It is useful to examine what partitioning implies with respect to the standard method of maximizing a differentiable function, which consists in solving the system of equations

$$\text{(III.1.1)} \qquad \frac{\partial G_0^*}{\partial Z_{tr}} = 0, \qquad r = 1, 2, \ldots, R_t; \qquad t = 1, 2, \ldots, T.$$

Since the function Φ_{01}^* involves only the first period variables Z_{1r}, its derivatives also must involve only these variables.

Hence the system of equations:

$$\frac{\partial G_0^*}{\partial Z_{1r}} = \frac{\partial \Phi_{01}^*}{\partial Z_{1r}} = 0$$ forms a determined sub-system of

(III.1.1) consisting of R_1 equations in the R_1 unknowns Z_{1r}, the solution of which will yield $\tilde{Z}_{1r}$. Thus the partitioning of the pay-off function corresponds to a partitioning of the system of equations (III.1.1).

It is probably impossible to find an exact economic counterpart to this analytical example. A reasonable approximation might be found in the case of a firm dealing in a non-storable commodity, which must be produced (or purchased) and disposed of within the same "period", and employing very little fiscal capital. For such a form decisions within each period would tend to depend exclusively on actual and anticipated conditions within this period. And knowledge about conditions in future periods would be of little value since such knowledge would not lead to current decisions different from those taken in its absence. To be sure, an anticipation of drastic changes might call for some special actions now; and under such conditions it might be worth while to devote resources to increase whatever rudimentary knowledge about future periods might already be available. But generally the relevant expectations and planning horizon would tend to be quite short.

Before proceeding to other examples we may note that the analytical example we have just given can be considerably generalized.

Clearly a sufficient condition for G_0^* to partition is that:

(a) G_0 can be written as the sum of two functions G_{0I} and G_{0II} such that all the components Z_{1t} appear in G_{0I}, and every other component appears either in G_{0I} or in G_{0II} but not in both.

(b) Any constraint in which a component entering in G_{0I} appears involves no components appearing in G_{0II} (or more precisely no components entering in any constraint in which a component of G_{0II} appears).

Note finally that if condition (b) holds, then partitioning will occur even if G_0 represents the product rather than the sum of G_{0I} and G_{0II}. For then

$$G_0^* = G_{0I}^* \times G_{0II}^*$$, when G_{0I}^* is obtained from G_{0I}, and the

constraints; and similarly for $\overset{I}{\phi}{}^{*}_{OII}$. Therefore the system of equations obtained by differentiating G_0^* with respect to the variables appearing in $\overset{I}{\phi}{}^{*}_{OI}$, say Z'_{tr}, will be of the form

$$(III.1.2) \qquad \frac{\partial G_0^*}{\partial Z'_{tr}} = \frac{\partial \overset{I}{\phi}{}^{*}_{OI}}{\partial Z'_{tr}} \cdot \overset{I}{\phi}{}^{*}_{OII} = 0,$$

since

$$\frac{\partial \overset{I}{\phi}{}^{*}_{OII}}{\partial Z'_{tr}} \equiv 0;$$

each of these equations involves both Z'_{tr} and the variables appearing only in $\overset{I}{\phi}{}^{*}_{OII}$, say Z''_{tr}. However since $\overset{I}{\phi}{}^{*}_{OII}$ is generally non-zero, a necessary and sufficient condition for (III.1.2) to be satisfied is that:

$$(III.1.2') \qquad \frac{\partial \overset{I}{\phi}{}^{*}_{OI}}{\partial Z'_{tr}} = 0 ,$$

a system of equations each of which involves only Z'_{tr}. Hence G_0^* can be partitioned as $G_0^* = G_{OI}^* \cdot G_{OII}^*$

where $G_{OI}^* = \overset{I}{\phi}{}^{*}_{OI}$, and $G_{OII}^* = \overset{I}{\phi}{}^{*}_{OII}$.

III.2 <u>Other analytical examples</u>. So far we have examined situations in which G_0^* can be expressed as a sum or product of functions having no variables in common. This is, however, not a necessary condition for partitioning.

Suppose the function G_0^* to be maximized unconditionally has the form:

$$(III.2.0) \qquad G_0^* = G(x_{11}, x_{12}, x_{21}, x_{22}/a,b,c,d,e,f,g,h,m) = ax_{11} + bx_{11}^2 + cx_{12} - dx_{12}^2 + e(x_{12} + x_{21}) - f(x_{12} + x_{21})^2 + gx_{21}^2 + h(x_{11} + x_{22}) - m(x_{11} + x_{22})^2 .$$

Here, x_{11} and x_{12} denote all the components of the first move which have to be determined immediately; x_{21} and x_{22} denote the components of the second (and only other) move which need not be determined until the beginning of the second period. The symbols a, . . ., m represent parameters and will be assumed to satisfy certain inequalities necessary to insure the fulfillment of second order maximum conditions.

The first order maximum conditions yield the equations:

$$\text{(III.2.1)} \quad \frac{\partial G}{\partial x_{11}} = a-2bx_{11} + h-2m(x_{11} + x_{22}) = 0$$

$$\text{(III.2.2)} \quad \frac{\partial G}{\partial x_{12}} = c-2dx_{12} + e-2f(x_{12} + x_{21}) = 0$$

$$\text{(III.2.3)} \quad \frac{\partial G}{\partial x_{21}} = e-2f(x_{12} + x_{21}) + 2gx_{21} = 0$$

$$\text{(III.2.4)} \quad \frac{\partial G}{\partial x_{22}} = h-2m(x_{11} + x_{22}) = 0$$

In principle, in order to find $\tilde{x}_{11}$ and $\tilde{x}_{12}$ it is necessary to solve this system of 4 simultaneous equations. Note however, that, because of equation (III.2.4), the last two terms of (III.2.1) drop out so that this equation reduces to:

$$\text{(III.2.1}'\text{)} \quad a-2bx_{11} = 0 ,$$

and no longer involves x_{22}. Since x_{22} does also not appear in equations (III.2.2) and (III.2.3) these equations together with (III.2.1′) form a determinate sub-system of three equations in the three unknowns, x_{11}, x_{12}, x_{21}, which is sufficient to determine $\tilde{x}_{11}$ and $\tilde{x}_{12}$.

It follows that the maximizing value of x_{11} and x_{12} does not depend on the parameters h and m which appear exclusively in (III.2.4). Thus, the decision problem at point zero can be solved without knowledge of these parameters and without regard to x_{22}. The only "decision" that need be made at point zero about x_{22} is that the optimum value will be chosen for this variable, at the time when such a choice has to be made, namely at the beginning of the second period. This optimum value is of course the value satisfying (III.2.4), given the value of the variable x_{11} which, by that time, will have become a constant; it is only at this point that knowledge of the parameters of (III.2.4) will be required.

The above results suggest that x_{22} and the parameters of (III.2.4) are "irrelevant" at point O. If this is the case we should be able to show that the function G of equation (III.2.0) partitions. This is indeed the case. G can be expressed as the sum of the functions

(III.2.5) $$G_I\,(x_{11},x_{12},x_{21};\ a,b,c,d,e,f) = ax_{11} - bx_{11}^2 + cx_{12} - dx_{12}^2 + e(x_{12} + x_{21}) - f(x_{12} + x_{21})^2 + gx_{21}^2$$

and

(III.2.6) $$G_{II}\,(x_{11},\ x_{22};\ h,m) = h(x_{11} + x_{22}) - m(x_{11} + x_{22})^2 .$$

Furthermore, it is easy to verify that the maximization of G_I with respect to x_{11}, x_{12} and x_{21}, and the maximization of G_{II} with respect to x_{22} after replacing x_{11} by $\tilde{x}_{11}$ yields a solution which is identical with the solution of equations (III.2.1) to (III.2.4).

Thus the functions G_I and G_{II} defined by equations (III.2.5) and (III.2.6) satisfy all the necessary conditions to be a partitioning of the function G; and therefore also the parameters h and m, and the component x_{22} which appear in G but not in G_I, are irrelevant according to our earlier definition.

Suppose the parameter g of the function G_0^* defined by equation (III.2.0) has the value zero, and consider what effect this will have on the system (III.2.1) to (III.2.4). It will be seen that, because of the new form of equation (III.2.3), equation (III.2.2) can be reduced to

(III.2.2′) $$c-2dx_{12} = 0,$$

an equation which no longer involves x_{21}. Equations (III.2.1) and (III.2.2′) now contain all of the first period variables, and only these variables. Hence, the solution of these two equations is sufficient for determining the maximizing value of the first period variables, and the variables and parameters of the second period, which appear in equations (III.2.3) and (III.2.4), are irrelevant. In fact, the function G can be written as the sum of two functions:

$$G_I\,(x_{11},\, x_{12};\, a,b,c,d) = ax_{11} - bx_{11}^2 + cx_{12} - dx_{12}^2$$

$$G_{II}\,(x_{11},x_{12},x_{21},x_{22};\quad e,f,h,m) = e(x_{12} + x_{21}) - f(x_{12} + x_{21})^2 +$$
$$h(x_{11} + x_{22}) - m(x_{11} + x_{22})^2$$

which satisfy both required conditions to be a partitioning of the function G.

The above result indicates that the relevance of certain parameters may depend on the value of one or more other "strategic" parameters. It follows, that the estimation of such strategic parameters is certainly warranted, while the estimation of the remaining parameters may or may not be warranted depending on the value of the first parameters. This suggests that both the amount of detail in expectations and the time period covered by these expectations may vary from point to point depending on the nature of certain other expectations.

We will not attempt to make a general statement about the class of continuous and differentiable functions which have the property of partitioning in the sense of our definition. In the example we have just discussed, the partitioning of the function G can be traced to the following property: equation (III.2.0) can be written as:

$$G_0^* = \varphi(x_{11},x_{22}) + \psi(x_{11}) + \lambda(x_{22}) + \chi(x_{11},x_{21})$$

where

$$\psi\,(x_{11}) = ax_{11} - bx_{11}^2\,; \qquad \lambda = 0$$

$$\varphi\,(x_{11},x_{22}) = h(x_{11} + x_{22}) - m(x_{11} + x_{22})^2\,;$$

and φ has the following essential property:

(III 2.7) $$\frac{\partial \varphi}{\partial x_{11}} = \frac{\partial \varphi}{\partial x_{22}} \ .$$

The first order maximum conditions $\frac{\partial G_0^*}{\partial x_{11}} = 0$, $\frac{\partial G_0^*}{\partial x_{22}} = 0$ become

(III.2.8) $$\frac{\partial \varphi}{\partial x_{11}} + \psi'(x_{11}) = 0 \ ;$$

(III.2.9) $$\frac{\partial \varphi}{\partial x_{22}} + \alpha = 0$$

But, because of (III.2.7), equation (III.2.9) enables us to replace the term $\frac{\partial \varphi}{\partial x_{11}}$ of (III.2.8), which generally involves x_{22}, by the constant $-\alpha$. Thus the value of x_{11} maximizing G_0^* is independent of x_{22} and of the parameters of the function φ, a result that can be traced to the property (III.2.7) of the function φ. The expression $\varphi(x_{11}, x_{22}) + \alpha x_{22}$ will be components of the function G_{0II}^*.

It seems possible to find many economic instances which have the same analytical structure as the examples just given, i.e., instances in which later actions can be adjusted with no loss of efficiency to earlier ones, whatever these might have been, so that these earlier actions may be chosen without regard to the later ones. This will be illustrated by two very simple examples.

Consider, first, a two-period problem with x_1 and x_2 denoting production in the first and second period, respectively. Suppose, that all sales will occur in the second period. Let the demand conditions be represented by a demand function $P = P(Z_2)$, where $Z_2 = x_1 + x_2$ denotes sales. Suppose, finally, that total cost depends only on total production and is independent of its distribution over time, say:

$C = C(x_1 + x_2)$. In this case the function G^* can be written as:

$G^* = (x_1 + x_2)\ P(x_1 + x_2) - C(x_1 + x_2)$, and the first order conditions become:

$$\frac{\partial G^*}{\partial x_1} = \frac{\partial G^*}{\partial x_2} = (x_1 + x_2) P' + P - C' = 0 .$$

These are two equations which are not independent. Therefore the maximizing values of x_1 and x_2 are not determined although their sum Z_2 is. In fact $\tilde{Z}_2$ is that value of Z_2 which satisfies the equation:

$$Z_2 P'(Z_2) + P - C'(Z_2) = 0.$$

The maximizing value of x_2 may be stated as $\tilde{x}_2 = \tilde{Z}_2 - x_1$, but for x_1 any value (provided it is less than $\tilde{Z}_2$) may be chosen. Thus, precise knowledge of second period demand conditions will not be required for first period decision. Such knowledge will be required only in the second period in order to set price and to determine second period production so as to make up any balance $\tilde{Z}_2 - x_1$.

First period moves, however, need not be indeterminate, as shown by the following modification of the previous example. Suppose that two commodities x_{11} and x_{12} are produced in the first period, and that x_{12} is demanded only in the first period. Let x_{11} and x_{12} have joint costs represented by the expression:

$$C_1(x_{11}, x_{12}) = a_0 x_{11} + a_1 x_{12} + \frac{b}{2} (x_{11} - x_{12})^2.$$

Suppose, further, that the first commodity can also be produced in the second period at constant marginal cost, that is:

$$C_2 (x_{21}) = a_2 x_{21}.$$

Let the demand conditions be described by:

$$P_{21} = P_{21} (Z_{21}); \qquad P_{12} = P_{12} (Z_{12}) ,$$

where

$Z_{12} \leq x_{12}$ denotes first period sales of the second commodity,

and

$Z_{21} \leq x_{11} + x_{21}$ denotes second period sales of the first commodity.

The pay-off function now becomes:

$$G = Z_{12} P_{12} + Z_{21} P_{21} - a_0 x_{11} - a_1 x_{12} - a_2 x_{21} - \frac{b}{2} (x_{11} - x_{12})^2 \qquad \text{(III.2.10)}$$

which is to be maximized subject to the constraints just listed. Since it is clear that the maximization of (III.2.10) requires $Z_{12} = x_{12}$; $Z_{21} = x_{11} + x_{21}$, the function G^* can be obtained by substituting the x's for the Z's in (III.2.10). The first order maximum conditions then yield the equations:

$$\frac{\partial G^*}{\partial x_{11}} = P_{21} + (x_{11} + x_{21}) P_{21}' - a_0 - b(x_{11} - x_{12}) = 0 \qquad \text{(III.2.11)}$$

$$\frac{\partial G^*}{\partial x_{12}} = P_{12} + x_{12} P_{12}' - a_1 + b(x_{11} - x_{12}) = 0 \qquad \text{(III.2.12)}$$

$$\frac{\partial G^*}{\partial x_{21}} = P_{21} + (x_{11} + x_{21}) P_{21}' - a_2 = 0 \qquad \text{(III.2.13)}$$

Because of (III.2.13), equation (III.2.11) reduces to:

$$a_2 - a_0 - b\,(x_{11} - x_{12}) = 0\,, \qquad \text{(III.2.11')}$$

an equation which does not involve x_{21}. The simultaneous solution of (III.2.11') and (III.2.12) will yield the maximizing solution (generally unique) for the first period productions, $\tilde{x}_{11}$, $\tilde{x}_{12}$, and sales $\tilde{Z}_{12} = \tilde{x}_{12}$. This solution is obtained without regard to x_{21} and without reference to second period demand conditions; the only second period information required is the marginal cost, a_2, of producing the first commodity in the second period. Knowledge of the demand function P_{21} will be required only in the second period for setting the price P_{21} and for choosing $\tilde{x}_{21}$, which will be the value of x_{21} that satisfies equation (III.2.13), given the value of x_{11} determined by the first period decision.[1]

As in previous examples, the function G^* can be written as the sum of the following functions which satisfy both conditions for partitioning:

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[1] It can be verified that the second order maximum conditions will be satisfied procided $b > 0$ and the demand functions have negative slopes.

$$G_I^* = x_{12}\, P_{12}\, (x_{12}) - (a_0 - a_2)\, x_{11} - \frac{b}{2}\, (x_{11} - x_{12})^2$$

$$G_{II}^* = (x_{11} + x_{21})\quad P_{21}\, (x_{11} + x_{21}) - a_2\, (x_{11} + x_{21})$$

The function P_{21} and the move x_{21} which appear only in G_{II}^* are thus irrelevant in terms of our earlier definition.

IV An Alternative Definition of Irrelevance.

The examples of the previous sections suggest the following alternative definition of irrelevance. The maximization of the function G^* can be logically reduced to the solution of a system of simultaneous relations involving all components of all future moves and all parameters of future constraints. However, at point zero the firm needs to be concerned only with the solution of this system for the K components of the first move, since these are the only ones on which it can and must act currently.

Suppose the system of simultaneous relations partitions in such a way that the solution for the first move can be gotten by solving a certain subset of the entire set of relations, then this subset is the only system which needs to be solved at point zero. We may then say that all those parameters and components of future moves which do not appear in this subset of relations are irrelevant at point zero. Conversely, those which do appear in this subset may be called "relevant".

It should be noted that the implications of relevance are somewhat different for parameters, as distinguished from future moves. Consider the maximizing solution for the components of the first move, $\bar{x}_{1k}$. Each of these $\bar{x}_{1k}$ will be a function of the parameters of the problem but since irrelevant parameters do not appear in the sub-system that needs to be solved in determining ~~again~~ $\bar{x}_{1k}$, these $\bar{x}_{1k}$ will not depend on irrelevant parameters. It is for this reason that it is not worthwhile devoting resources to the estimation of such parameters.

The implications of relevants are somewhat different with respect to future moves. If the firm has to decide only on the first move then it appears unnecessary to make explicit plans, i.e., to solve explicitly for any future move whether relevant or irrelevant; in this sense all components except those of the first move appear irrelevant. While it is true that relevant future moves appear in the sub-system that has to be solved to determine the first move, it is always possible to solve this system explicitly for just some of the unknowns and not for the remaining ones. Even so, there remains a very significant difference between the role of relevant and irrelevant future moves.

In the first place, it is well known that once a system has been solved for some of its unknowns the remaining ones can be frequently obtained at a low marginal cost. In the second place, all the information necessary to solve for the relevant components should be available since it is required in the solution for the first move. Finally, the most economical way of solving for the first move may involve solving first for certain other components even if they are not the ones of primary interest.

To solve for the irrelevant moves, on the other hand, would require solving a different sub-system and, furthermore, in order to provide the solution it would be necessary to secure information on future parameters which are of no value for the first period decision. Hence plans with respect to irrelevant moves would appear definitely unwarranted.

But would it be worthwhile to compute plans for relevant components except in so far as they are a bi-product of the solution for the first move? To answer this question let us first look at the possible role and significance of these plans. They are certainly not final decisions as to the actual level of future components. Such decisions will be made at the proper time, namely at the time at which they must actually be implemented; and when that time comes, the actual level of these components will not be determined by the level planned at an earlier point of time, but by the level which appears best in the light of information available at the later point of time. It seems, therefore, more accurate to think of plans not as decisions about future actions, but as the best judgment that can be made at point 0 as to what the best value of these components is eventually going to be; and this judgment is made for the purpose of deciding the first move and not for the purpose of deciding future moves.

If we stick to our assumption that all relevant parameters are estimated at point zero and that to these estimates a subjective probability of 1 is attached, then the planned value of any relevant component would also be expected, with a probability of 1, to turn out to be the best move when the time comes. It would then be worthwhile to incur the marginal expenditure necessary to make the plan explicit if this is cheaper than solving the problem again at a later point. Even if we drop the assumption of subjective certainty then the marginal expenditure might still be worthwhile because of the probability that at least part of the plan will eventually be worth implementing. That is to say, although part of the plan may be discarded, the salvage value of the whole may still be greater than the cost of making it explicit. We will not pursue this line further, however, because the explicit recognition of uncertainty introduces a number of problems that need to be considered separately.

V Non-Negativity of Variables as a Source of Partitioning.

In many problems arising in economics the conditions of the problem are such that certain variables such as production, inventories, sales, cannot be negative. The non-negative character of these variables seems to be an important source of partitioning as it may be sufficient to sever the links between the nearer and the further future. This point will be illustrated by two examples.

Let x_1, x_2, Z_1, Z_2 denote, respectively, production and sales in each of two periods, and let the demand conditions in each period be described by the demand functions:

$$P_1 = P_1 (Z_1) ; \qquad P_2 = P_2 (Z_2) .$$

Suppose that production occurs at constant marginal cost, though the marginal cost may be different in the two periods. Then the total cost of any program x_1, x_2 can be described by:

$$\text{(V.1)} \qquad C(x_1, x_2) = C_1 x_1 + C_2 x_2 + a$$

We assume that the firm wants to maximize the profit function:

$$G = Z_1 P_1 (Z_1) + Z_2 P_2 (Z_2) - C_1 x_1 - C_2 x_2 - a$$

subject to the constraints:

$$Z_1, Z_2, x_1, x_2 \geq 0$$

$$x_1 - Z_1 = h_1 \geq 0; \quad x_2 + h_1 - Z_2 = h_2 \geq 0$$

where h_1 and h_2 represent inventories at the end of each period. We will not go through the details of the solution which is rather tedious in view of the nature of the constraints. It can be shown, however, that the solution is as follows:

(I) If $C_1 \geq C_2$, then:

(I.1) $Z_1 = \tilde{Z}_1$, where $\tilde{Z}_1$ is the root of the equation

$$\frac{d}{dZ_1}\left[Z_1 P_1 (Z_1)\right] = C_1$$

(I.2) $\tilde{x}_1 = \tilde{Z}_1$; $\tilde{h}_1 = 0$; $\tilde{P}_1 = P_1 (\tilde{Z}_1)$

(I.3) $Z_2 = \tilde{Z}_2$, where $\tilde{Z}_2$ is the root of the equation

$$\frac{d}{dZ_2}\left[Z_2 P_2 (Z_2)\right] = C_2$$

(I.4) $\tilde{x}_2 = \tilde{Z}_2$; $\tilde{h}_2 = 0$; $\tilde{P}_2 = P_2 (\tilde{Z}_2)$

(II) If $C_1 < C_2$, then :

(II.1) $Z_1 = \tilde{Z}_1$, where $\tilde{Z}_1$ is given by (I.1)

(II.2) $Z_2 = \tilde{Z}_2$, where $\tilde{Z}_2$ is given by the root of the equation

$$\frac{d}{dZ_2}\left[Z_2 P_2 (Z_2)\right] = C_1$$

(II.3) $\tilde{x}_1 = \tilde{Z}_1 + \tilde{Z}_2$; $\tilde{h}_1 = \tilde{x}_1 - \tilde{Z}_1 = \tilde{Z}_2$; $\tilde{x}_2 = 0$; $\tilde{h}_2 = 0$.

Thus, in case (II), sales and prices in both periods should be set at the level which equates marginal revenue with first-period marginal costs, and all production should occur in the first period only. The first-period decision therefore requires at least some knowledge of second-period cost conditions, and full knowledge of second-period demand conditions. But suppose that costs in the second period are expected to be lower than in the first period, a case included under (I). If we disregarded the fact that inventories cannot be negative, then the solution would be symmetrical to that for case (II); in each period price should be set at the point where marginal revenue equals second-period marginal cost (which would therefore have to be known exactly at point O), and all production should occur in the second period only. But this would imply negative end-inventories for the first period ($h_1 = -\tilde{Z}_1$), i.e., selling commodities that have not yet been produced and which will only be produced in the second period. When we take into account the inventory constraint, the best feasible plan consists in producing in the first period exclusively on the basis of first-period demand and cost conditions. Thus, as a result of the inventory constraint, we get a partitioning of the pay-off function. Second-period demand conditions and the specific level of costs are irrelevant at point zero.

If we make the problem more realistic by assuming that there are costs attached to carrying inventories, and we denote by α the cost of carrying one unit for one period, then in order for partitioning to occur it is enough that $C_1 > C_2 - \alpha$ [1]. In other words partitioning will occur even if second-period costs are expected to be higher than the first-period costs provided the difference is less than storage costs.

The above example, incidentally, illustrates again how the relevance of certain future parameters may itself depend on the value of certain other parameters, in this case second period costs. It also shows that expectations

[1] This result follows because under these assumptions a term αh_1 is added to the right hand side of (V.1). See Appendix, section 1.

about a future period may change substantially without having any effect on current actions.

Our last example will be discussed very briefly here as it is treated more fully in the appendix to this paper. In this example the parameters of future constraints are the quantities demanded in each future period, which quantities are assumed independent of the actions of the firm. It is assumed that the firm wants to meet this demand and the problem of maximizing the pay-off function reduces to that of minimizing the cost of meeting the given demand. In the appendix we give a general analytical solution to this problem of minimizing a certain function of T variables subject to linear inequalities, and examine the effect of the constraints on the relevant expectation and planning horizon at point zero.

First we analyze the problem disregarding storage costs; and our results can be stated approximately as follows (see Appendix, section 2.3): if there is any point in the horizon, say t', beyond which average sales are expected to be not larger than average sales up to t', then the specific level of sales in each of the periods following t' is irrelevant for the first period decision; and all moves beyond t' are also irrelevant.

Consideration of storage costs tends to shrink very much further the horizon of relevant expectations and plans. Our results, by and large, indicate that sales in the further future can only be relevant if they are expected to exceed substantially sales in the nearer future; and the further the future, the more must be this excess (see Appendix, section 3.2). Thus, the presence of significant storage costs tends to cut down the length of the relevant horizon.

Finally, we consider a situation in which sales are subject to seasonal variation (see Appendix, section 4). We find that, at any point within a given seasonal cycle, the relevant planning and expectation horizon will tend to extend up to the peak of the given cycle or shortly beyond it, but all further seasonal cycles will normally tend to be irrelevant especially if storage costs (including deterioration, obsolescence, etc.) are significant. This result checks with certain characteristics of observed planning and expectation horizons noted at the beginning of this paper, e.g., the length of the horizon is a periodic function of time, linked with the seasonal cycle.

We may note, once more, that while the relevant horizon for the production plan may terminate with the end of the current season, the length of the relevant expectation horizon may itself depend on certain strategic expectations. Suppose that expectations for the current season are such that in order to meet expected demand, or in order to meet it at lower costs, it would be necessary to add to or improve productive capacity. Then, in order to reach an optimal decision on this matter, it may become necessary to estimate demand and technological conditions over a much longer horizon (although this estimate may require much less detail than would be necessary in planning production). If, on the other hand, current expectations do not warrant any action with respect to productive facilities, then the horizon of relevant expectations may itself not extend beyond the current season.