COWLES COMMISSION DISCUSSION PAPER: ECONOMICS NO. 281

Administrative Aspects of Allocative Efficiency

by Herbert A. Simon

April 25, 1950

It has been shown by Koopmans and others that in a rational system for allocating resources, the location of decision-making functions is unessential. That is, an optimum allocation may be produced by centralized decision-making or by decentralized decision-making with appropriate rules and shadow prices. Evidently the allocative mechanisms thus far studied abstract from the specifically "administrative" aspects of the situation.

This paper will suggest in a preliminary manner three possible ways of introducing administrative factors into a formal theory of allocation. As yet these three approaches are fragmentary and are not fitted into an over-all general theory.

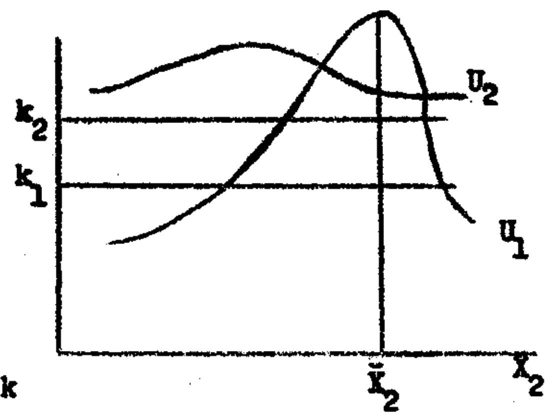
I. Non-identity of interests

In the theory of Koopmans and others, it is assumed that decentralized decision makers will accept rules as laid down by "umpire"--that all participants wish to maximize same objective function .

A. If we now consider that each participant has a separate objective function we are led to some generalization of the von Neumann game theory.

B. In particular we attempt to construct a "game" that embodies the traditional relationship between employer and employee. We assume objective functions U_1, U_2 for each, respectively, which are functions of the behavior (strategy) of the employee only, i.e., $U_1 = U_2(X_2)$. In return for a wage, the employee accepts authority, i.e. permits the employer, within certain limits, to select the value of X_2. Presumably the employer will select X_2 to maximize U_1, but then U_2 will not in general, be maximized.

How can we reconcile such behavior with assumptions of rationality? One possibility is that U_2 is nearly constant over a certain set of values of X_2. Assume that employee will accept employment if $U_2 > k_2$ for this set; and that employer will hire employee if there exists an X_2' in this set such that $U_1(X_2') > k_1$. Then employer will fix X_2 at $\bar{X}_2$ (see chart)



II. Division of Work

This formalization was suggested by Reiter's investigations of returns to scale. We suppose there are a number of (elementary) tasks to be performed and that we wish to minimize the number of workers required to perform them. We will call any set of elementary tasks a "job." We define a positive scalar set function $t(A)$ on all jobs, which represents the <u>time</u> required to perform the tasks in this job. Let W be the time available from each employee. Then we want a partitioning of the elementary tasks into a minimum number of jobs such that for each job, A_i, $t(A_i) \leq W$. Let N_j be the number of jobs in a particular partitioning. Then we require $N_j = \bar{N}$, a minimum.

A. Consider the case where $t(A + B) = t(A) + t(B)$ (tasks are additive), and $t(a_i)$ E for all elementary tasks a_i. Then we can prove that:

$$\frac{T}{W-\mathcal{E}} + 1 \geq \bar{N} \geq \frac{T}{W}, \qquad \text{where } T = \sum_i t(a_i)$$

B. We can elaborate this scheme for the case where all the W_j are not equal, and where jobs are not additive. We can also introduce the supervisory time required, and the affects of organizational units by again partitioning our set of jobs into "units," and these into larger organizational entities—i.e., by introducing a hierarchy of partitionings and making $T(A)$ dependent on the entire hierarchy.

III. Comunications

Along the lines of Klahr's earlier paper, we may introduce an explicit communication system into the model. Here the "reaction characteristics" of the communication system take the place of the helmsman and shadow prices in earlier allocative models. This is obviously analogous to Samuelson's dynamic models for approach to equilibrium.

We may take as example, the following model. Let I be inventories of finished goods, O, orders received in a given week, p, goods produced in that week. Then:

$$\frac{dI}{dt} = P - O.$$

Further, let $P = kX$, where X is the "level of activity" in production. We wish to construct a communication system that will make X "appropriately" responsive to O and I, e.g., that will keep I close to zero.

Consider the system:

$$X = \alpha O + \beta I, \quad \alpha > 0, \qquad \beta < 0.$$

Then

$$\frac{dI}{dt} = k(\alpha O + \beta I) - O = \beta kI + (\alpha k - 1) O$$

$$\frac{d^2I}{dt^2} = \beta k \frac{dI}{dt} + (\alpha k - 1) \frac{dO}{dt} \quad .$$

We may evaluate the characteristics of this communication system by servomechanism methods. Assume, for example, that $\frac{dO}{dt} = 0$.

Then $I = A\,1^{\alpha t}$, where

$$-\beta k < 0 .$$

Hence, the rapidity of response of the system is proportional to βk.