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Jacob Marschak: THE RATIONALE OF THE DEMAND FOR MONEY AND "MONEY ILLUSION."

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DEFINITIONS, ASSUMPTIONS AND RESULTS

PLAN of the a-th Person ($a=1,\ldots,A$) for the n-th Good ($n=1,\ldots,N$). (Subscripts a, n omitted.) Time unit = length of interval between two successive marketing dates. Planning date, $t=0$. Horizon (= latest marketing) date, $t=T$.

Restrictions on Signs	Stocks Brought	Stocks Retained	Stocks Sold	At Price		Consumed
	≥ 0, $< \infty$	≥ 0, $< \infty$	$> -\infty$, $< \infty$	> 0, $< \infty$		≥ 0 $< \infty$
Date of Marketing					**Period of Consumption**	
0	$\bar{y}$	y^0	$z^0 = \bar{y} - y^0$	p^0	$(0,1)$	x^0
1	$y^0 - x^0$	y^1	$z^1 = y^0 - x^0 - y^1$	p^1	$(1,2)$	x^1
...	..	..	....	..	...	..
t	$y^{t-1} - x^{t-1}$	y^t	$z^t = y^t - x^{t-1} - y^t$	p^t	$(t,t+1)$	x^t
...	..	..	.....	..	...	..
T-1	$y^{T-2} - x^{T-2}$	y^{T-1}	$z^{T-1} = y^{T-2} - x^{T-2} - y^{T-1}$	p^{T-1}	$(T-1,T)$	x^{T-1}
T	$y^{T-1} - x^{T-1}$	y^T	$z^T = y^{T-1} - x^{T-1} - y^T$	p^T	—	—

Find values for sets $\left\{y_{na}^{T}\right\}$, $\left\{y_{na}^{t}\right\}$, $\left\{x_{na}^{t}\right\}$, $n=1,\ldots,N$, $t=0,\ldots,T-1$, that would maximize the

utility $u_{(a)}\left(\left\{x_{na}^{t}\right\}, \quad \left\{y_{na}^{T}\right\}\right)$, subject to above restrictions on signs and to

(1) budget restrictions $\sum_n z_{na}^t p_{na}^t = 0$, and

(2) market imperfection $p_{na}^t = f_{na}^t\,(z_{na}^t)$, $n=1,\ldots,N$; $t=0,\ldots,T$, where the N-th good is numéraire and legal tender: $p_{Na}^t = f_{Na}^t = 1$ for all a, t.

The givens are: initial stocks $\left\{\bar{y}_{na}\right\}$; functions $u_{(a)}$, $\left\{f_{na}^{t}\right\}$.

The set $\left\{f_{na}^t\right\}$ is restricted by "clear-the-market" condition:

(3) $\sum_a z_{na}^o = 0$ for all n.

Marginal utilities: $u_{na}^T = \partial u_{(a)}/\partial y_n^T$; $u_{na}^t = \partial u_{(a)}/\partial x_n^t$, $t = 0,\dots,T-1$; all non-negative

Illiquidities: $g_{na}^t = df_{na}^t/dz_{na}^t$, $t = 0,\dots,T$; all non-positive.

Special, mutually independent, assumptions (omitting subscript a).

(I) Static case: $T=0$; or $T=1$, $u_n^T = 0$, implying $x_n^o - y_n^o - y^1 = 0$ for all n.

(II) Paper money: $u_N^t = 0$, $t=0,\dots,T$, implying $y_N^T = 0 = x_N^t$, $t=0,\dots,T-1$.

(III) Perfect markets: $g_n^t = 0$ (perfect liquidity) for all n, t.

Method. Express sign restrictions (see PLAN) by equations

(4) $0 = x_n^t - (r_n^t)^2 = y_n^T - (r_n^T)^2 = y_n^t - x_n^t - (s_n^t)^2$, $t=0,\dots,T-1$,

where r_n^t, r_n^T, s_n^t are real. The λ, μ, ν are Lagrange multipliers.

General result (non-static, imperfect markets):

(5) $u_n^t + \mu_n^t = \lambda^t (p_n^t + z_n^t g_n^t) = \lambda^{t+1} (p^{t+1} + z_n^{t+1} g_n^{t+1}) + \nu_n^t$,

(6) $\mu_n^t r_n^t = \nu_n^t s_n^t = 0 = \nu_n^T = \lambda_n^{T+1}$; $t=0,\dots,T$.

Result for Special Cases:

(I).(II) Static, perfect: $p_n^o = u_n^o/u_n^o$; $x_N^o \geq 0$ ("classical case").

(I).(II).(III) Static, perfect, with paper money: $x_N^o = 0 = y_N^o$ ("money of account").

(III) Non-static, perfect: If prices change ($p_n^{t+1} \neq p_n^t$ for all $n \neq N$) then all stocks but one are "unloaded", down to consumption needs of next period.

(III).(II) Non-static, perfect, with paper money: If prices change then money stock is either zero or absorbs all resources except those needed for current consumption.

Money Illusion. Solve (1), (2), (4)-(6) for all supplies (demands) of the a-th Person, z_{na}^t. In case of perfect markets (III) each z_n^t is a function of all "absolute" prices -- p_n^o, $p_{na}^1,\dots,p_{na}^T$, $n = 1,\dots,N-1$; not of the "relative" prices -- p_n^o/p_1^o, $p_{na}^1/p_{1a}^1,\dots,p_{na}^T/p_{na}^T$, $n = 2,\dots,N-1$. (With imperfect markets, the givens are $\left\{f_{na}^t\right\}$, not $\left\{p_{na}^t\right\}$; but the statement remains true.)

Generalizations not yet discussed: Uncertainty; Production; Borrowing.