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Invention and Cost Reduction in Technological Change

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This paper is a preliminary report on the application of an economic model with linear production functions (requiring fixed proportions of the factors of production) to problems of technological change. The first part is a discussion of some definitional difficulties, the second a description of the model, and the remaining parts are concerned with the application of the model.

The paper will perhaps raise more questions than it answers. Nevertheless, even a preliminary study of models of the sort described here indicates that such models mirror many of the interesting processes in the real world that are involved in technological change. In particular, these models permit the study of possible "trigger" effects of technological change--for example, a sudden replacement of an old process by a new, or a very large increase in national income resulting from a reduction in cost of a commodity of only moderate importance. Such "trigger" effects are obscured in models whose production functions permit substitution of one factor for another without process change, and, moreover, it is almost impossible to study effects in the large in models of the latter sort without strong assumptions about the form of the production functions.

I. Definitions of Technological Change

At least three different meanings for the term technological change can be found in the literature of economics.

These are:

(1) a shift in a production function--where the production function is an expression relating the quantity of output of a commodity to quantities of input of various factors of production;

(2) a shift in a cost curve--where the cost curve is an expression that relates quantity of output of a commodity to the minimum cost (using optimal combinations of the factors) of producing that quantity;

(3) a change in a production process--e.g., the substitution of machine for hand methods in making cigars.

These three concepts are by no means equivalent. Only the first is purely technological--i.e., independent of relative prices. It relates to the discovery of new production possibilities that permit a given output to be produced with a smaller quantity of one or more of the factors of production. This type of technological change will be called innovation.

The second type of technological change--a shift in the cost curve--may result from an innovation, or it may be due to a lowering in the price of one of the factors of production. The cost curve corresponds only to that portion of the production function which represents optimal combinations of the factors of production for any given output at given prices. Hence, through price changes, the cost curve may change (and different combinations of factors may--but need not--become optimal) without any change in the production function. Conversely, a portion of the production function not representing an optimal combination at given prices may be altered without change in the cost curve. Technological change in the second sense will be called cost reduction.

It may be objected that cost reduction has nothing to do with technological change, unless it results from innovation. On the contrary, innovation in one industry (say, electric power production) may reduce cost in another industry (say, aluminum reduction), bringing about a secondary effect in the second industry. The subject of technological change should certainly encompass indirect effects of this sort, and writers on the history of technology have in fact included such effects. This brings us to the third aspect of technological change--process change.

The third type of technological change--the introduction of new processes--is essentially a qualitative concept, because what is to be considered "the same" and "different" processes is not a precise matter. For example, if wheat may be grown either with little or with much fertilization, these may be considered two processes, or simply two different combinations of factors for producing with the same process. Corresponding to each process will be a distinct production function, but we can always combine these into a single production function by selecting for each combination of factors that process which gives the greatest output. Nevertheless, in the discussion of technological change, it is often convenient to deal with the production functions for the individual processes rather than with the composite production function for the commodity. Technological change in the third sense, then, is any change which makes a new process function optimal, in place of the one that has previously been optimal. This will be referred to as a <u>process change</u>.

The reasons for making these distinctions may be clarified by a few hypothetical examples.

<u>Example 1</u>. Suppose there are two processes for mining coal, which we may call "hand" and "machine" processes. With some combinations of factors the hand method may yield the greater output, with some other combinations, the machine method. If there is a sufficient reduction in the price of a factor that is more important in machine than in hand production (e.g., capital), this will result in a cost reduction, and may result in a process change. There would, however, be no innovation involved. A great deal of the technological change occuring in the industrialization of a primitive economy is of this character, for it results primarily from the shift in relative price of labor and capital.

<u>Example 2</u>. Suppose that at a time when the hand process is employed, a small innovation is made in the machine process--not sufficient to cause a process change. There would then be no cost reduction. However, this innovation might advance the date at which a later process change, resulting from change in relative price, would take place.

<u>Example 3</u>. If the innovation in Example 2 took place <u>after</u> the shift in relative prices had brought about the process change, it would result in an immediate cost reduction.

<u>Example 4</u>. If the innovation in Example 2 took place before the process change, but was of sufficient magnitude to bring about the process change without any change in relative prices, then there would also be an immediate cost reduction.

These examples afford no particular difficulty with out present terminology, but it can be seen that ambiguities result if the term "technological change" is applied indiscriminately to all three phenomena.

There are some further ambiguities in the concept of innovation that deserve at least brief mention. First, when there is knowledge of a process, say, in the United States, but the knowledge needed to introduce this process is lacking, say, in China, shall we include this process in the Chinese production function, or exclude it? If the latter, the importation of persons into China capable of employing the process is an innovation; if the former, it is an increase in the supply of a factor of production, i.e., a scarce skill.

Second, process change may be a motivating factor in invention. If machine methods become economical, or nearly economical, where they have not been before, their new practical importance may lead to rapid improvement in them. Is this improvement to be treated as genuine innovation, or is the production function for an uneconomical process (uneconomical because of relative prices) to include an allowance for the improvements that will certainly take place in it once this process is considered to be within the realm of practicability?

It will probably depend upon the problem at hand as to which way of resolving these two ambiguities will be most convenient, and this difficulty will not be considered further here.

II. Linear Models for the Study of Technological Change

In view of the distinctions made above, it is useful to have models for the study of technological change in which innovations, cost reductions, and process changes can be considered separately. So long as the production function of each commodity is assumed to be homogeneous of the first degree in the factors of production, taken as a whole, the process functions in such a model can be assumed to be linear and to require fixed combinations of factors, without loss of generality.

As has been pointed out by Koopmans (Cowles Commission Discussion Papers, Economics, No. 215 (February 10, 1948), page 3):

> ". . . within one process, there are no diseconomies of scale, because what one firm can do can be multiplied by any integer by having more firms of the same kind operating. Decreasing returns for the economy as a whole are seen to arise from basic limitations on the amounts of primary factors available to the economy."

Two cases may be distinguished:

A. Where the factors are continuously substitutable for one another, the production function is the infinite pencil of lines through the origin representing the infinite family of processes. The dimensionality of this pencil in the factor and product space is equal to the number of factors.

B. Where the factors are not continuously substitutable for one another, the production function consists of the axes for the factors, the lines through the origin representing the (finite number of) production processes, and the outermost planes determined by these lines. This surface will have the same dimensionality as in case A, but will be made up of a series of facets having discontinuous derivatives at their boundaries.

It is clear that in either case the production function is a convex figure, for if it contains any two lines through the origin (processes), it also lies above or on the plane determined by them (attainable by averaging the two processes).

Systems with linear production functions have been studied by Wald, von Neumann, Leontieff, Koopmans, and others. A system sufficiently general for the study of many problems of technological change is the following:

Let λ represent an index of real income; z_i the quantity of the i^{th} commodity directly consumed; x_i the total quantity produced of the i^{th} commodity; ${}_i\alpha_\rho$ the percentage of the total output of the i^{th} commodity produced by the ρ^{th} production process; ${}_\rho x_{ji}$ the quantity of the j^{th} commodity used in the production of the i^{th} commodity by the ρ^{th} process; ${}_\rho y_{ki}$ the quantity of the k^{th} factor of production used in the production of the i^{th} commodity by the ρ^{th} process; y_k the total quantity of the k^{th} factor used in producing commodities; and $\bar{y}_k$ the maximum quantity of the k^{th} factor available for the period. All constants are required to be non-negative.

$$(1)\quad \lambda = a_{0i}\, z_i \qquad (i = 1, \ldots, m)$$

$$(2)\quad x_i = \sum_{j=1}^{m} \sum_{\rho=1}^{r_i} {}_\rho x_{ij} + z_i \qquad (i = 1, \ldots, m)$$

$$(3)\quad {}_i\alpha_\rho\, x_i = {}_\rho a_{ji}\; {}_\rho x_{ji} = {}_\rho b_{ki}\; {}_\rho y_{ki} \qquad (i, j = 1, \ldots, m;\ k = 1, \ldots, m;\ \rho = 1, \ldots, r_i;\ {}_\rho a_{ii} = 0)$$

$$(4)\quad \sum_{\rho=1}^{r_i} {}_i\alpha_\rho = 1 \qquad (i = 1, \ldots, m)$$

$$(5)\quad \bar{y}_k \geq y_k = \sum_{i=1}^{m} \sum_{\rho=1}^{r_i} {}_\rho y_{ki} \qquad (k = 1, \ldots, m)$$

Equations (1) are the consumption functions, equations (3) the production functions. The system does not permit price substitution of commodities in consumption, but it does in production (by variation of the α's, i.e., by process changes).

Equations (3) may be used to eliminate the $\{{}_{\rho}x_{ij}\}$ in (2) and the $\{{}_{\rho}y_{ki}\}$ in (5), giving:

$$x_i = z_i + \sum_{j=1}^{m}\sum_{\rho=1}^{r_j} \frac{{}_{i}\alpha_{\rho}\, x_j}{{}_{\rho}a_{ji}} \qquad (i = 1, \ldots, m) \tag{2a}$$

$$\bar{y}_k \geq y_k = \sum_{i=1}^{m}\sum_{\rho=1}^{r_i} \frac{{}_{i}\alpha_{\rho}\, x_i}{{}_{\rho}b_{ik}} \qquad (k = 1, \ldots, n) \tag{5a}$$

For convenience, we introduce new symbols $\{B_{ij}\}$, all non-negative functions of the a's and α's, and rewrite (2a):

$$x_i = z_i + \sum_{\substack{j=1 \\ j \neq i}}^{m} B_{ij}\, x_j \qquad (i = 1, \ldots, m) \tag{2b}$$

We call the system (2b) **admissible** for some set $\{z_i\}$ (all $z_i > 0$) if the system possesses a solution $\{x_i\}$ such that all $x_i \geq 0$. If such a solution exists, it can immediately be induced from (2b) that we must have $x_i > 0$ for all x_i. Since the B_{ij} depend upon the α's, we note that the system may be admissible for certain sets $\{\alpha_\rho\}$ and not for others. For the moment we assume $\{{}_{i}\alpha_{\rho}\}$ to be given.

A. Theorems on Admissibility

We define $B_{ii} = -1$, $D = |-B_{ij}|$. I conjecture (although I do not have a proof for $n > 4$) that a necessary and sufficient condition that (2b) be admissible for all $\{z_i\}$ is that D and its principal minors be all positive. This will always be so if the non-diagonal elements of $\|-B_{ij}\|$ are sufficiently small. The economic meaning of this condition is this: If k units of commodity (A) are needed to produce one unit of commodity (B), then less than one unit of commodity (B) must be required to produce k units of commodity (A)--otherwise the production of these two commodities would be impossible. Analogous conditions must hold for triads of commodities, and so forth.

It has already been pointed out that a system may be admissible for certain values (which we will call admissible values) of the parameters $\{_i\alpha_\rho\}$ and not for other values.

B. Existence of Solutions Satisfying (5a)

If (2b) is admissible, we may write:

(2c) $$x_i = \lambda\, \phi_i\left[\{a_{ij}\}, \{t_j\}\right] \qquad (i=1,\ldots,n)$$

from whence,

(5c) $$\bar{y}_k \geq y_k = \lambda\, \psi_k\left[\{a_{oj}\}, \{_\rho a_{ij}\}, \{_i\alpha_\rho\}\right] \qquad (k=1,\ldots,m)$$

We can always find λ's sufficiently small that the inequalities in (5c) will be satisfied. Let $\bar{\lambda}$ be the least upper bound of such λs. Then $\bar{\lambda}$ will be a function of $\{\psi_i\}$, and hence of $\{_i\alpha_\rho\}$. Consider now the set of $\{\bar{\lambda}\}$ corresponding to all admissible sets $\{_i\alpha_\rho\}$. The upper bound of this set we will designate an optimum $\bar{\lambda}$, and the solution of (1)-(5) corresponding to this $\bar{\lambda}$, an optimum solution. We conclude that if (2b) is admissible for at least one set $\{_i\alpha_\rho\}$, then an optimum solution of (5c) exists.

C. Scarce Factors and Economical Processes

Suppose that we have obtained an optimum solution, and consider the system derived from (1)-(5) by eliminating all elements of (3) for which $_i\alpha_\rho = 0$, and all elements of (5) for which $\bar{y}_k > y_k$. We will call this the reduced system. The equations (3) in this system correspond to economical processes, and the variables y_k in equations (5) to scarce factors.

Let T be the number of parameters $_i\alpha_\rho$, and M the number of factors $\bar{y}_k$ in the reduced system. Then, in order for this system to have a unique solution, we must have in general (that is, for almost all sets $\{\bar{y}_k\}$)

(6) $$(n + M) = (T + 1)$$

That is, the number of products plus the number of scarce factors must exceed by one the number of economical production processes. In particular, if $T = n$ (i.e., a unique process for each product), then $M = 1$ (i.e., there can exist only one scarce factor).

But in the case of $T = n$, our model is substantially identical with that of Leontieff (The Structure of American Economy, 1919-1929). Hence Leontief's model is self-consistent only if there is a single scarce factor of production. If a model is to represent a world in which there are several scarce factors, it must admit a sufficient number of alternative economical processes of production.

III. Some Applications of the Model

A. Diminishing Returns

From (5c) we deduce that an increase in the supply of a scarce factor (say, the r^{th}) will produce a less than proportionate increase in $\tilde{\lambda}$. For if the increase were proportionate, or greater, we would have:

$$(7) \qquad d\log\lambda \geq d\log y_k \qquad (k = 1, \ldots, M)$$

Hence,

$$(8) \qquad d\log\psi_k = d\log y_k - d\log\lambda \leq 0,$$ (with the strong inequality holding at least for the r^{th} equation).

But this contradicts the assumption that the ψ_k give an optimum solution. (Koopmans has proved the equivalent theorem for his model.) This can be generalized to show diminishing returns for a proportionate increase in any proper subset of the scarce factors.

B. A One-Commodity Model

We may specialize (1)-(5) to the case of one commodity, x_1 produced by $\Upsilon_1 = T$ processes. Then we have: $M = T$, i.e., the number of economical processes will (in general) equal the number of scarce factors. For the further specialization $M = 2$, the entire system can be represented graphically. In Figure 1, y_1 and y_2 are quantities of (not necessarily scarce) factors. AA' is a contour on the production surface for $x_1 = \lambda$. The production processes B, C, D, E employ y_1 and y_2 in varying proportions. All but C are economical for a particular range of factor prices. If the ratio of available y_1 to available y_2 (i.e., $\bar{y}_1/\bar{y}_2$) is as shown by the line Y, then D and E will both be economical processes, B will not. The ratio of factor prices will be given by the slope of the segment of AA' joining D to E.

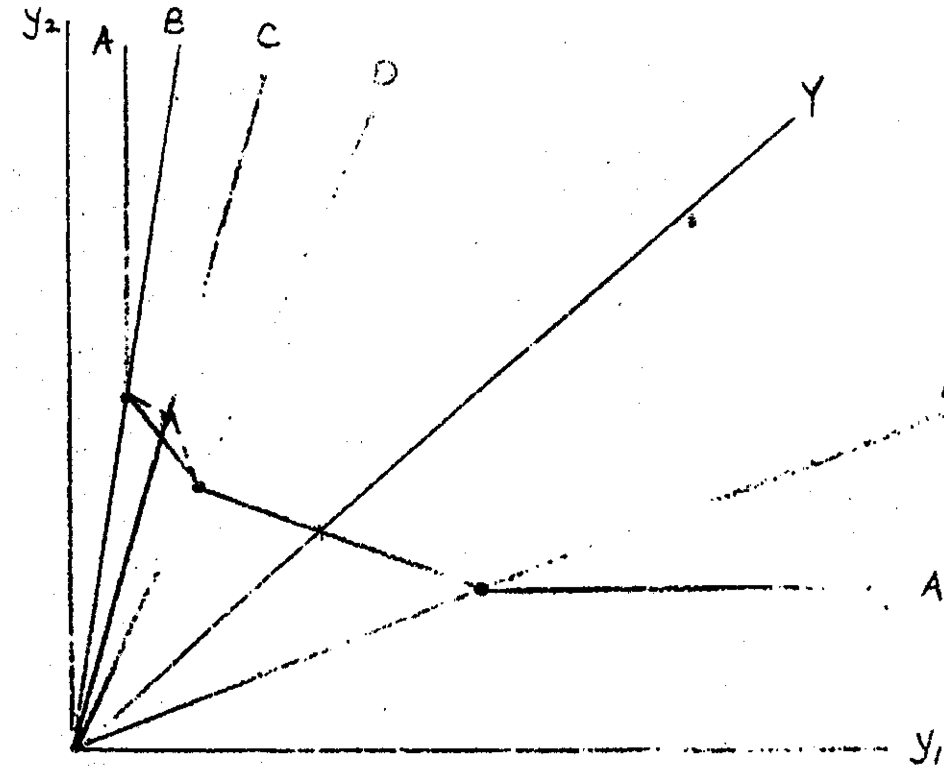

Figure 1

From the figure, we can immediately read off a number of conclusions for this model. These conclusions are easily generalized to the case of one commodity and many factors.

a. With the factor supply as shown by Y, a greater product will result from the combined use of processes D and E, in suitable proportions, than from either alone.

b. The points of intersection of AA' with the process functions BCD- depend upon the technical coefficients in these functions. Any change in the coefficients can be represented by the combination of a movement of the line representing the function through some arc (change in the proportions of the factors) combined with a movement of the point of intersection along the line. Process changes can come about either through changes in relative factor scarcity (movement of Y) or changes in the technical coefficients (movements of the process functions and their intersections with AA').

c. We call a process <u>feasible</u> if it is economical for some factor ratio. (B, D, E are feasible, C is not). A sufficient downward movement of the intersection of the contour AA' with C will make C feasible. A somewhat greater movement will make D infeasible (in which case C and E would be the economical processes), and a still greater movement will make B infeasible.

d. If Y were to move sufficiently to the left, B and D would replace D and E as the economical processes. The same would occur if, instead, D moved sufficiently to the right.

e. Of the feasible processes, those tend to be economical which employ the factors in ratios closest to the actual ratio of factor availability. This offers a possible explanation for the great stability of the ratio C/L noted by Douglas in his inter-industry studies of the production function.

f. If Y moves sufficiently to the right (or E sufficiently to the left), E will be the only economical process, and y_1 will become a free factor.

g. If a new process becomes feasible it either: (1) has no immediate effect (if it lies to the right of E or to the left of D); (2) displaces one or both processes previously used (if it lies between D and E); or (3) makes scarce a factor previously free (If Y lies to the right of E, and the new process to the right of Y).

h. Suppose only the process B were known. Then y_1 would be free. Suppose now the feasible process D is invented. Then D will replace B, and y_1 will remain free. Suppose, however, that E were invented instead of D. Then B would not be displaced, but both B and E would be economical and y_1 would become scarce. We have the somewhat paradoxical situation that the less "revolutionary" invention completely displaces the old process, while the more "revolutionary" invention only partially displaces it.

These examples will perhaps serve to illustrate some of the characteristic properties of our model.

C. <u>Prices and Cost Curves</u>

Thus far we have not taken explicit account of prices. Accounting prices can be introduced into the reduced system by the equations:

$$(9) \quad p_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{p_j}{\rho a_{ij}} + \sum_{k=1}^{m} \frac{p'_k}{\rho b_{ik}} \qquad (\rho = 1, \ldots, T;\ i = 1, \ldots, n)$$

There will be T such equations, and $(n + M) = (T - 1)$ unknown prices, p_i. Hence, since the equations are homogeneous in the p's they will in general have unique solutions up to a factor of proportionality (numeraire).

It will be noticed that the supplies of the factors affect prices only indirectly, that is, by determining which factors are scarce and which processes are economical. Given the scarce factors and economical processes, prices do not vary with small changes in the supply of factors. Changes in factor supply, within these limits, affect only the intensities of the various processes.

There is an alternative, but equivalent, graphic representation, using prices, of the model with one commodity and two factors. The price of y_1, p'_1, is taken as numeraire. In Figure 2, let D represent the cost curve for x (as a function of p'_2) with process D, and E the cost curve with process E. The composite cost curve is given by the heavy portions of the two curves. If D and E are both economical, we must have $p'_2 = a$, $p_x = b$.

The cost curves D', D", E', E" correspond to various types of technological improvements in the processes D and E. Such improvements, if both D and E remained economical, would bring about changes in the equilibrium prices (the intersection of the two cost curves) that can be read off the figure.

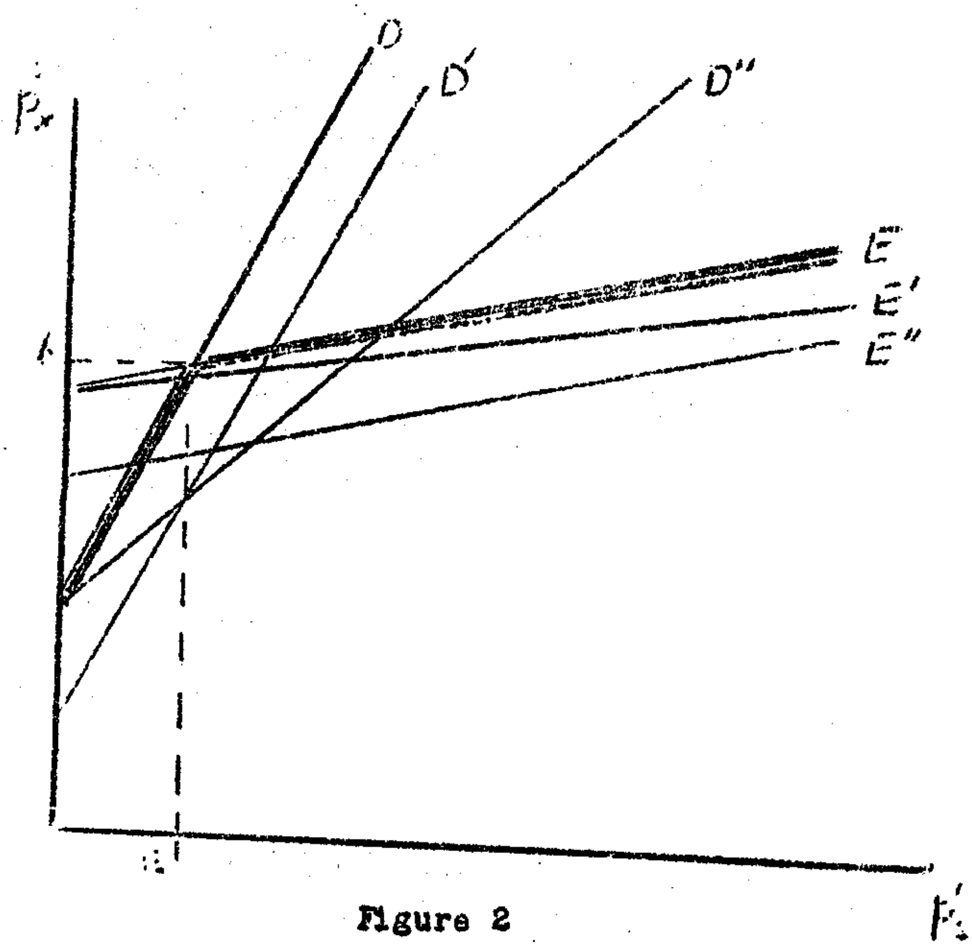

Figure 2

We see, for example, that the technological improvements represented by D' and D"" will increase the prices of x and y_2, while the improvements represented by E' and E" will decrease those prices.

E. <u>Income Effects of Technological Change</u>

Thus far, the model presented here has proved far less tractable in yielding propositions about the income effects of technological change than in yielding propositions about the economy of production processes. One possible line of attack would be to attempt to extend the correspondence between process changes and changes in relative factor scarcity that was shown in Figure 1. This might lead to a theorem about "diminishing returns of technological progress" comparable to the theorem regarding diminishing returns with derive such a theorem.

Another line of approach would be to construct special cases to demonstrate that very large "trigger" effects of technological change are possible in such models. By consideration of the conditions that have to be imposed upon the model to produce such trigger effects, more general theorems might be suggested or derived.

Still a third approach would be to study technological changes of "moderate" size in the reduced system. This is, in fact, what Leontief has done, for the special case of a system with only one scarce factor.

A fourth approach would be to study the limiting case where a technological change is just large enough to alter the set of economical processes and scarce factors.

Models of the sort described here have already shown their value in the study of linear programming. It would seem that they have equal potentialities for the study of the effects of technological change in the large.