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A letter from L. Klein dated June 9, 1948, the Hague, includes the comment, "It seems that in maximizing a profit integral under imperfect competition, it is not possible to avoid the occurrence of derivatives under the integral in an essential way so that the calculus of variation methods must be used." The insert which he suggests for page 13 of chapter II of the manuscript "Economic Fluctuations in the U. S." is given below:

When the possibility of making capital gains from inventory speculation is properly taken into account, however, the generalization to conditions of imperfect competition has a more essential influence on the equations of the system. If the relation between anticipated prices and output is given by the formula in (2.1.15), the logical interpretation of the formula for $an\mathring{p}$ (associated with capital gains) would be

$$(2.1.18) \quad an\mathring{p} = \frac{\partial p}{\partial x} \mathring{x} \quad .$$

From the production function (2.1.1), $\dot{x}$ is given by

$$\mathring{x} = \frac{\partial f}{\partial n} \mathring{n} + \frac{\partial f}{\partial d} \mathring{d} + \frac{\partial f}{\partial t} \quad .$$

The maximization of profits (2.1.3) subject to (2.1.1), (2.1.15), and (2.1.18) is a true problem in the calculus of variations because both derivatives and levels of variables with respect to which profits are to be maximized enter the profit integral. The only derivative under the integral sign for perfect competition is $an\mathring{p}$, which is taken as given in the maximization process for the single firm.

The well-known Euler equations for the maximum of an integral in the calculus of variations take the form

$$(2.1.19) \qquad \mathrm{anp}\,\frac{\partial x}{\partial n}\left(1-\frac{1}{\eta}\right)-\mathrm{an}\;w-\frac{\partial p}{\partial x}\,\frac{\partial x}{\partial n}\,(\dot{h}-\rho h)=0\,,$$

$$(2.1.20) \qquad \mathrm{an}\;p\,\frac{\partial x}{\partial d}\left(1-\frac{1}{\eta}\right)-\mathrm{an}\;q-\frac{\partial p}{\partial x}\,\frac{\partial x}{\partial d}\,(\dot{h}-\rho h)=0,$$

$$(2.1.6) \qquad \frac{\partial\left(\,\varepsilon\,(h,\,u_3)\,\right)}{\partial h}=\mathrm{an}\;\dot{p}\,,$$

for the problem of maximizing (2.1.3) subject to (2.1.1), (2.1.15) and (2.1.18). The first-order condition with respect to inventory variation remains the same, but an essentially new term appears in the first-order conditions associated with labor and capital variation. The size of the new term depends on the divergence between the change in inventories and a percentage of the stock of inventories — the percentage being the discount rate. The more specific form of the production function (2.1.1a) and the elasticity transformation enable us to write (2.1.19) and (2.1.20) in the forms

$$(2.1.19a) \qquad \mathrm{an}\;wn=\alpha_1\left(1-\frac{1}{\eta}\right)\;\mathrm{an}\;px+\frac{\alpha_1}{\eta}\;\mathrm{an}\;p\,(\dot{h}-\rho h),$$

$$(2.1.20a) \qquad d=\alpha_2\left(1-\frac{1}{\eta}\right)\mathrm{an}\;\frac{px}{q}+\frac{\alpha_2}{\eta}\;\mathrm{an}\;\frac{p}{q}\,(\dot{h}-\rho h).$$

These equations can be written in terms of observed aggregates[1] by an application of preceding methods on the empirical work; equations of the form (2.1.19a) and (2.1.20a) with the extra terms on the right hand side have not yet been used. The main obstacle for empirical work here is to find satisfactory data on the size of the discount rate, $\rho$.

1. Observed values of $\rho$ are not easy to determine, however.

While equation (2.1.6) remains in the same form as in the perfectly competitive theory, the form to be used in statistical calculation is charged because the function used to convert an $\dot{p}$ into observed quantities is different. It is given by (2.1.18) instead of (2.1.10). In aggregative discrete variables a linear approximation will now be

$$(2.1.6C^*) \quad H = \eta_0^* + \eta_1^* p_{-1} + \eta_2^* p_{-2} + \eta_3^* X + \eta_4^* X_{-1} + u_9^* ,$$

which can be considered as an alternative to (2.1.6C).