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CAPITAL ACCUMULATION AND THE END OF PROSPERITY

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I. THE PROBLEM. Prosperity is characterized by rapid accumulation of capital. Does capital accumulation as such bring prosperity to an end?

This question has appeared in economic literature a number of times, particularly in the works of Marx, Hobson, Keynes, Hansen, Kalecki, Kaldor and others.

II. SIMPLIFIED KEYNES SYSTEM.

(1.1) $Y = C + S$ (1.4) $I = S$

(1.2) $S = F(Y)$ (1.5) $Y = P$,

(1.3) $I = g(Y, E)$

where $\underline{Y}$: national income; $\underline{C}$: consumption; $\underline{S}$: saving; $\underline{I}$: investment; $\underline{P}$: productive capacity of the economy, in this case a constant; all understood in net sense per unit of time. $\underline{E}$: exogenous causes.

The system is over-determined and, in the general case, breaks down because of the contradiction between (1.1)-(1.4) and (1.5). It is a static system, and capital accumulation is not expressed explicitly, though referred to a number of times in the General Theory.

III. HOBSON SYSTEM, also found in Marx.

There are two systems here.

1. For $\underline{K \leq K_n}$, where $\underline{K}$ is capital stock, the Keynes system is repeated but with equation (1.3) omitted. Equilibrium is maintained.

2. For $\underline{K > K_n}$, (1.3) is brought back in the form of $\underline{I} = \underline{a}$, where $\underline{a}$ is some small constant. Now the equilibrium is destroyed. But the nature of $\underline{K_n}$ the "critical" value of capital, is rather vague.

The essence of the Hobson-Marx system lies (1) in the assumption that equilibrium is maintained for a period of time, and (2) in the demonstration that the resulting accumulation of capital produces the breakdown.

IV. COMBINED SYSTEM.

<u>A. Equilibrium conditions</u>. The system is based on the dual nature of investment, namely, that it not only generates income, but also increases productive capacity. (See my papers in <u>Econometrica</u>, April 1946, and <u>American Economic Review</u>, March 1947.) A somewhat similar approach has been used by M. Kalecki.

(2.1) $Y = C + S$

(2.2) $S = F(Y)$

(2.3) $\frac{dP}{dt} = f(I, s)$

(2.4) $I = S$

(2.5) $\frac{dY}{dt} = \frac{dP}{dt}$

where $Y_0 = P_0$, t is time, and s the constant ratio of potential output (per unit of time) to capital, required by existing techniques of production. The equation (2.3) is a substitute for a production function.

Expressing for simplicity (2.2) as

(2.6) $S = \beta + \alpha Y, \quad (\beta \leq 0, \ 0 < \alpha < 1)$

and (2.3) as

(2.7) $\frac{dP}{dt} = Is,$

we derive the solution

(2.8) $I = I_0 e^{\alpha st},$

which also yields

(2.9) $\frac{\frac{dP}{dt}}{P} = \frac{\frac{dI}{dt}}{I - \beta}$

<u>B. Possible breakdowns.</u>

1. In reality, I may be expressed by a function such as

(2.10) $I = H(Y, K, E)$

which (when $\underline{Y}$, $\underline{K}$ and $\underline{E}$ are expressed as functions of $\underline{t}$) need not be identical with (2.8). This is the Keynesian case considered above. We should add that whenever $\dfrac{\frac{dI}{dt}}{I} < \alpha s$, an overaccumulation of capital takes place in the sense that $\dfrac{\frac{dY}{dt}}{\frac{dK}{dt}} < s$. This is supposed to inhibit future investment.

2. Equation (2.3) and also (2.9) may be subject to the restriction that

$$(2.11) \qquad \frac{dP}{dt} * \text{ Is only for } \frac{\frac{dP}{dt}}{P} \ll r,$$

where $\underline{r}$ is the maximum rate of growth the economy can achieve. If due to this restriction $\dfrac{\frac{dI}{dt}}{I} < \alpha S$, an over-accumulation of capital again takes place. But unlike the preceding case it is due to physical factors: shortage of labor and insufficiently rapid technological progress.

Assuming that the breakdown is not due to the first cause as expressed by (2.10), the question arises whether the restriction given in (2.11) is actually likely to bring prosperity to an end.

C. Possible objections

1. Rejection of the whole approach on the grounds that $\underline{s}$ is not a meaningful concept. A slowly declining rate of profit resulting from

$$\frac{\frac{d^2Y}{dt^2}}{\frac{d^2K}{dt^2}} < 0$$ will hardly have cyclical significance.

2. If exogenous factors play a predominant role in the investment function, past capital accumulation is of little importance.

3. The effects of excessive capital accumulation and capital losses on investment depend on the distribution of these losses among firms and industries. New investment need not necessarily be deterred.

4. Most important is the fact that we have hardly any empirical proof that the restriction (2.11) has been effective, particularly over a relatively short period of time, such as five-seven years. The presence of unemployment in the past is not a sufficient proof that $\frac{\frac{dI}{dt}}{1-\beta} > r$.

D. Possible solutions.

1. If $\frac{\frac{dI}{dt}}{1-\beta} > r$, three measures can be suggested:

a. Reduction of α;

b. Increase of $|\beta|$;

c. Reduction of s, by developing industries requiring large capital outlays per unit of output.

d. Inflation.

2. If, on the other hand, $\frac{\frac{dI}{dt}}{1-\beta} < r$, a full employment equilibrium can be maintained by a positive guarantee that Y will grow at the relative rate of $\frac{\frac{dI}{dt}}{1-\beta}$. This is an extremely interesting case.

An empirical study of the magnitudes of α, β, s and r is highly desirable.