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STRUCTURAL EQUATIONS FOR THE INVESTIGATION OF THE RELATION OF CORN YIELD TO WEATHER

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Introduction:

In Cooper's model of the agricultural sector of the economy, the production function involves a variable labeled weather. In the course of investigation of what functional(s) of the various weather variables should be used as weather in that function, it became apparent that we must construct in agriculture, as in economics, a system of structural equations to explain the phenomena. This paper concerns itself with the yield per acre of corn in a homogeneous plot.

The variables involved:

Endogenous

H: Effective time of harvest (the time at which growth ceases).

D(t): Development of crop at time t.

G(t): Growth of crop at time t.

C: Size of crop harvested.

f(t): Fraction of crop fertilized at time t.

k(t): Fraction of crop sterilized at time t.

W(t): Effective ground water level at time t.

M(t): Mineral content of the soil at time t.

Exogenous

T(t): Temperature at time t (degrees Fahrenheit).

R(t): Rainfall at time t.

L(t): Length of day at time t.

$u(t)$: Wind speed at time t.

$e_s(t)$: Saturation pressure of water vapor at time t.

$e(t)$: Pressure of atmospheric water vapor at time t.

$E(t)$: Rate of evaporation at time t (Approximately proportional to $u(e_s - e)$).

Time is measured from planting time.

<u>The equations of the system:</u>

(1) $$D(H) = 1$$

This equation determines the scale of D.

(2) $$C = G(H)f(H).$$

(3) $$\dot{G} = \phi_3(W, G, D, T, L, M).$$

suggested form:

$$\dot{G} = \phi_{31}(L)\phi_{32}(T)\psi_3(W, G, D, M)$$

where

$$\phi_{11}(L) = a_{31} + a_{32}L.$$

$$\phi_{12}(T) \sim 2^{\frac{T}{18}}$$ for moderate T (van't Hoff's law).

(But it has a maximum.)

(4) $$\dot{D} = \phi_4(D, T, \dot{G}, L)$$

Suggested form

$$\dot{D} = \phi_{41}(L)\phi_{42}(T)\,\psi_4(D, \dot{G})$$

where

$$\phi_{41}(L) = a_{41} + a_{42}L.$$

$$\phi_{42}(T) \sim 2^{\frac{T}{18}}$$ for moderate T.

If we assume

$$\psi_4(D, \dot{G}) = \psi_{43}(D)\,\psi_{44}(\dot{G}),$$

we may define

$$D^* = \frac{\int_0^D \frac{dx}{\psi_{43}(x)}}{\int_0^1 \frac{dx}{\psi_{43}(x)}}$$

Then D* satisfies (1). However, if D* is used instead of D, ψ_{43} becomes a constant. A suggested form for ψ_{44} is

$$\psi_{44}(\dot{G}) = a_{43} + a_{44}\dot{G} + a_{45}\dot{G}^2$$

(5) $\dot{M} = \phi_{51}(M, R) + \phi_{52}(\dot{G}, D).$

ϕ_{51} represents the leaching effect of rain, ϕ_{52} the effect of the use by the plant, on minerals.

A suggested form is

$$\phi_{51} = -a_{51}MR,$$
$$\phi_{52} = -a_{52}\dot{G}.$$

(6) $\dot{W}$ = -R + runoff + evaporation + use by plant for growth.

Suggested forms:

$$\text{runoff} = \frac{a_{61}R}{a_{61} + W},$$

$$\text{evaporation} = E(a_{62} + a_{63}G^{2/3}),$$

$$\text{use by plant for growth} = a_{64}\dot{G}.$$

Putting these together, we obtain

$$\dot{W} = \frac{-WR}{a_{61} + W} + E(a_{62} + a_{63}G^{2/3}) + a_{64}\dot{G}.$$

(7) $\dot{f} = (1 - f - k)\phi_7(D)$

(8) $\dot{k} = (1 - f - k)\phi_7(D)\phi_8(E).$

Suggested forms:

$$\phi_7 = \begin{cases} a_{71}(D - a_{72})^{a_{73}}(D - a_{74})^{a_{75}}, & a_{72} \leq D \leq a_{74}, \\ 0, & D \leq a_{72} \text{ or } a_{74} \leq D. \end{cases}$$

$$a_{73},\ a_{75} > 1,$$

$$0 \leq a_{72} < a_{74} \leq 1.$$

$$\phi_8 = a_{81} E$$

These last two equations reflect the rate of fertilization and the rate of destruction of reproductive elements by drying.

A suggested method for introducing random elements into the above scheme is to consider that each equation differs from actuality by a term which is the "derivative" of an everywhere continuous differential process. E.g., equation (8) may become

$$\text{(9)} \qquad k = \int (1 - f - k)\, \phi_7\, (i)\, \phi_8\, (E)\, dt + \epsilon(t)$$

where $\epsilon(t_1) - \epsilon(t_1')$ is independent of $\epsilon(t_2) - \epsilon(t_2')$ whenever the intervals (t_1, t_1') and (t_2, t_2') are non-overlapping, and where $\epsilon(t)$ is everywhere continuous.